

GRE Mathematics Practice Test 02

Advanced Practice

66 questions 170 minutes A–E

Instructions

Choose the single best answer to each question. No calculator is permitted. Begin timing when you start Question 1. Use the blank space on the question pages for scratch work.

All angles are measured in radians unless stated otherwise. The notation $\ln$ denotes the natural logarithm, and $\mathbb{N} = \{1, 2, 3, \dots\}$. Norms are Euclidean unless specified. I_n is the identity matrix, $M_n(\mathbb{R})$ the real $n \times n$ matrices, and $\mathbb{F}_p$ the field with p elements.

Difficulty

An advanced form with shorter multistep calculations and conceptual questions.

Answer key

The complete answer key follows Question 66. Correct choices are provided without worked solutions.

Coverage

33 calculus questions, 17 algebra questions, and 16 questions on additional topics.

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100 practice tests, complete answer keys, and future updates.

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Questions 1–3

1. What is the value of

$$\lim_{x \rightarrow 0} \frac{\ln(1 + \sin x) - x + \frac{1}{2}x^2}{x^3} ?$$

- (A) $\frac{1}{6}$
(B) $-\frac{1}{3}$
(C) $-\frac{1}{6}$
(D) $\frac{1}{3}$
(E) $\frac{1}{2}$

2. The value of

$$\int_0^1 \frac{x^3}{x^2 + (1-x)^2} dx$$

is

- (A) $\frac{4 - \pi}{8}$
(B) $\frac{\pi - 2}{4}$
(C) $\frac{\pi}{8}$
(D) $\frac{6 - \pi}{4}$
(E) $\frac{6 - \pi}{8}$

3. Let V be the real vector space of polynomials of degree at most 4. Define $T : V \rightarrow V$ by

$$(Tp)(x) = p'(x) + p(0)x^2.$$

If I_V is the identity transformation on V , what is $\det(I_V + T)$?

- (A) 3
(B) 1
(C) 2
(D) 6
(E) -1

Questions 4–6

4. Let U and V be independent random variables, each uniformly distributed on $[0, 1]$. What is

$$\mathbb{P}\left(U + V > 1 \mid \max(U, V) > \frac{3}{4}\right)?$$

- (A) $\frac{3}{7}$
- (B) $\frac{3}{4}$
- (C) $\frac{6}{7}$
- (D) $\frac{7}{8}$
- (E) $\frac{4}{7}$

5. Let

$$D = \{(x, y) : x \geq y \geq 0, 1 \leq x + y \leq 2\}.$$

What is the value of $\iint_D (x^2 - y^2) dx dy$?

- (A) $\frac{15}{16}$
- (B) $\frac{15}{8}$
- (C) $\frac{5}{4}$
- (D) $\frac{7}{12}$
- (E) $\frac{7}{16}$

6. Let $F(x) = x + \ln x$ for $x > 0$, and let G be its inverse. What is $G^{(3)}(1)$?

- (A) $\frac{3}{32}$
- (B) $\frac{1}{32}$
- (C) $-\frac{1}{32}$
- (D) $-\frac{1}{16}$
- (E) 0

Questions 7–9

7. A real 3×3 matrix A satisfies

$$\operatorname{tr} A = 2, \quad \operatorname{tr}(A^2) = 6, \quad \det A = -2.$$

What is $\det(A^2 + I_3)$?

- (A) 12
- (B) 20
- (C) 36
- (D) 24
- (E) 16

8. For a real parameter p , consider the series

$$\sum_{n=2}^{\infty} \frac{\sqrt{n+1} - \sqrt{n}}{n^p (\ln n)^2}.$$

For exactly which values of p does the series converge?

- (A) $p > \frac{1}{2}$
- (B) $p \geq 1$
- (C) $p \geq \frac{1}{2}$
- (D) $p > 1$
- (E) $p > 0$

9. Real numbers x and y satisfy

$$x^2 + 4y^2 = 4, \quad y \geq 0.$$

What is the maximum possible value of x^2y ?

- (A) $\frac{4}{3}$
- (B) 2
- (C) $\frac{8\sqrt{3}}{3}$
- (D) $\frac{4\sqrt{3}}{9}$
- (E) $\frac{8\sqrt{3}}{9}$

Questions 10–12

10. Suppose y satisfies the initial value problem

$$y'' - 4y' + 4y = e^{2x}, \quad y(0) = 0, \quad y'(0) = 1.$$

What is $y(1)$?

