

GRE Mathematics Practice Test 03

Advanced Practice

66 questions 170 minutes A–E

Instructions

Choose the single best answer to each question. No calculator is permitted. Begin timing when you start Question 1. Use the blank space on the question pages for scratch work.

All angles are measured in radians unless stated otherwise. The notation $\ln$ denotes the natural logarithm, and $\mathbb{N} = \{1, 2, 3, \dots\}$. Norms are Euclidean unless specified. I_n is the identity matrix, $M_n(\mathbb{R})$ the real $n \times n$ matrices, and $\mathbb{F}_p$ the field with p elements.

Difficulty

An advanced form at the same difficulty tier as Practice Test 02.

Answer key

The complete answer key follows Question 66. Correct choices are provided without worked solutions.

Coverage

33 calculus questions, 17 algebra questions, and 16 questions on additional topics.

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100 practice tests, complete answer keys, and future updates.

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Questions 1–3

1. What is the value of

$$\lim_{x \rightarrow 0} \frac{\int_0^x \ln(1+t^2) dt - \frac{x^3}{3}}{x^5} ?$$

- (A) $\frac{1}{10}$
- (B) $-\frac{1}{6}$
- (C) $-\frac{1}{10}$
- (D) $-\frac{1}{5}$
- (E) $\frac{1}{5}$

2. For which real value of a does the system

$$\begin{pmatrix} 1 & a & a \\ a & 1 & a \\ a & a & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

have infinitely many solutions?

- (A) 1
- (B) 0
- (C) $\frac{1}{2}$
- (D) $-\frac{1}{2}$
- (E) -1

3. One of two coins is selected with equal probability. The probabilities of heads are $\frac{1}{3}$ for the first coin and $\frac{2}{3}$ for the second. The selected coin is tossed repeatedly, independently conditional on its identity. Given that exactly two of the first three tosses are heads, what is the probability that the fourth toss is heads?

- (A) $\frac{2}{3}$
- (B) $\frac{5}{9}$
- (C) $\frac{7}{9}$
- (D) $\frac{1}{2}$
- (E) $\frac{4}{9}$

Questions 4–6

4. The value of

$$\int_0^1 \frac{\arctan x}{(1+x)^2} dx$$

is

- (A) $\frac{\ln 2}{2}$
(B) $\frac{\ln 2}{4}$
(C) $\frac{\pi}{8}$
(D) $\frac{\pi}{4} - \frac{\ln 2}{2}$
(E) $\frac{\pi}{8} - \frac{\ln 2}{4}$

5. Let

$$D = \{(x, y) : x \geq 0, y \geq 0, 1 \leq x^2 - y^2 \leq 3, 0 \leq 2xy \leq 4\}.$$

What is $\iint_D (x^2 + y^2) dx dy$?

- (A) 8
(B) 16
(C) 1
(D) 2
(E) 4

6. Let

$$A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}.$$

What is $\min_{v \in \mathbb{R}^2} \|Av - b\|^2$, where $\|\cdot\|$ is the Euclidean norm?

- (A) $\frac{6}{5}$
(B) $\frac{4}{5}$
(C) $\frac{2}{5}$
(D) $\frac{8}{5}$
(E) 2

Questions 7–9

7. For $f(x) = \ln(1 + x + x^2)$, what is $f^{(6)}(0)$?

- (A) 720
- (B) 240
- (C) 120
- (D) -120
- (E) -240

8. Let $f_n : [0, 1] \rightarrow \mathbb{R}$ satisfy

$$|f_n(x) - f_n(y)| \leq |x - y| \quad \text{for all } x, y \in [0, 1] \text{ and all } n.$$

Suppose $f_n(r) \rightarrow 0$ for every rational $r \in [0, 1]$. Which statements must be true?

- I. $f_n \rightarrow 0$ uniformly on $[0, 1]$.
- II. $\int_0^1 |f_n(x)| dx \rightarrow 0$.
- III. $\sum_{n=1}^{\infty} f_n(1/2)$ converges.

