

GRE Mathematics Practice Test 04

Bridge-Level Practice

66 questions 170 minutes A–E

Instructions

Choose the single best answer to each question. No calculator is permitted. Begin timing when you start Question 1. Use the blank space on the question pages for scratch work.

All angles are measured in radians unless stated otherwise. The notation $\ln$ denotes the natural logarithm, and $\mathbb{N} = \{1, 2, 3, \dots\}$. Norms are Euclidean unless specified. I_n is the identity matrix, $M_n(\mathbb{R})$ the real $n \times n$ matrices, and $\mathbb{F}_p$ the field with p elements.

Difficulty

A bridge from advanced practice to standard timed-test difficulty.

Answer key

The complete answer key follows Question 66. Correct choices are provided without worked solutions.

Coverage

33 calculus questions, 17 algebra questions, and 16 questions on additional topics.

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100 practice tests, complete answer keys, and future updates.

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Questions 1–3

1. What is the value of

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+2x} - 1 - x}{x^2} ?$$

- (A) $-\frac{1}{4}$
- (B) $\frac{1}{2}$
- (C) $-\frac{1}{2}$
- (D) 0
- (E) -1

2. For exactly which real values of a does

$$x^4 - 5x^2 + a = 0$$

have four distinct real solutions?

- (A) $0 < a < \frac{25}{4}$
- (B) $a < \frac{25}{4}$
- (C) $a < 0$
- (D) $0 \leq a \leq \frac{25}{4}$
- (E) $a > \frac{25}{4}$

3. A bag contains five red balls and three blue balls. Three balls are chosen uniformly at random without replacement. Given that at least one chosen ball is blue, what is the probability that exactly two are blue?

- (A) $\frac{15}{46}$
- (B) $\frac{5}{23}$
- (C) $\frac{15}{56}$
- (D) $\frac{5}{14}$
- (E) $\frac{3}{8}$

Questions 4–6

4. The value of

$$\int_0^1 x \ln(1 + x^2) dx$$

is

- (A) $\frac{1}{2}$
(B) $-\frac{1}{2} + \ln(2)$
(C) $1 - \ln(2)$
(D) $\frac{\ln(2)}{2}$
(E) $-1 + 2 \ln(2)$

5. Let

$$F(x) = \int_0^{\sin x} \cos(t^2) dt.$$

What is $F''(\pi/2)$?

- (A) $-\sin(1)$
(B) $\sin(1)$
(C) $-\cos(1)$
(D) 0
(E) $\cos(1)$

6. Let V be the real vector space of polynomials p of degree at most 5 satisfying

$$p(1) = p(-1) = 0, \quad p'(0) = 0.$$

What is $\dim V$?

- (A) 1
(B) 3
(C) 5
(D) 4
(E) 2

Questions 7–9

7. What is the sum of the series

$$\sum_{n=1}^{\infty} \frac{n(n+1)}{4^n}?$$

- (A) $\frac{4}{3}$
- (B) $\frac{8}{9}$
- (C) $\frac{16}{27}$
- (D) $\frac{20}{27}$
- (E) $\frac{32}{27}$

8. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be differentiable and satisfy $|f'(x)| \leq 3$ for every real x . Which statements must be true?

I. f is uniformly continuous on $\mathbb{R}$.

II. f is bounded on $\mathbb{R}$.

III. $\lim_{x \rightarrow +\infty} \frac{f(x)}{x^2} = 0$.

- (A) I only
- (B) I, II, and III
- (C) II and III only
- (D) I and II only
- (E) I and III only

9. Let $f(x, y, z) = \ln(x^2 + y^2 + z^2)$. What is the directional derivative of f at $(1, 2, 2)$ in the direction from $(1, 2, 2)$ toward $(3, 1, 4)$?

- (A) $\frac{2}{3}$
- (B) $\frac{4}{27}$
- (C) $\frac{8}{27}$
- (D) $\frac{8}{9}$
- (E) $\frac{4}{9}$

Questions 10–12

10. Let y solve

$$y' + \frac{y}{1+x} = (1+x)^2, \quad y(0) = 1.$$

What is $y(1)$?