- (A) $\frac{e^2}{2}$
- (B) $\frac{3e^2}{2}$
- (C) $\frac{5e^2}{2}$
- (D) $2e^2$
- (E) e^2

11. Regard $\mathbb{Z}_m$ as the additive group of integers modulo m . How many cyclic subgroups of order 6 does

$$\mathbb{Z}_6 \times \mathbb{Z}_6$$

have?

- (A) 10
- (B) 24
- (C) 6
- (D) 12
- (E) 8

12. A three-element subset of $\{1, 2, 3, 4, 5, 6\}$ is chosen uniformly at random. Let S be the sum of its elements. What is $\text{Var}(S)$?

- (A) $\frac{35}{12}$
- (B) $\frac{7}{4}$
- (C) 7
- (D) $\frac{21}{4}$
- (E) $\frac{35}{4}$

Questions 13–15

13. A smooth function $z = z(x, y)$ is defined near $(1, 1)$ by

$$x^2 + y^2 + z^2 + xyz = 4, \quad z(1, 1) = 1.$$

What is $\frac{\partial^2 z}{\partial x \partial y}(1, 1)$?

- (A) -1
- (B) 0
- (C) $-\frac{1}{3}$
- (D) $-\frac{2}{3}$
- (E) $\frac{1}{3}$

14. For which real values of x does the power series

$$\sum_{n=2}^{\infty} \frac{(x-2)^n}{3^n n \ln n}$$

converge?

- (A) $[-1, 5)$
- (B) $(-1, 5)$
- (C) $(-3, 3)$
- (D) $[-1, 5]$
- (E) $(-1, 5]$

15. How many integers x with $0 \leq x < 120$ satisfy

$$x^2 \equiv 1 \pmod{120}?$$

- (A) 16
- (B) 4
- (C) 24
- (D) 12
- (E) 8

Questions 16–18

16. An entire function f has real part

$$\operatorname{Re} f(x + iy) = x^3 - 3xy^2 + 2xy$$

for all real x, y , and $f(0) = i$. What is $f(1 + i)$?

- (A) $-3i$
- (B) $-2 + 3i$
- (C) $2 + 3i$
- (D) $3i$
- (E) $2 + i$

17. The value of

$$\int_0^1 \int_{x^{1/3}}^1 e^{y^2} dy dx$$

is

- (A) $\frac{1}{2}$
- (B) $\frac{e-1}{3}$
- (C) $\frac{e-2}{2}$
- (D) $\frac{e-1}{2}$
- (E) $\frac{1}{3}$

18. Let S be the upper hemisphere

$$x^2 + y^2 + z^2 = 1, \quad z \geq 0,$$

with outward-pointing unit normal $\mathbf{n}$. For $\mathbf{F}(x, y, z) = (x, y, z^2)$, what is $\iint_S \mathbf{F} \cdot \mathbf{n} dS$?

- (A) $\frac{3\pi}{2}$
- (B) 2π
- (C) $\frac{5\pi}{3}$
- (D) $\frac{4\pi}{3}$
- (E) $\frac{11\pi}{6}$

Questions 19–21

19. A real 5×5 matrix A satisfies

$$A^2 = 3A + 4I_5, \quad \text{tr } A = 5.$$

What is $\det(2A + I_5)$?

- (A) 81
- (B) -81
- (C) -9
- (D) -729
- (E) 729

20. For $x \geq 0$, define

$$f_n(x) = \frac{n}{n^2 + x^2}.$$

Which of the following statements are true?

- I. f_n converges uniformly to zero on $[0, \infty)$.
- II. $\lim_{n \rightarrow \infty} \int_0^{\infty} f_n(x) dx = 0$.
- III. $\lim_{n \rightarrow \infty} \int_0^{\infty} f_n(x)^2 dx = 0$.

- (A) II only
- (B) I, II, and III
- (C) I only
- (D) I and III only
- (E) I and II only

21. What is the arc length of the graph

$$y = \frac{x^2}{4} - \frac{1}{2} \ln x, \quad 1 \leq x \leq 2?$$

- (A) $\frac{1}{2} + \frac{1}{2} \ln 2$
- (B) $\frac{3}{4} + \ln 2$
- (C) $\frac{3}{4} - \frac{1}{2} \ln 2$
- (D) $\frac{3}{4} + \frac{1}{2} \ln 2$
- (E) $\frac{3}{2} + \frac{1}{2} \ln 2$

Questions 22–24

22. Let C be the counterclockwise boundary of the region enclosed by $y = x^2$ and $y = 2x$. What is

$$\oint_C (y^2 dx + x^2 dy) ?$$

- (A) $\frac{16}{5}$
- (B) $\frac{8}{5}$
- (C) $-\frac{8}{5}$
- (D) $-\frac{16}{5}$
- (E) 0

23. Let $\mathbb{F}_5$ be the field with five elements, and let

$$R = \mathbb{F}_5[x]/(x^3 - x).$$

How many elements $r \in R$ satisfy $r^2 = r$?