- (A) II and III only
- (B) II only
- (C) I only
- (D) I and II only
- (E) I, II, and III

9. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be twice continuously differentiable and satisfy

$$f_{xx}(0, 2) + f_{yy}(0, 2) = 3.$$

Define $u(s, t) = f(s^2 - t^2, 2st)$. What is $u_{ss}(1, 1) + u_{tt}(1, 1)$?

- (A) 24
- (B) 18
- (C) 12
- (D) 6
- (E) 48

Questions 10–12

10. Let y solve

$$y' + 2xy = xe^{-x^2}, \quad y(0) = 1.$$

What is $y(1)$?

- (A) $\frac{e}{2}$
- (B) $\frac{1}{e}$
- (C) $\frac{2}{e}$
- (D) $\frac{1}{2e}$
- (E) $\frac{3}{2e}$

11. In the symmetric group S_6 , let

$$\sigma = (1\ 2\ 3)(4\ 5\ 6).$$

How many permutations $\tau \in S_6$ satisfy $\tau\sigma = \sigma\tau$?

- (A) 18
- (B) 36
- (C) 12
- (D) 6
- (E) 9

12. How many permutations of $\{1, 2, \dots, 7\}$ have exactly two fixed points?

- (A) 840
- (B) 1260
- (C) 924
- (D) 1008
- (E) 1050

Questions 13–15

13. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^3 + x$, and let $g = f^{-1}$. What is

$$\int_0^2 [g(y)]^2 dy?$$

- (A) $\frac{3}{5}$
- (B) $\frac{7}{15}$
- (C) $\frac{14}{15}$
- (D) $\frac{5}{4}$
- (E) $\frac{2}{3}$

14. Smooth functions $u = u(x, y)$ and $v = v(x, y)$ are defined near $(2, 2)$ by

$$u + v^2 = x, \quad u^2 + v = y, \quad u(2, 2) = v(2, 2) = 1.$$

What is $\frac{\partial u}{\partial x}(2, 2) + \frac{\partial v}{\partial y}(2, 2)$?

- (A) $\frac{4}{3}$
- (B) $-\frac{2}{3}$
- (C) 2
- (D) $-\frac{1}{3}$
- (E) $\frac{2}{3}$

15. A real 5×5 matrix A has characteristic polynomial and minimal polynomial

$$p_A(t) = (t - 2)^3(t + 1)^2, \quad m_A(t) = (t - 2)^2(t + 1),$$

respectively. What is $\text{rank}(A - 2I_5)$?

- (A) 3
- (B) 5
- (C) 4
- (D) 2
- (E) 1

Questions 16–18

16. Independent random variables X, Y each have density e^{-x} for $x > 0$, and zero otherwise. What is

$$\mathbb{P}\left(\frac{X}{X+Y} < \frac{1}{3} \mid X+Y > 2\right)?$$

- (A) e^{-2}
- (B) $\frac{1}{4}$
- (C) $\frac{1}{2}$
- (D) $\frac{1}{3}$
- (E) $\frac{2}{3}$

17. For real parameters p and q , consider

$$\int_0^1 \frac{[\ln(1+x)]^p}{x^{3/2}(1-x)^q} dx.$$

Which condition is necessary and sufficient for this improper integral to converge?

- (A) $p > -\frac{1}{2}, q < 1$
- (B) $p > \frac{1}{2}, q \leq 1$
- (C) $p \geq \frac{1}{2}, q < 1$
- (D) $p < \frac{1}{2}, q > 1$
- (E) $p > \frac{1}{2}, q < 1$

18. Nonnegative real numbers x, y, z satisfy

$$x + 2y + z = 4.$$

What is the maximum possible value of xyz^2 ?

- (A) 8
- (B) 1
- (C) 2
- (D) $\frac{16}{27}$
- (E) 4

Questions 19–21

19. Let V be the real vector space of all polynomials p of degree at most 7 that satisfy

$$p(1-x) = p(x) \quad \text{for every real } x, \quad p(0) = p\left(\frac{1}{2}\right) = 0.$$

What is $\dim V$?

