

GRE Mathematics Practice Test 01

High-Difficulty Practice

66 questions 170 minutes A–E

Instructions

Choose the single best answer to each question. No calculator is permitted. Begin timing when you start Question 1. Use the blank space on the question pages for scratch work.

All angles are measured in radians unless stated otherwise. The notation $\ln$ denotes the natural logarithm, and $\mathbb{N} = \{1, 2, 3, \dots\}$. Norms are Euclidean unless specified. I_n is the identity matrix, $M_n(\mathbb{R})$ the real $n \times n$ matrices, and $\mathbb{F}_p$ the field with p elements.

Difficulty

A challenging practice form with extended reasoning and demanding conceptual questions.

Answer key

The complete answer key follows Question 66. Correct choices are provided without worked solutions.

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100 practice tests, complete answer keys, and future updates.

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Questions 1–3

1. What is the value of

$$\lim_{x \rightarrow 0} \frac{\left(\frac{\sin x}{x}\right)^{1/x^2} - e^{-1/6}}{x^2} ?$$

- (A) $-\frac{e^{-1/6}}{90}$
(B) 0
(C) $-\frac{e^{-1/6}}{360}$
(D) $\frac{e^{-1/6}}{180}$
(E) $-\frac{e^{-1/6}}{180}$

2. The value of

$$\int_0^1 \frac{x(1-x)^2}{(\ln x)^2} dx$$

is

- (A) $2 \ln \frac{4}{3}$
(B) $\ln \frac{4}{3}$
(C) $\ln \frac{32}{27}$
(D) $\ln \frac{3}{2}$
(E) $10 \ln 2 - 6 \ln 3$

3. Let $T : \mathbb{R}^{12} \rightarrow \mathbb{R}^{12}$ be a linear transformation satisfying

$$T^4 = 0, \quad \text{rank } T = 8, \quad \text{rank } T^2 = 5.$$

What is $\text{rank } T^3$?

- (A) 0
(B) 2
(C) 1
(D) 3
(E) 4

Questions 4–6

4. Let (a_n) be a sequence of positive real numbers such that

$$\lim_{n \rightarrow \infty} n a_n = 1.$$

Which of the following statements must be true?

- I. $\sum_{n=1}^{\infty} a_n^2$ converges.
- II. $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.
- III. $\sum_{n=1}^{\infty} \left(a_n - \frac{1}{n}\right)$ converges.

- (A) II only
(B) I and II only
(C) I, II, and III
(D) I only
(E) III only

5. What is the value of

$$\lim_{n \rightarrow \infty} n^2 \left[\int_0^1 \left(1 + \frac{x}{n}\right)^n dx - (e - 1) + \frac{e - 2}{2n} \right] ?$$

- (A) $\frac{11e - 24}{24}$
(B) $\frac{7e - 16}{24}$
(C) $\frac{11e - 24}{12}$
(D) $\frac{5e - 12}{12}$
(E) $\frac{e - 2}{4}$

6. Define

$$F(x) = \int_0^x e^{t^2} dt,$$

and let G be the inverse of F in a neighborhood of the origin. What is $G^{(5)}(0)$?

- (A) 40
(B) 28
(C) 20
(D) 56
(E) 12

Questions 7–9

7. Let A be a real symmetric 6×6 matrix with characteristic polynomial

$$(t - 1)^3(t + 1)^2t.$$

Let V be the real vector space of all real symmetric 6×6 matrices, and define

$$T : V \rightarrow V, \quad T(X) = AX + XA.$$

What is $\text{rank } T$?

- (A) 18
- (B) 14
- (C) 15
- (D) 7
- (E) 12

8. For a real parameter a , define

$$f_n(x) = n^a x^2(1 - x)^n, \quad 0 \leq x \leq 1.$$

For exactly which values of a does f_n converge uniformly to zero on $[0, 1]$, while f'_n does not converge uniformly to zero on $[0, 1]$?

- (A) $a \leq 1$
- (B) $a < 1$
- (C) $1 < a \leq 2$
- (D) $1 \leq a < 2$
- (E) $a \geq 2$

9. Real numbers x, y, z satisfy

$$x + y + z = 1, \quad x^2 + y^2 + z^2 = 3.$$

What is the maximum possible value of xyz ?