- (A) 2
- (B) 4
- (C) 5
- (D) 25
- (E) 8

24. Let X and Y be independent standard normal random variables. Define

$$U = 2X - Y, \quad V = X + 2Y.$$

What is $\mathbb{E}[U^2V^2]$?

- (A) 75
- (B) 9
- (C) 5
- (D) 25
- (E) 20

Questions 25–27

- 25.** Suppose y satisfies

$$x^2y'' + xy' - y = x^2, \quad x > 0,$$

with $y(1) = 0$ and $y'(1) = 1$. What is $y(2)$?

- (A) $\frac{2}{3}$
- (B) $\frac{4}{3}$
- (C) $\frac{7}{6}$
- (D) 1
- (E) $\frac{3}{2}$

- 26.** How many local minimum points does the function

$$f(x, y) = x^4 + y^4 - 2(x - y)^2$$

have on $\mathbb{R}^2$?

- (A) 2
- (B) 1
- (C) 4
- (D) 3
- (E) 0

- 27.** How many elements of the symmetric group S_6 have order exactly 4?

- (A) 180
- (B) 360
- (C) 90
- (D) 120
- (E) 240

Questions 28–30

28. How many surjective functions

$$f : \{1, 2, 3, 4, 5\} \longrightarrow \{1, 2, 3\}$$

satisfy both $f(1) \neq f(2)$ and $f(3) \neq f(4)$?

- (A) 60
- (B) 96
- (C) 108
- (D) 72
- (E) 84

29. The value of the improper integral

$$\int_0^1 \frac{\ln x}{(1+x)^3} dx$$

is

- (A) $-\frac{1}{4} - \ln 2$
- (B) $-\frac{1}{4} - \frac{1}{2} \ln 2$
- (C) $-\frac{1}{2} - \frac{1}{2} \ln 2$
- (D) $-\ln 2$
- (E) $\frac{1}{4} - \frac{1}{2} \ln 2$

30. What is the area inside the right-hand loop of the lemniscate

$$r^2 = 2 \cos(2\theta)$$

but outside the circle $r = 1$?

- (A) $1 - \frac{\pi}{4}$
- (B) $\frac{\sqrt{3}}{2} - \frac{\pi}{6}$
- (C) $\frac{\sqrt{3}}{2} - \frac{\pi}{3}$
- (D) $\frac{\pi}{3} - \frac{\sqrt{3}}{2}$
- (E) $\sqrt{3} - \frac{\pi}{3}$

Questions 31–33

31. Let $u, v \in \mathbb{R}^5$ be linearly independent. What is the dimension of the real vector space

$$W = \{A \in M_5(\mathbb{R}) : A^T = -A, Au = 0, Av = 0\}?$$

- (A) 1
- (B) 5
- (C) 3
- (D) 6
- (E) 2

32. For a positive integer n , let

$$T_n = \frac{1}{n} \left[\frac{0^4 + 1^4}{2} + \sum_{k=1}^{n-1} \left(\frac{k}{n} \right)^4 \right]$$

be the composite trapezoidal approximation to $\int_0^1 x^4 dx$. What is $\lim_{n \rightarrow \infty} n^2 \left(T_n - \frac{1}{5} \right)$?

- (A) $\frac{1}{2}$
- (B) $-\frac{1}{3}$
- (C) $\frac{1}{6}$
- (D) $\frac{1}{12}$
- (E) $\frac{1}{3}$

33. Let $f(x, y, z) = x^2y + yz$, and let $P = (1, 1, 1)$. Among all unit vectors $\mathbf{u}$ parallel to the plane $x + y + z = 3$, what is the largest value of $D_{\mathbf{u}}f(P)$?

- (A) 3
- (B) $\frac{\sqrt{3}}{3}$
- (C) $\sqrt{2}$
- (D) $\frac{\sqrt{6}}{3}$
- (E) $\frac{\sqrt{6}}{2}$

Questions 34–36

34. Let

$$\Omega = \{(x, y, z) : x, y, z \geq 0, x + y + z \leq 2\}.$$

What is $\iiint_{\Omega} (x + y + z)^2 dV$?