- (A) 2
- (B) 4
- (C) 1
- (D) 3
- (E) 5

20. Under the transformation

$$w = \frac{z-1}{z+1},$$

what is the image of $\{z \in \mathbb{C} : |z| < 1, \operatorname{Im} z > 0\}$?

- (A) $\{w : \operatorname{Re} w > 0, \operatorname{Im} w < 0\}$
- (B) $\{w : \operatorname{Re} w < 0, \operatorname{Im} w < 0\}$
- (C) $\{w : \operatorname{Re} w < 0, \operatorname{Im} w > 0\}$
- (D) $\{w : \operatorname{Re} w > 0, \operatorname{Im} w > 0\}$
- (E) $\{w : |w| < 1, \operatorname{Im} w > 0\}$

21. For each $t > 0$, the tangent line to $y = x^{-2}$ at (t, t^{-2}) meets the positive coordinate axes at points A_t and B_t .

What is the minimum possible length of the segment $A_t B_t$?

- (A) $\frac{3\sqrt{6}}{2}$
- (B) $\frac{3\sqrt{3}}{2}$
- (C) $\sqrt{6}$
- (D) 3
- (E) $\frac{3\sqrt{2}}{2}$

Questions 22–24

22. What is the value of

$$\lim_{n \rightarrow \infty} \left[\prod_{k=1}^n \left(1 + \frac{k^2}{n^2} \right) \right]^{1/n} ?$$

- (A) $2e^{\pi/2-2}$
- (B) $2e^{2-\pi/2}$
- (C) $e^{\pi/2-2}$
- (D) $2e^{\pi/4-1}$
- (E) $e^{\pi/2-1}$

23. Let G be a group of order 99. Which statements must be true?

- I. G is abelian.
- II. G is cyclic.
- III. G has an element of order 33.

- (A) I, II, and III
- (B) II and III only
- (C) I only
- (D) II only
- (E) I and III only

24. Let $f, g : [0, 1] \rightarrow \mathbb{R}$ be bounded functions that differ at at most countably many points. Suppose f is Riemann integrable. Which statements must be true?

- I. g is Riemann integrable.
- II. If g is Riemann integrable, then $\int_0^1 g = \int_0^1 f$.
- III. $|g|$ is Riemann integrable.

- (A) I and II only
- (B) I only
- (C) I, II, and III
- (D) II only
- (E) II and III only

Questions 25–27

25. Let y solve

$$y' = y(1 - y) \cos x, \quad y(0) = \frac{1}{3}.$$

What is $y(\pi/2)$?

- (A) $\frac{1}{e+2}$
- (B) $\frac{e}{e+1}$
- (C) $\frac{2e}{2e+1}$
- (D) $\frac{e}{e+2}$
- (E) $\frac{2}{e+2}$

26. For every continuous function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$, which expression equals

$$\int_0^1 \int_{x^2}^{2-x} f(x, y) \, dy \, dx ?$$

- (A) $\int_0^1 \int_0^{y^2} f(x, y) \, dx \, dy + \int_1^2 \int_0^{2-y} f(x, y) \, dx \, dy$
- (B) $\int_0^2 \int_0^{\sqrt{y}} f(x, y) \, dx \, dy$
- (C) $\int_0^1 \int_0^{\sqrt{y}} f(x, y) \, dx \, dy + \int_1^2 \int_{2-y}^1 f(x, y) \, dx \, dy$
- (D) $\int_0^1 \int_{y^2}^{2-y} f(x, y) \, dx \, dy + \int_1^2 \int_0^{2-y} f(x, y) \, dx \, dy$
- (E) $\int_0^1 \int_0^{\sqrt{y}} f(x, y) \, dx \, dy + \int_1^2 \int_0^{2-y} f(x, y) \, dx \, dy$

27. Consider the quadratic form

$$q(x, y, z) = x^2 + 4xy + 2y^2 + 6yz + 4z^2.$$

After an invertible real linear change of variables makes q diagonal, what are the numbers of positive and negative diagonal coefficients, respectively?