- (A) 0
- (B) -1
- (C) $\frac{1}{9}$
- (D) $\frac{1}{3}$
- (E) $\frac{5}{27}$

Questions 10–12

10. Let

$$D = \left\{ (x, y) : x > 0, y > 0, 1 \leq xy \leq 4, 1 \leq \frac{y}{x} \leq 9 \right\}.$$

What is the value of

$$\iint_D (x^2 + y^2) dx dy?$$

- (A) $\frac{20}{3}$
- (B) $\frac{200}{3}$
- (C) $\frac{40}{3}$
- (D) $\frac{50}{3}$
- (E) $\frac{100}{3}$

11. Let A be a real 4×4 matrix such that

$$\det(tI_4 - A) = t^4 - 2t^3 - t^2 + 3t + 1.$$

What is the value of

$$\operatorname{tr}((A - I_4)^{-2})?$$

- (A) $-\frac{3}{4}$
- (B) $-\frac{5}{4}$
- (C) $\frac{5}{4}$
- (D) $\frac{1}{4}$
- (E) $\frac{3}{4}$

12. In the usual topology on $\mathbb{R}$, let

$$K = \{0\} \cup \left\{ \frac{1}{n} : n \in \mathbb{N} \right\} \cup \left\{ \frac{1}{n} + \frac{1}{m} : n, m \in \mathbb{N} \right\}.$$

For $S \subseteq \mathbb{R}$, let S' denote the set of all accumulation points of S . Which of the following is $(K)'$?

- (A) K
- (B) $\{0\}$
- (C) $\{0\} \cup \left\{ \frac{1}{n} : n \in \mathbb{N} \right\}$
- (D) $\emptyset$
- (E) $[0, 2]$

Questions 13–15

13. A smooth function $z = z(x, y)$ is defined implicitly near $(0, 0)$ by

$$z + xz^2 + ye^z = 1, \quad z(0, 0) = 1.$$

What is the value of

$$\frac{\partial^3 z}{\partial x^2 \partial y}(0, 0)?$$

- (A) $-21e$
- (B) $21e$
- (C) $-9e$
- (D) $9e$
- (E) $-15e$

14. Suppose y satisfies

$$x^2 y'' - 3xy' + 4y = x^2 \ln x, \quad x > 0,$$

with

$$y(1) = 0, \quad y'(1) = 1.$$

What is $y(e)$?

- (A) $\frac{e^2}{2}$
- (B) $\frac{7e^2}{6}$
- (C) e^2
- (D) $\frac{3e^2}{2}$
- (E) $\frac{e^2}{6}$

15. Regard $\mathbb{Z}_m$ as the additive group of integers modulo m . How many surjective group homomorphisms are there from

$$\mathbb{Z}_{12} \times \mathbb{Z}_{18}$$

to $\mathbb{Z}_6$?

- (A) 18
- (B) 24
- (C) 12
- (D) 30
- (E) 36

Questions 16–18

16. Let C be the circle $|z| = 2$, oriented counterclockwise. What is the value of

$$\frac{1}{2\pi i} \oint_C \frac{e^z}{(z-1)^2(z+1)} dz?$$

- (A) $\frac{e + e^{-1}}{4}$
- (B) $\frac{e + e^{-1}}{2}$
- (C) $\frac{e}{4}$
- (D) $\frac{e - e^{-1}}{2}$
- (E) $\frac{e - e^{-1}}{4}$

17. What is the volume of the solid

$$\Omega = \left\{ (x, y, z) : x^2 + y^2 + z^2 \leq 4, \quad z \geq \sqrt{x^2 + y^2}, \quad z \geq 1 \right\}?$$

- (A) $\frac{\pi}{3}(8 - 4\sqrt{2})$
- (B) $\frac{\pi}{3}(16 - 9\sqrt{2})$
- (C) $\frac{\pi}{3}(15 - 8\sqrt{2})$
- (D) $\frac{\pi}{3}(15 - 4\sqrt{2})$
- (E) $\frac{\pi}{3}(16 - 8\sqrt{2})$