- (A) $\frac{4}{5}$
- (B) $\frac{32}{5}$
- (C) $\frac{8}{3}$
- (D) $\frac{16}{5}$
- (E) $\frac{8}{5}$

35. In the field

$$F = \mathbb{F}_3[x]/(x^2 + 1),$$

let α be the residue class of x . What is the multiplicative order of $1 + \alpha$ in $F^\times$?

- (A) 8
- (B) 6
- (C) 3
- (D) 4
- (E) 2

36. Let C be the circle $|z| = 2$, oriented counterclockwise. What is

$$\frac{1}{2\pi i} \oint_C \frac{ze^z}{z^2 + 1} dz?$$

- (A) $\sin 1$
- (B) $\cos 1$
- (C) $\cosh 1$
- (D) $2 \cos 1$
- (E) $\sinh 1$

Questions 37–39

37. What is the value of

$$\lim_{n \rightarrow \infty} n \left(\sum_{k=1}^n \frac{1}{n+k} - \ln 2 \right) ?$$

- (A) $-\frac{1}{4}$
(B) $\frac{1}{2}$
(C) 0
(D) $\frac{1}{4}$
(E) $-\frac{1}{2}$

38. Let y satisfy

$$xy' + y = \sin(\ln x), \quad x > 0, \quad y(1) = 0.$$

What is $y(e^{\pi/2})$?

- (A) $\frac{1 - e^{-\pi/2}}{2}$
(B) $\frac{1 + e^{-\pi/2}}{2}$
(C) $1 - e^{-\pi/2}$
(D) $e^{-\pi/2}$
(E) $1 + e^{-\pi/2}$

39. A finite group G has order 45. If H and K are subgroups of G with orders 9 and 15, respectively, what is $|H \cap K|$?

- (A) 3
(B) 9
(C) 1
(D) 5
(E) 15

Questions 40–42

40. A coin is tossed independently and repeatedly, with probability $2/3$ of heads on each toss. Let N be the number of tosses completed when two consecutive tosses first have the same outcome, either HH or TT . What is $\mathbb{E}[N]$?

- (A) $\frac{18}{7}$
(B) $\frac{9}{2}$
(C) 3
(D) $\frac{10}{3}$
(E) $\frac{20}{7}$

41. As a ranges over all real numbers, what is the minimum value of

$$\int_0^1 x |x - a| dx ?$$

- (A) $\frac{3 - 2\sqrt{2}}{6}$
(B) $\frac{2 - \sqrt{2}}{6}$
(C) $\frac{2 - \sqrt{2}}{3}$
(D) $\frac{\sqrt{2} - 1}{6}$
(E) $\frac{1}{12}$

42. For $n \geq 1$, define $H_n = \sum_{k=1}^n 1/k$. What is the sum of the series

$$\sum_{n=1}^{\infty} \frac{H_n}{2^n} ?$$

- (A) $1 + \ln 2$
(B) $2 - \ln 2$
(C) $2 \ln 2$
(D) $\ln 2$
(E) 2

Questions 43–45

43. A real polynomial p of degree at most 3 satisfies

$$p(1) = 1, \quad p(2) = \frac{1}{2}, \quad p(3) = \frac{1}{3}, \quad p(4) = \frac{1}{4}.$$

What is $p(0)$?

- (A) $\frac{13}{6}$
(B) $\frac{11}{6}$
(C) 2
(D) $\frac{25}{12}$
(E) $\frac{3}{2}$
44. Let f_n be continuously differentiable real-valued functions on $[0, 1]$, and suppose that f_n converges uniformly to zero. Which statements must be true?
- I. $f'_n(x) \rightarrow 0$ for every $x \in [0, 1]$.
II. $\int_0^1 f_n(x) dx \rightarrow 0$.
III. There exist $c_n \in (0, 1)$ such that $f'_n(c_n) \rightarrow 0$.
- (A) I only
(B) II and III only
(C) I, II, and III
(D) II only
(E) I and II only

45. Define

$$f(x, y) = \begin{cases} \frac{x^2 y}{x^2 + y^2}, & (x, y) \neq (0, 0), \\ 0, & (x, y) = (0, 0). \end{cases}$$

Which statements are true?