- (A) (1, 2)
- (B) (1, 1)
- (C) (3, 0)
- (D) (2, 1)
- (E) (2, 0)

Questions 28–30

28. Let $n \geq 2$ and $\theta > 0$. Suppose $X_1, \dots, X_n$ are independent and uniformly distributed on $[0, \theta]$. Define

$$M = \max(X_1, \dots, X_n), \quad T = \frac{n+1}{n}M.$$

What is $\mathbb{E}[(T - \theta)^2]$?

- (A) $\frac{n\theta^2}{(n+1)^2(n+2)}$
- (B) $\frac{\theta^2}{(n+1)(n+2)}$
- (C) $\frac{\theta^2}{n(n+2)}$
- (D) $\frac{\theta^2}{n^2}$
- (E) $\frac{\theta^2}{n(n+1)}$

29. What is the sum of the series

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)3^n}?$$

- (A) $2 - 3 \ln \frac{3}{2}$
- (B) $\ln \frac{3}{2} - \frac{1}{3}$
- (C) $1 - 2 \ln \frac{3}{2}$
- (D) $2 \ln \frac{3}{2}$
- (E) $1 - \ln \frac{3}{2}$

30. A solid of uniform density occupies

$$\Omega = \{(x, y, z) : x^2 + y^2 + z^2 \leq 4, z \geq 1\}.$$

What is the z -coordinate of its center of mass?

- (A) $\frac{7}{5}$
- (B) $\frac{27}{20}$
- (C) $\frac{9}{5}$
- (D) $\frac{3}{2}$
- (E) $\frac{4}{3}$

Questions 31–33

31. How many ideals, including the zero ideal and the whole ring, does

$$R = \mathbb{F}_3[x] / (x^2(x-1)^3)$$

have?

- (A) 18
- (B) 8
- (C) 12
- (D) 10
- (E) 6

32. A lattice path from $(0, 0)$ to $(5, 4)$ consists only of steps $(1, 0)$ and $(0, 1)$. How many such paths never pass above the line $y = x$?

- (A) 42
- (B) 84
- (C) 28
- (D) 126
- (E) 14

33. Define

$$f(x, y) = \begin{cases} \frac{xy^3}{x^2 + y^2}, & (x, y) \neq (0, 0), \\ 0, & (x, y) = (0, 0). \end{cases}$$

What is the ordered pair

$$\left(\frac{\partial}{\partial y} \left(\frac{\partial f}{\partial x} \right) (0, 0), \quad \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial y} \right) (0, 0) \right)?$$

- (A) $(0, 1)$
- (B) $(-1, 1)$
- (C) $(0, 0)$
- (D) $(1, 0)$
- (E) $(1, 1)$

Questions 34–36

34. For $a > 0$, define

$$F(a) = \int_0^1 \ln(1 + ax^2) dx.$$

What is $F''(1)$?

- (A) $\frac{\pi}{4} - 1$
- (B) $\frac{1}{2} - \frac{\pi}{4}$
- (C) $\frac{\pi}{8} - \frac{1}{4}$
- (D) $\frac{5}{4} - \frac{3\pi}{8}$
- (E) $\frac{3\pi}{8} - \frac{5}{4}$

35. A real 6×6 matrix A satisfies

$$A^T A = I_6, \quad A^T = -A.$$

What is $\det(I_6 + A)$?

- (A) -8
- (B) 8
- (C) 4
- (D) 16
- (E) 0

36. Let (a_n) be a sequence of real numbers such that $\sum_{n=1}^{\infty} a_n$ converges. Which statements must be true?

- I. $\sum_{n=1}^{\infty} \frac{a_n}{n}$ converges.
- II. $\sum_{n=1}^{\infty} a_n^2$ converges.
- III. $\sum_{n=1}^{\infty} \frac{|a_n|}{n}$ converges.