- (A) $\frac{15}{8}$
 - (B) $\frac{19}{4}$
 - (C) $\frac{17}{8}$
 - (D) $\frac{5}{2}$
 - (E) $\frac{19}{8}$
11. Regard $\mathbb{Z}_m$ as the additive group of integers modulo m . How many group homomorphisms $\varphi : \mathbb{Z}_{18} \rightarrow \mathbb{Z}_{12}$ have a kernel with exactly three elements?
- (A) 2
 - (B) 4
 - (C) 6
 - (D) 1
 - (E) 3
12. How many distinct seven-letter strings can be formed using exactly three letters A , two letters B , and two letters C , if the two letters B are not adjacent?
- (A) 120
 - (B) 210
 - (C) 150
 - (D) 140
 - (E) 180

Questions 13–15

13. A smooth function $y = y(x)$ is defined near $x = 1$ by

$$x^2 + xy + y^2 = 7, \quad y(1) = 2.$$

What is $y''(1)$?

- (A) $-\frac{21}{125}$
- (B) $\frac{42}{125}$
- (C) $-\frac{4}{5}$
- (D) $-\frac{6}{25}$
- (E) $-\frac{42}{125}$

14. For exactly which real values of x does

$$\sum_{n=1}^{\infty} \frac{(2x-1)^n}{n^2 2^n}$$

converge?

- (A) $[-\frac{1}{2}, \frac{3}{2})$
- (B) $[-1, 1]$
- (C) $(-\frac{1}{2}, \frac{3}{2}]$
- (D) $(-\frac{1}{2}, \frac{3}{2})$
- (E) $[-\frac{1}{2}, \frac{3}{2}]$

15. Linear transformations $A : \mathbb{R}^6 \rightarrow \mathbb{R}^4$ and $B : \mathbb{R}^5 \rightarrow \mathbb{R}^6$ have ranks 3 and 4, respectively. What is the smallest possible rank of $A \circ B$?

- (A) 4
- (B) 2
- (C) 0
- (D) 3
- (E) 1

Questions 16–18

16. Let C be the circle $|z| = 2$, oriented counterclockwise. What is

$$\frac{1}{2\pi i} \oint_C \frac{z \cos z}{(z-1)^2} dz?$$

- (A) $-\sin(1)$
- (B) $\cos(1)$
- (C) $-\sin(1) + \cos(1)$
- (D) $-\sin(1) + 2\cos(1)$
- (E) $\cos(1) + \sin(1)$

17. Let $D = \{(x, y) : x \geq 0, y \geq 0, x + y \leq 1\}$. What is

$$\iint_D \frac{x}{x+y} dx dy?$$

The integrand is defined to be zero at the origin.

- (A) $\frac{1}{4}$
- (B) $\frac{1}{2}$
- (C) $\frac{1}{6}$
- (D) $\frac{1}{8}$
- (E) $\frac{1}{3}$

18. What is the maximum possible value of $x + 2y$ for real x, y satisfying

$$x^2 + xy + y^2 = 3?$$

- (A) 4
- (B) $2\sqrt{2}$
- (C) $2\sqrt{3}$
- (D) $\sqrt{15}$
- (E) 3

Questions 19–21

19. Let $V = M_2(\mathbb{R})$, and let

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}.$$

Define $T : V \rightarrow V$ by $T(X) = AX$. What is the determinant of T as a linear transformation of the four-dimensional space V ?

- (A) 4
(B) -2
(C) 2
(D) 16
(E) -4
20. For $x \in \mathbb{R}$, define $f_n(x) = \frac{nx}{1 + n^2x^2}$. Which statements are true?
- I. f_n converges pointwise to zero on $\mathbb{R}$.
II. f_n converges uniformly to zero on $\mathbb{R}$.
III. f_n converges uniformly to zero on $[1, \infty)$.
- (A) I and III only
(B) I and II only
(C) III only
(D) I, II, and III
(E) I only

21. What is the arc length of

$$\mathbf{r}(t) = (3 \cos t, 3 \sin t, 4t), \quad 0 \leq t \leq \pi?$$

- (A) 25π
(B) 7π
(C) 4π
(D) 5π
(E) 3π

Questions 22–24

22. A ladder of length 10 meters rests against a vertical wall on level ground. Its lower end moves away from the wall at 2 meters per second. When the lower end is 6 meters from the wall, at what rate, in square meters per second, is the area of the triangle formed by the ladder, wall, and ground changing?