18. Let S be the portion of the paraboloid

$$z = 4 - x^2 - y^2$$

lying above the plane $z = 1$, oriented with upward-pointing unit normal $\mathbf{n}$. If

$$\mathbf{F}(x, y, z) = (xz, yz, z^2),$$

what is the flux

$$\iint_S \mathbf{F} \cdot \mathbf{n} \, dS?$$

- (A) 42π
- (B) 33π
- (C) 48π
- (D) 39π
- (E) 36π

Questions 19–21

19. How many integers x with $0 \leq x < 360$ satisfy

$$x^3 \equiv x \pmod{360}?$$

- (A) 45
- (B) 27
- (C) 54
- (D) 81
- (E) 36

20. Let U and V be independent random variables, each uniformly distributed on $[0, 1]$. What is

$$\mathbb{E} \left[\frac{\min(U, V)}{\max(U, V)} \mid U + V > 1 \right] ?$$

- (A) $2 - 2 \ln 2$
- (B) $1 - \ln 2$
- (C) $\ln 2$
- (D) $2 \ln 2 - 1$
- (E) $\frac{3}{2} - \ln 2$

21. The value of

$$\int_0^1 \int_{\sqrt{x}}^{x^{1/3}} xy e^{y^2} dy dx$$

is

- (A) $\frac{3e - 8}{4}$
- (B) $\frac{3e - 8}{2}$
- (C) $\frac{e - 2}{4}$
- (D) $\frac{8 - 3e}{4}$
- (E) $\frac{3e - 7}{4}$

Questions 22–24

22. Let C be the closed curve formed by the intersection of the cylinder

$$x^2 + y^2 = 1$$

and the plane

$$z = 1 + x + 2y.$$

Orient C so that its projection onto the xy -plane is traversed counterclockwise. What is the value of

$$\oint_C (-yz^2 dx + xz^2 dy + xy dz) ?$$

- (A) 3π
- (B) 5π
- (C) 9π
- (D) 6π
- (E) 7π

23. For a real parameter a , define the quadratic form

$$q_a(x, y, z) = x^2 + 2y^2 + 3z^2 + 2axy.$$

For exactly which values of a is q_a strictly positive at every nonzero point of the plane $x + y + z = 0$?

- (A) $|a| < 2\sqrt{5}$
- (B) $|a| < \sqrt{2}$
- (C) $|a + 3| < \sqrt{5}$
- (D) $a > -3$
- (E) $|a + 3| < 2\sqrt{5}$

24. Let

$$f : [0, \infty) \rightarrow [0, \infty)$$

be continuous, with

$$\int_0^\infty f(x) dx < \infty.$$

Which of the following statements must be true?

I. $\liminf_{x \rightarrow \infty} f(x) = 0.$

II. $\int_0^\infty f(x)^2 dx < \infty.$

III. If f is uniformly continuous on $[0, \infty)$, then $\lim_{x \rightarrow \infty} f(x) = 0.$

- (A) III only
- (B) I, II, and III
- (C) I and III only
- (D) I only
- (E) II only

Questions 25–27

25. Consider the initial value problem

$$y' = xy^2 + \frac{y}{x}, \quad x > 0, \quad y(1) = 1.$$

What is the right endpoint of the maximal interval of existence of its solution containing $x = 1$?

- (A) $\sqrt{3}$
- (B) $\sqrt[3]{2}$
- (C) $\sqrt[3]{3}$
- (D) 2
- (E) $\sqrt[3]{4}$

26. A plane curve is parametrized by

$$x = t - \frac{t^3}{3}, \quad y = t^2, \quad t > 0.$$

Let P and Q be the points at which the tangent lines are parallel to $y = x$ and $y = -x$, respectively. What is the arc length along the curve from P to Q ?

- (A) 8
- (B) $8\sqrt{2}$
- (C) $\frac{20}{3}$
- (D) $\frac{16}{3}$
- (E) $4\sqrt{2}$

27. How many elements of the symmetric group S_7 have order exactly 6?

- (A) 1470
- (B) 1050
- (C) 1680
- (D) 1260
- (E) 840

Questions 28–30

28. How many functions

$$f : \{1, 2, \dots, 8\} \rightarrow \{1, 2, 3\}$$

are surjective and satisfy

$$f(i) \neq f(i+1), \quad i = 1, 2, \dots, 8,$$

where $f(9)$ is defined to be $f(1)$?