- I. f is continuous at the origin.
II. Every directional derivative of f at the origin exists.
III. f is differentiable at the origin.
- (A) II only
(B) I, II, and III
(C) I only
(D) I and III only
(E) I and II only

Questions 46–48

46. Let C be the intersection of

$$x^2 + y^2 + z^2 = 9 \quad \text{and} \quad x + y + z = 3.$$

Orient C by the right-hand rule with normal vector $(1, 1, 1)$. What is $\oint_C (z \, dx + x \, dy + y \, dz)$?

- (A) $6\sqrt{3}\pi$
- (B) 18π
- (C) 6π
- (D) $9\sqrt{3}\pi$
- (E) $3\sqrt{3}\pi$

47. Let $G = \mathbb{Z}_{12} \times \mathbb{Z}_8$, written additively, and let H be the subgroup generated by $(4, 2)$. Which group is isomorphic to G/H ?

- (A) $\mathbb{Z}_2 \times \mathbb{Z}_{12}$
- (B) $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$
- (C) $\mathbb{Z}_4 \times \mathbb{Z}_2$
- (D) $\mathbb{Z}_4 \times \mathbb{Z}_4$
- (E) $\mathbb{Z}_8$

48. How many ordered quadruples of integers (x_1, x_2, x_3, x_4) satisfy

$$x_1 + x_2 + x_3 + x_4 = 10, \quad 0 \leq x_i \leq 4 \quad (1 \leq i \leq 4)?$$

- (A) 80
- (B) 60
- (C) 72
- (D) 68
- (E) 64

Questions 49–51

49. For a real parameter p , consider

$$\int_0^{\infty} \frac{\sin(x^2)}{x^p} dx.$$

For exactly which values of p does this improper integral converge but not converge absolutely? The limits at 0 and $+\infty$ are taken separately.

- (A) $0 < p \leq 1$
- (B) $-1 < p \leq 1$
- (C) $-1 < p < 3$
- (D) $1 < p < 3$
- (E) $-1 < p < 1$

50. What is the volume of the solid

$$\Omega = \{(x, y, z) : x^2 + y^2 + z^2 \leq 4, x^2 + y^2 \geq 1\}?$$

- (A) $\frac{8\sqrt{3}\pi}{3}$
- (B) $\frac{4\pi}{3}(8 - 3\sqrt{3})$
- (C) $2\sqrt{3}\pi$
- (D) 8π
- (E) $4\sqrt{3}\pi$

51. Real 7×7 matrices P, Q satisfy

$$P^T = P, \quad P^2 = P, \quad Q^T = Q, \quad Q^2 = Q, \quad PQ = QP,$$

and

$$\operatorname{tr} P = 3, \quad \operatorname{tr} Q = 4, \quad \operatorname{tr}(PQ) = 2.$$

What is $\operatorname{rank}(P - Q)$?

- (A) 2
- (B) 7
- (C) 1
- (D) 5
- (E) 3

Questions 52–54

52. A simple undirected graph has vertex set $\{1, 2, 3, 4\}$ and edge set

$$\{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}\}.$$

How many spanning trees does this graph have?

- (A) 6
- (B) 8
- (C) 5
- (D) 10
- (E) 9

53. For $a > 0$, define

$$F(a) = \int_0^a \ln(1 + at) \, dt.$$

What is $F''(1)$?

- (A) $2 - 2 \ln 2$
- (B) 1
- (C) 0
- (D) $1 + \ln 2$
- (E) $2 \ln 2$

54. A sequence is defined by

$$a_1 = 1, \quad a_{n+1} = \frac{a_n}{1 + a_n^2} \quad (n \geq 1).$$

What is $\lim_{n \rightarrow \infty} \sqrt{n} a_n$?

- (A) $\sqrt{2}$
- (B) 1
- (C) $\frac{1}{2}$
- (D) $\frac{1}{\sqrt{2}}$
- (E) 2

Questions 55–57

55. How many invertible 2×2 matrices over $\mathbb{F}_3$ have trace zero?

- (A) 12
- (B) 18
- (C) 24
- (D) 27
- (E) 16

56. Give

$$K = \{0\} \cup \{1/n : n \in \mathbb{N}\}$$

the subspace topology inherited from $\mathbb{R}$. Which subset is both open and closed in K ?