- (A) I only
- (B) I, II, and III
- (C) II only
- (D) I and II only
- (E) I and III only

Questions 37–39

37. Let y_1, y_2 be solutions of

$$y'' + 2xy' + (1 + x^2)y = 0$$

with $y_1(0) = 1$, $y_1'(0) = 0$, $y_2(0) = 0$, and $y_2'(0) = 1$. What is

$$\det \begin{pmatrix} y_1(1) & y_2(1) \\ y_1'(1) & y_2'(1) \end{pmatrix} ?$$

- (A) 1
- (B) e^{-2}
- (C) e^{-1}
- (D) e
- (E) 0

38. For exactly which real values of x does the series

$$\sum_{n=1}^{\infty} \frac{(x+1)^{2n}}{n3^n}$$

converge?

- (A) $[-1 - \sqrt{3}, -1 + \sqrt{3}]$
- (B) $(-1 - \sqrt{3}, -1 + \sqrt{3})$
- (C) $[-1 - \sqrt{3}, -1 + \sqrt{3})$
- (D) $(-1 - \sqrt{3}, -1 + \sqrt{3}]$
- (E) $(-\sqrt{3}, \sqrt{3})$

39. What is the smallest positive integer k such that

$$2^k \equiv 1 \pmod{77} ?$$

- (A) 15
- (B) 10
- (C) 30
- (D) 60
- (E) 5

Questions 40–42

40. A fair coin is tossed six times independently. Let N be the number of indices $i \in \{1, 2, 3, 4, 5\}$ for which both toss i and toss $i + 1$ are heads. What is $\text{Var}(N)$?

- (A) $\frac{23}{16}$
- (B) $\frac{25}{16}$
- (C) $\frac{5}{4}$
- (D) $\frac{15}{16}$
- (E) $\frac{19}{16}$

41. Let S be the sphere $x^2 + y^2 + z^2 = 4$, with outward unit normal $\mathbf{n}$. If

$$\mathbf{F}(x, y, z) = (x^3, y^3, z^3),$$

what is $\iint_S \mathbf{F} \cdot \mathbf{n} \, dS$?

- (A) $\frac{192\pi}{5}$
- (B) $\frac{384\pi}{5}$
- (C) 64π
- (D) $\frac{128\pi}{5}$
- (E) 96π

42. Let f be twice continuously differentiable on $[0, 1]$, with

$$f(0) = f(1) = 0, \quad f\left(\frac{1}{2}\right) = 1.$$

Which value must $f''(c)$ equal for at least one $c \in (0, 1)$?

- (A) -16
- (B) -8
- (C) 8
- (D) 0
- (E) -4

Questions 43–45

43. Let F be a field with 64 elements. How many elements $a \in F$ satisfy

$$a^{16} = a?$$

- (A) 2
- (B) 1
- (C) 8
- (D) 16
- (E) 4

44. Let X be a compact topological space, let Y be Hausdorff, and let $f : X \rightarrow Y$ be a continuous surjection. Which statements must be true?

- I. f maps closed subsets of X to closed subsets of Y .
- II. $f^{-1}(\{y\})$ is compact for each $y \in Y$.
- III. f maps open subsets of X to open subsets of Y .

- (A) I, II, and III
- (B) I and II only
- (C) I and III only
- (D) II only
- (E) I only

45. Let C be the triangular path traversed in the order

$$(1, 0, 0) \longrightarrow (0, 2, 0) \longrightarrow (0, 0, 3) \longrightarrow (1, 0, 0).$$

What is $\oint_C (z^2 dx + x^2 dy + y^2 dz)$?

- (A) $-\frac{23}{3}$
- (B) $\frac{26}{3}$
- (C) $\frac{13}{3}$
- (D) $\frac{23}{3}$
- (E) $\frac{17}{3}$

Questions 46–48

46. What is the arc length of the curve

$$\mathbf{r}(t) = \left(t, \frac{t^2}{2}, \frac{t^3}{6} \right), \quad 0 \leq t \leq 2?$$

- (A) $\frac{8}{3}$
- (B) $\frac{10}{3}$
- (C) 4
- (D) $\frac{14}{3}$
- (E) $2\sqrt{3}$

47. Let

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}.$$

What is the entry in row 1, column 3 of A^6 ?