- (A) 240
- (B) 384
- (C) 252
- (D) 126
- (E) 258

29. What is the sum of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}n}{(2n-1)(2n+1)(2n+3)}?$$

- (A) $\frac{4-\pi}{8}$
- (B) $\frac{\pi-2}{16}$
- (C) $\frac{\pi-3}{4}$
- (D) $\frac{\pi-2}{8}$
- (E) $\frac{4-\pi}{16}$

30. The polar curves

$$r = 1 + \cos \theta, \quad r = 1 - \cos \theta$$

each enclose a region. What is the area common to the two regions?

- (A) $\frac{3\pi}{2} - 2$
- (B) $2\pi - 4$
- (C) $\frac{3\pi}{4} - 2$
- (D) $\pi - 2$
- (E) $\frac{3\pi}{2} - 4$

Questions 31–33

31. Let $\mathbb{F}_3$ be the field with three elements. How many units are there in the quotient ring

$$R = \mathbb{F}_3[x] / ((x^2 + 1)^2(x - 1))?$$

- (A) 144
- (B) 216
- (C) 108
- (D) 96
- (E) 162

32. How many zeros of the polynomial

$$p(z) = z^8 + 6z^5 + z^2 + 1$$

lie in the annulus

$$1 < |z| < 2?$$

Count zeros with multiplicity.

- (A) 4
- (B) 1
- (C) 3
- (D) 2
- (E) 5

33. For a real parameter a , define

$$f_a(x, y) = x^4 + 2ax^2y + y^2 + y^4.$$

Which of the following is the set of all values of a for which the origin is a strict local minimum of f_a ?

- (A) $\{0\}$
- (B) $(-\infty, -1) \cup (1, \infty)$
- (C) $[-1, 1]$
- (D) $\mathbb{R}$
- (E) $(-1, 1)$

Questions 34–36

34. Let

$$f(x, y, z) = xy + yz + zx.$$

Consider all unit vectors $\mathbf{u}$ parallel to the tangent plane to the ellipsoid

$$x^2 + 2y^2 + 3z^2 = 6$$

at $P = (1, 1, 1)$. What is the maximum possible value of the directional derivative $D_{\mathbf{u}}f(P)$?

- (A) $\sqrt{6}$
 (B) $2\sqrt{3}$
 (C) $\sqrt{\frac{18}{7}}$
 (D) $\sqrt{\frac{12}{7}}$
 (E) $\sqrt{\frac{6}{7}}$

35. Let

$$F = \mathbb{F}_2[x]/(x^3 + x + 1),$$

and let α denote the residue class of x in F . Which of the following is the minimal polynomial over $\mathbb{F}_2$ of

$$\beta = \alpha + \alpha^{-1}?$$

- (A) $t^2 + t + 1$
 (B) $t^3 + t^2 + 1$
 (C) $t^3 + 1$
 (D) $t^3 + t^2 + t + 1$
 (E) $t^3 + t + 1$

36. Newton's method is applied to

$$f(x) = (x - 1)^3 e^x,$$

giving

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad x_0 > 1.$$

What is the value of

$$\lim_{n \rightarrow \infty} \frac{x_{n+1} - 1 - \frac{2}{3}(x_n - 1)}{(x_n - 1)^2}?$$

- (A) $\frac{1}{27}$
 (B) $\frac{1}{9}$
 (C) $\frac{1}{3}$
 (D) $\frac{2}{3}$
 (E) $\frac{1}{6}$

Questions 37–39

37. Let

$$D = \{(x, y) : x^2 + 4y^2 < 1\}.$$

What is the value of the improper integral

$$\iint_D \frac{x^2 y^2}{\sqrt{1 - x^2 - 4y^2}} dx dy?$$

- (A) $\frac{\pi}{120}$
- (B) $\frac{\pi}{60}$
- (C) $\frac{\pi}{90}$
- (D) $\frac{\pi}{30}$
- (E) $\frac{\pi}{15}$

38. Let the ellipse

$$C : \frac{x^2}{4} + y^2 = 1$$

be oriented counterclockwise. What is the value of

$$\oint_C \left(-\frac{y}{x^2 + y^2} - y^3 \right) dx + \left(\frac{x}{x^2 + y^2} + x^3 \right) dy?$$

- (A) $\frac{15\pi}{2}$
- (B) $\frac{21\pi}{2}$
- (C) $\frac{17\pi}{2}$
- (D) $\frac{19\pi}{2}$
- (E) $\frac{23\pi}{2}$

39. How many group automorphisms does

$$\mathbb{Z}_8 \times \mathbb{Z}_2$$

have?