- (A) $\{1/(2n) : n \in \mathbb{N}\}$
- (B) $\{0\} \cup \{1/(2n-1) : n \in \mathbb{N}\}$
- (C) $\{1/n : n \in \mathbb{N}\}$
- (D) $\{0\} \cup \{1/n : n \geq 3\}$
- (E) $\{0\}$

57. As a, b range over all real numbers, what is the minimum value of

$$\int_0^1 (x^2 - ax - b)^2 dx ?$$

- (A) $\frac{1}{90}$
- (B) $\frac{1}{60}$
- (C) $\frac{1}{180}$
- (D) $\frac{1}{120}$
- (E) $\frac{1}{360}$

Questions 58–60

58. Let

$$\mathbf{F}(x, y, z) = (2xy + z^2, x^2 + 2yz, 2xz + y^2).$$

If C is any piecewise smooth path from $(1, 0, 1)$ to $(0, 2, 3)$, what is $\int_C \mathbf{F} \cdot d\mathbf{r}$?

- (A) 9
- (B) 10
- (C) 11
- (D) 13
- (E) 12

59. Vectors $u, v \in \mathbb{R}^5$ satisfy

$$u^T u = 2, \quad v^T v = 5, \quad u^T v = 2.$$

What is $\det(I_5 + uu^T + vv^T)$?

- (A) 14
- (B) 18
- (C) 10
- (D) 16
- (E) 12

60. Let U_1, U_2, U_3, U_4 be independent, uniformly distributed random variables on $[0, 1]$, with order statistics

$$U_{(1)} \leq U_{(2)} \leq U_{(3)} \leq U_{(4)}.$$

What is

$$\mathbb{P}\left(U_{(4)} - U_{(1)} < \frac{1}{2} \mid U_{(4)} < \frac{3}{4}\right)?$$

- (A) $\frac{20}{27}$
- (B) $\frac{2}{3}$
- (C) $\frac{4}{9}$
- (D) $\frac{5}{16}$
- (E) $\frac{16}{27}$

Questions 61–63

61. The curve $x = t^2$, $y = t^3$, with $t > 0$, has a unique point at which its tangent is parallel to $y = x$. What is the curvature at that point?

- (A) $\frac{9}{16}$
- (B) $\frac{3}{8}$
- (C) $\frac{9\sqrt{2}}{16}$
- (D) $\frac{9\sqrt{2}}{32}$
- (E) $\frac{3\sqrt{2}}{16}$

62. Consider the boundary value problem

$$y'' + \lambda y = 0, \quad y(0) = 0, \quad y'(\pi) = 0.$$

What is the third smallest positive value of λ for which a nonzero solution exists?

- (A) $\frac{49}{4}$
- (B) 9
- (C) $\frac{25}{4}$
- (D) 4
- (E) $\frac{9}{4}$

63. How many integers x with $0 \leq x < 65$ satisfy

$$x^6 \equiv 1 \pmod{65}?$$

- (A) 6
- (B) 12
- (C) 24
- (D) 8
- (E) 16

Questions 64–66

64. Eight distinct people, including A, B, C, D , are seated around a circular table. How many arrangements have neither A next to B nor C next to D ? Arrangements differing only by rotation are considered the same; reflections are considered different.

(A) 2520
(B) 2880
(C) 2400
(D) 3360
(E) 2640

65. Define

$$f(x) = \frac{e^x}{1-x^2}, \quad |x| < 1.$$

What is $f^{(5)}(0)$?

(A) 141
(B) 121
(C) 151
(D) 161
(E) 131

66. Let A, B be real 4×4 matrices such that $AB = BA$, and suppose A has four distinct real eigenvalues. Which statements must be true?

- I. B is diagonalizable over $\mathbb{R}$.
- II. $A + B$ has four distinct eigenvalues.
- III. There is a real polynomial p of degree at most 3 such that $B = p(A)$.

(A) II only
(B) I, II, and III
(C) I and III only
(D) I and II only
(E) I only

Answer Key

Practice Test 02 — correct choices only.

Question	Answer	Question	Answer	Question	Answer
1	A	23	E	45	E
2	E	24	D	46	A
3	A	25	C	47	C
4	C	26	A	48	D
5	A	27	A	49	B
6	C	28	E	50	E
7	B	29	B	51	E
8	C	30	B	52	B
9	E	31	C	53	E
10	B	32	E	54	D
11	D	33	D	55	B
12	D	34	D	56	D
13	C	35	A	57	C
14	A	36	B	58	C
15	A	37	A	59	A
16	D	38	B	60	E
17	A	39	A	61	D
18	E	40	E	62	C
19	B	41	B	63	B
20	D	42	C	64	E
21	D	43	D	65	A
22	C	44	B	66	C

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