- (A) 480
- (B) 320
- (C) 192
- (D) 160
- (E) 240

48. A finite connected simple graph is embedded in the plane so that the boundary of every face, including the unbounded face, is a triangle. Every vertex has degree 5 or 6. How many vertices have degree 5?

- (A) 10
- (B) 6
- (C) 8
- (D) 14
- (E) 12

Questions 49–51

49. The arc

$$y = x^2, \quad 0 \leq x \leq \frac{\sqrt{3}}{2},$$

is rotated through one complete revolution about the y -axis. What is the area of the surface generated by the arc?

- (A) $\frac{7\pi}{6}$
- (B) $\frac{7\pi}{3}$
- (C) $\frac{3\pi}{4}$
- (D) $\frac{4\pi}{3}$
- (E) $\frac{7\pi}{12}$

50. Let $D = \{(x, y) : x^2 + 4y^2 \leq 4\}$. What is

$$\iint_D \frac{1}{1 + x^2 + 4y^2} dx dy ?$$

- (A) $\frac{\pi}{2} \ln 5$
- (B) $\frac{\pi}{2} \ln 2$
- (C) $\pi \ln 5$
- (D) $\pi \ln 2$
- (E) $\frac{\pi}{4} \ln 5$

51. Which is the minimal polynomial over $\mathbb{Q}$ of $\sqrt{2} + \sqrt{5}$?

- (A) $t^4 - 10t^2 + 9$
- (B) $t^4 - 14t^2 + 49$
- (C) $t^4 - 14t^2 + 9$
- (D) $t^4 - 7t^2 + 3$
- (E) $t^4 - 14t^2 - 9$

Questions 52–54

52. For $h \neq 0$, define

$$D_h = \frac{e^h - e^{-h}}{2h}, \quad R_h = \frac{4D_{h/2} - D_h}{3}.$$

What is $\lim_{h \rightarrow 0} \frac{R_h - 1}{h^4}$?

- (A) $\frac{1}{480}$
- (B) $-\frac{1}{24}$
- (C) $\frac{1}{120}$
- (D) $-\frac{1}{480}$
- (E) $-\frac{1}{120}$

53. Let y solve

$$x^2 y'' - xy' + 2y = 0, \quad x > 0, \quad y(1) = 0, \quad y'(1) = 1.$$

What is $y(e^{\pi/2})$?

- (A) $e^{\pi/2}$
- (B) 0
- (C) $e^{-\pi/2}$
- (D) $\frac{\pi}{2}e^{\pi/2}$
- (E) $-e^{\pi/2}$

54. What is the minimum Euclidean distance between a point on the paraboloid

$$z = x^2 + y^2$$

and a point on the plane $z = 2x + 2y - 3$?

- (A) 1
- (B) $\frac{\sqrt{2}}{3}$
- (C) $\frac{2}{3}$
- (D) $\frac{1}{\sqrt{3}}$
- (E) $\frac{1}{3}$

Questions 55–57

55. Let r_1, r_2, r_3 be the roots, counted with multiplicity, of

$$t^3 - 3t + 1 = 0.$$

What is $r_1^6 + r_2^6 + r_3^6$?

- (A) 63
- (B) 57
- (C) 27
- (D) 81
- (E) 54

56. Let C be the circle $|z| = 1$, oriented counterclockwise. What is

$$\frac{1}{2\pi i} \oint_C z^2 \sin\left(\frac{1}{z}\right) dz?$$

- (A) -1
- (B) 0
- (C) 1
- (D) $\frac{1}{6}$
- (E) $-\frac{1}{6}$

57. What is the value of

$$\lim_{n \rightarrow \infty} n^2 \int_0^1 \frac{x^n(1-x)}{1+x} dx?$$

- (A) 0
- (B) 1
- (C) $\frac{1}{4}$
- (D) 2
- (E) $\frac{1}{2}$

Questions 58–60

58. Let C be the positively oriented boundary of the annulus $1 \leq x^2 + y^2 \leq 4$; thus the outer circle is traversed counterclockwise and the inner circle clockwise. What is

$$\oint_C \left(-\frac{2y^3}{3} dx + \frac{x^3}{3} dy \right) ?$$

- (A) $\frac{45\pi}{4}$
- (B) $\frac{45\pi}{2}$
- (C) $\frac{17\pi}{4}$
- (D) $\frac{15\pi}{4}$
- (E) $\frac{15\pi}{2}$

59. Let $V = M_3(\mathbb{R})$ and define $T : V \rightarrow V$ by

$$T(X) = X^T + (\operatorname{tr} X)I_3.$$

What is the determinant of T as a linear transformation of the 9-dimensional vector space V ?