- (A) 16
- (B) 12
- (C) 32
- (D) 8
- (E) 24

Questions 40–42

40. Let $n \geq 3$, and let $X_1, \dots, X_n$ be independent, identically distributed random variables with finite fourth moment and

$$\mathbb{E}[X_1] = 0, \quad \mathbb{E}[X_1^2] = 1, \quad \mathbb{E}[X_1^3] = 2.$$

Define

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i, \quad S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2.$$

What is $\text{Cov}(\bar{X}, S^2)$?

- (A) $\frac{1}{n}$
- (B) $\frac{2}{n^2}$
- (C) $\frac{2}{n}$
- (D) 0
- (E) $\frac{2}{n-1}$

41. A continuous function $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies

$$f(x) = e^x + \int_0^x (x-t)f(t) dt \quad \text{for every } x \in \mathbb{R}.$$

What is $f(\ln 2)$?

- (A) $\ln 2 + \frac{13}{8}$
- (B) $\ln 2 + \frac{9}{8}$
- (C) $\ln 2 + \frac{7}{4}$
- (D) $\frac{\ln 2}{2} + \frac{13}{8}$
- (E) $2 \ln 2 + \frac{1}{4}$

42. Among all continuously differentiable real-valued functions f on $[0, 1]$ satisfying

$$f(0) = f(1) = 0, \quad \int_0^1 [f'(x)]^2 dx = 1,$$

what is the maximum possible value of

$$\int_0^1 xf(x) dx?$$

- (A) $\frac{1}{6}$
- (B) $\frac{\sqrt{5}}{15}$
- (C) $\frac{\sqrt{3}}{6}$
- (D) $\frac{1}{15}$
- (E) $\frac{\sqrt{5}}{30}$

Questions 43–45

43. Let A be a real 6×6 matrix satisfying

$$A^2 + A + I_6 = 0.$$

Define

$$W = \{X \in M_6(\mathbb{R}) : AX = XA\}.$$

What is the dimension of W as a real vector space?

- (A) 27
 - (B) 12
 - (C) 18
 - (D) 9
 - (E) 15
44. A coin is tossed independently and repeatedly. On each toss, the probability of heads H is $2/3$, and the probability of tails T is $1/3$. Let N be the total number of tosses completed when the consecutive pattern HTH first appears. What is $\mathbb{E}[N]$?
- (A) 6
 - (B) 9
 - (C) $\frac{27}{4}$
 - (D) $\frac{33}{4}$
 - (E) $\frac{15}{2}$
45. For real parameters p, q , consider the improper integral

$$\int_0^\infty \frac{\sin x}{x^p(1+x)^q} dx.$$

Which of the following is a necessary and sufficient condition for the integral to converge but not converge absolutely? The usual improper limits at 0 and $+\infty$ are taken separately.

- (A) $p < 1, \quad 0 < p + q \leq 1$
- (B) $p \leq 2, \quad 0 < p + q \leq 1$
- (C) $p < 2, \quad p + q > 1$
- (D) $p < 2, \quad 0 < p + q \leq 1$
- (E) $p < 2, \quad 0 < p + q < 1$

Questions 46–48

46. For which real value of λ does the boundary value problem

$$y'' + 4y = x \cos(2x) + \lambda \sin(2x), \quad y(0) = y(\pi) = 0$$

have at least one twice continuously differentiable solution?

- (A) $\frac{1}{4}$
- (B) $-\frac{1}{4}$
- (C) $-\frac{1}{2}$
- (D) $\frac{1}{2}$
- (E) 0

47. Among all monic cubic polynomials in $\mathbb{F}_5[x]$, how many have exactly one distinct root in $\mathbb{F}_5$?
Roots are counted without multiplicity.

- (A) 50
- (B) 65
- (C) 60
- (D) 55
- (E) 45

48. A point P is chosen uniformly with respect to area from the unit disk

$$x^2 + y^2 \leq 1.$$

Let

$$A = \left(-\frac{1}{2}, 0\right), \quad B = \left(\frac{1}{2}, 0\right).$$

What is the probability that triangle APB is acute?

- (A) $\frac{1}{4} + \frac{\sqrt{3}}{2\pi}$
- (B) $\frac{1}{3} + \frac{\sqrt{3}}{4\pi}$
- (C) $\frac{1}{12} + \frac{\sqrt{3}}{2\pi}$
- (D) $\frac{1}{3}$
- (E) $\frac{1}{6} + \frac{\sqrt{3}}{4\pi}$

Questions 49–51

49. For $a, b > 0$, define

$$f_{a,b}(x, y) = \begin{cases} \frac{|x|^a |y|^b}{x^2 + y^4}, & (x, y) \neq (0, 0), \\ 0, & (x, y) = (0, 0). \end{cases}$$

Which condition is necessary and sufficient for $f_{a,b}$ to be differentiable at the origin?

- (A) $2a + b > 5$
- (B) $a + b > 3$
- (C) $a + b > 4$
- (D) $a + b > 3$ and $2a + b > 5$
- (E) $a > 2$ and $b > 1$

50. The value of

$$\int_0^{2\pi} \frac{1 + \cos \theta}{(5 + 4 \cos \theta)^2} d\theta$$

is

- (A) $\frac{2\pi}{27}$
- (B) $\frac{\pi}{27}$
- (C) $\frac{\pi}{9}$
- (D) $\frac{4\pi}{27}$
- (E) $\frac{\pi}{54}$

51. Let $u, v \in \mathbb{R}^5$ satisfy

$$u^T u = 2, \quad v^T v = 3, \quad u^T v = 1.$$

Define

$$A = I_5 + uv^T + vu^T.$$

What is $\text{tr}(A^{-1})$?

- (A) 3
- (B) 2
- (C) 1
- (D) -1
- (E) 0

Questions 52–54

52. Among all connected, undirected, simple graphs with 12 vertices and 18 edges, what is the maximum possible number of bridges? A bridge is an edge whose removal disconnects the graph.

(A) 7
(B) 6
(C) 8
(D) 4
(E) 5

53. For a real parameter a , the equation

$$e^x = ax^2$$

has exactly three distinct real solutions if and only if

(A) $a = \frac{e^2}{4}$
(B) $a \geq \frac{e^2}{4}$
(C) $0 < a < \frac{e^2}{4}$
(D) $a > 0$
(E) $a > \frac{e^2}{4}$

54. The sequence (a_n) is defined by

$$a_1 = 1, \quad a_{n+1} = \ln(1 + a_n).$$

What is the value of

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{a_n} - \frac{n}{2}}{\ln n}?$$

(A) $\frac{1}{3}$
(B) 0
(C) $-\frac{1}{3}$
(D) $-\frac{1}{6}$
(E) $\frac{1}{6}$

Questions 55–57

55. How many integers a satisfying

$$1 \leq a \leq 90, \quad \gcd(a, 91) = 1$$

have the property that the smallest positive integer k for which

$$a^k \equiv 1 \pmod{91}$$

is $k = 12$?

- (A) 24
- (B) 48
- (C) 36
- (D) 32
- (E) 12

56. Define a topology on $\mathbb{R}$ by

$$\mathcal{T} = \{\emptyset\} \cup \{U \subseteq \mathbb{R} : \mathbb{R} \setminus U \text{ is at most countable}\}.$$

Which of the following statements about $(\mathbb{R}, \mathcal{T})$ are true?

- I. Every compact subset is finite.
- II. Every convergent sequence is eventually constant and equal to its limit.
- III. The space is connected.

- (A) II and III only
- (B) II only
- (C) I, II, and III
- (D) I and II only
- (E) I only

57. Let y be the solution of the initial value problem

$$y'' + 2y' + y = \frac{e^{-x}}{1+x^2}, \quad y(0) = y'(0) = 0.$$

What is the value of

$$\lim_{x \rightarrow \infty} \left(e^x y(x) - \frac{\pi x}{2} + \ln x \right)?$$

- (A) $\frac{\pi}{2}$
- (B) -1
- (C) -2
- (D) 0
- (E) 1

Questions 58–60

58. Define

$$F(a) = \int_0^1 \frac{\arctan(ax)}{x} dx.$$

What is $F'''(1)$?