- (A) -3
- (B) -1
- (C) 4
- (D) -4
- (E) 1

60. Let X, Y be independent standard normal random variables. What is the correlation coefficient between X^2 and $(X + Y)^2$?

- (A) $\frac{\sqrt{3}}{2}$
- (B) $\frac{1}{\sqrt{2}}$
- (C) $\frac{1}{2}$
- (D) $\frac{1}{4}$
- (E) 0

Questions 61–63

61. Functions $x = x(t)$ and $y = y(t)$ satisfy

$$\frac{dx}{dt} = x + 2y, \quad \frac{dy}{dt} = 2x + y, \quad x(0) = 1, \quad y(0) = 0.$$

What is $x(\ln 2)$?

- (A) $\frac{17}{4}$
- (B) $\frac{9}{2}$
- (C) 4
- (D) $\frac{15}{4}$
- (E) $\frac{5}{2}$

62. The value of the improper integral

$$\int_0^{\infty} \frac{(e^{-x} - e^{-3x})^2}{x} dx$$

is

- (A) $\ln \frac{9}{8}$
- (B) $\ln \frac{4}{3}$
- (C) $\ln 2$
- (D) $\ln \frac{3}{2}$
- (E) $2 \ln \frac{3}{2}$

63. What is the largest integer $k \geq 0$ for which

$$3^k \text{ divides } \binom{100}{50}?$$

- (A) 3
- (B) 4
- (C) 2
- (D) 6
- (E) 5

Questions 64–66

64. Let X, Y be arbitrary nonempty sets and let $f : X \rightarrow Y$. Which of the following conditions is, by itself, equivalent to f being bijective?

- I. $f(X \setminus A) = Y \setminus f(A)$ for every $A \subseteq X$.
- II. $f^{-1}(f(A)) = A$ for every $A \subseteq X$.
- III. $f(f^{-1}(B)) = B$ for every $B \subseteq Y$.

- (A) II only
- (B) I, II, and III
- (C) I only
- (D) III only
- (E) II and III only

65. What is the surface area of the portion of the cone

$$z = \sqrt{x^2 + y^2}$$

that lies inside the vertical cylinder $x^2 + y^2 \leq 2x$?

- (A) $\sqrt{2}\pi$
- (B) π
- (C) 2π
- (D) $2\sqrt{2}\pi$
- (E) 4π

66. For exactly which real values of a does

$$x^2 - ax + a - \frac{3}{4} = 0$$

have two distinct real roots, both strictly between 0 and 1?

- (A) $1 < a < 2$
- (B) $0 < a < 1$
- (C) $a > 3$
- (D) $0 < a < \frac{3}{4}$
- (E) $\frac{3}{4} < a < 1$

Answer Key

Practice Test 03 — correct choices only.

Question	Answer	Question	Answer	Question	Answer
1	C	23	E	45	D
2	D	24	D	46	B
3	B	25	D	47	E
4	B	26	E	48	E
5	D	27	D	49	A
6	D	28	C	50	A
7	E	29	C	51	C
8	D	30	B	52	D
9	A	31	C	53	A
10	E	32	A	54	E
11	A	33	D	55	B
12	C	34	E	56	E
13	C	35	B	57	E
14	B	36	A	58	A
15	A	37	C	59	D
16	D	38	D	60	C
17	E	39	C	61	A
18	C	40	A	62	B
19	A	41	B	63	B
20	C	42	B	64	C
21	B	43	E	65	A
22	A	44	B	66	E

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