- (A) $\frac{\pi - 3}{4}$
- (B) $\frac{4 - \pi}{2}$
- (C) $\frac{4 - \pi}{4}$
- (D) $\frac{\pi - 4}{2}$
- (E) $\frac{\pi - 3}{2}$

59. Let A be a real orthogonal 4×4 matrix satisfying

$$\det A = -1, \quad \operatorname{tr} A = 1.$$

What is $\operatorname{tr}(A^3)$?

- (A) 2
- (B) 0
- (C) -2
- (D) 1
- (E) -1

60. Let U_1, U_2, U_3, U_4 be independent random variables, each uniformly distributed on $[0, 1]$, and let

$$U_{(1)} \leq U_{(2)} \leq U_{(3)} \leq U_{(4)}$$

be their order statistics. What is

$$\mathbb{P}\left(U_{(3)} - U_{(2)} > \frac{1}{2} \mid U_{(4)} - U_{(1)} < \frac{3}{4}\right)?$$

- (A) $\frac{1}{63}$
- (B) $\frac{1}{21}$
- (C) $\frac{1}{27}$
- (D) $\frac{5}{81}$
- (E) $\frac{5}{189}$

Questions 61–63

61. What is the curvature of the plane curve

$$x^3 + y^3 = 3xy$$

at the point $(\frac{3}{2}, \frac{3}{2})$?

- (A) $\frac{16}{3}$
- (B) $\frac{16\sqrt{2}}{3}$
- (C) $\frac{32}{3}$
- (D) $\frac{8\sqrt{2}}{3}$
- (E) $\frac{4\sqrt{2}}{3}$

62. The region

$$R = \{(x, y) : 0 \leq x \leq 1, x^2 \leq y \leq x\}$$

is revolved through one complete revolution about the line

$$x + y + 1 = 0$$

in its plane. What is the volume of the resulting solid?

- (A) $\frac{19\pi}{60}$
- (B) $\frac{19\sqrt{2}\pi}{120}$
- (C) $\frac{19\pi}{30}$
- (D) $\frac{19\sqrt{2}\pi}{60}$
- (E) $\frac{19\sqrt{2}\pi}{30}$

63. Let A be a real 10×10 matrix satisfying

$$A^4 = I_{10}, \quad \text{tr } A = 0, \quad \text{tr}(A^2) = 2.$$

What is the value of

$$\det(I_{10} + A + A^2)?$$

- (A) 81
- (B) 243
- (C) 9
- (D) 729
- (E) 27

Questions 64–66

64. How many four-element subsets of $\{1, 2, \dots, 11\}$ contain no two consecutive integers and have an even sum of their elements?

(A) 32
(B) 35
(C) 38
(D) 34
(E) 36

65. The value of the improper integral

$$\int_0^{\infty} \frac{\ln x}{x^2 + 2x + 4} dx$$

is

(A) $\frac{\pi \ln 2}{3\sqrt{3}}$
(B) $\frac{2\pi \ln 2}{3\sqrt{3}}$
(C) $\frac{\pi \ln 2}{3}$
(D) $\frac{\pi \ln 2}{6\sqrt{3}}$
(E) $\frac{\pi \ln 2}{\sqrt{3}}$

66. Let $f : [-1, 1] \rightarrow \mathbb{R}$ be continuous, and suppose that

$$\int_{-1}^1 x^{2n} f(x) dx = 0$$

for every integer $n \geq 0$. Which of the following statements must be true?

I. $f(-x) = -f(x)$ for every $x \in [-1, 1]$.

II. $\int_{-1}^1 f(x)e^x dx = 0$.

III. $f(0) = 0$.

(A) III only
(B) I and III only
(C) I only
(D) II only
(E) I, II, and III

Answer Key

Practice Test 01 — correct choices only.

Question	Answer	Question	Answer	Question	Answer
1	E	23	E	45	D
2	E	24	C	46	A
3	B	25	E	47	D
4	D	26	C	48	C
5	A	27	A	49	D
6	B	28	C	50	A
7	B	29	E	51	C
8	D	30	E	52	B
9	E	31	A	53	E
10	E	32	C	54	D
11	C	33	C	55	D
12	B	34	D	56	C
13	A	35	B	57	B
14	B	36	B	58	E
15	B	37	B	59	C
16	A	38	D	60	E
17	C	39	A	61	D
18	D	40	C	62	D
19	A	41	A	63	E
20	A	42	B	64	C
21	A	43	C	65	A
22	E	44	D	66	B

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