- (A) $\frac{3}{2}$
- (B) $\frac{7}{2}$
- (C) $-\frac{7}{2}$
- (D) 8
- (E) 7

23. What is the sum of all integers x with $0 \leq x < 30$ that satisfy

$$14x \equiv 8 \pmod{30}?$$

- (A) 15
- (B) 29
- (C) 14
- (D) 22
- (E) 37

24. A tree has 20 vertices. Exactly four vertices have degree 3, and every other vertex has degree 1 or 2. How many vertices have degree 1?

- (A) 5
- (B) 4
- (C) 8
- (D) 6
- (E) 10

Questions 25–27

25. Let y satisfy

$$y' = y(1 - y), \quad y(0) = \frac{1}{4}.$$

At what positive value of x does $y(x)$ first equal $1/2$?

- (A) $\ln(2)$
- (B) $\ln(3)$
- (C) $2 \ln(3)$
- (D) $2 \ln(2)$
- (E) $\frac{\ln(3)}{2}$

26. What is the minimum value on $\mathbb{R}^2$ of

$$f(x, y) = x^2 + y^2 + xy - 2x + 4y?$$

- (A) -8
- (B) -10
- (C) $-\frac{20}{3}$
- (D) $-\frac{14}{3}$
- (E) $-\frac{28}{3}$

27. How many nonzero elements of the ring $\mathbb{Z}/45\mathbb{Z}$ are zero divisors?

- (A) 20
- (B) 21
- (C) 24
- (D) 30
- (E) 15

Questions 28–30

28. Four balls are placed independently into three boxes, each ball choosing each box with probability $1/3$. What is the expected number of boxes that contain at least one ball?

- (A) $\frac{7}{3}$
(B) $\frac{80}{27}$
(C) $\frac{65}{27}$
(D) $\frac{19}{9}$
(E) $\frac{8}{3}$

29. The value of

$$\int_0^{\pi/2} \sin^3 x \cos^2 x \, dx$$

is

- (A) $\frac{1}{5}$
(B) $\frac{2}{15}$
(C) $\frac{4}{15}$
(D) $\frac{1}{15}$
(E) $\frac{1}{6}$

30. What is the volume of the solid bounded below by $z = x^2 + y^2$ and above by $z = 6 - x^2 - y^2$?

- (A) 12π
(B) 18π
(C) 9π
(D) 6π
(E) $\frac{9\pi}{2}$

Questions 31–33

- 31.** A real 6×6 matrix P satisfies $P^2 = P$ and $\text{rank } P = 2$. What is $\det(I_6 + 2P)$?
- (A) 64
(B) 27
(C) 9
(D) 3
(E) 1
- 32.** In how many ways can a 2×6 rectangular board be tiled by 1×2 dominoes? Dominoes may be placed horizontally or vertically; all squares must be covered exactly once.
- (A) 16
(B) 21
(C) 12
(D) 13
(E) 8
- 33.** For $f(x, y) = x^3 + y^3 - 3xy$, which statement correctly describes all of its critical points?
- (A) $(0, 0)$ is a strict local minimum, and $(1, 1)$ is a saddle point.
(B) $(1, 1)$ is the only critical point.
(C) $(0, 0)$ and $(1, 1)$ are both saddle points.
(D) $(0, 0)$ is a saddle point, and $(1, 1)$ is a strict local minimum.
(E) $(0, 0)$ is a strict local maximum, and $(1, 1)$ is a strict local minimum.

Questions 34–36

34. What is the value of

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{\sqrt{n^2 + kn}}?$$

- (A) $2 - \sqrt{2}$
- (B) $-1 + \sqrt{2}$
- (C) $-2 + 2\sqrt{2}$
- (D) $\frac{\sqrt{2}}{2}$
- (E) $\ln(2)$

35. As A ranges over all real symmetric 3×3 matrices with $\text{tr } A = 6$, what is the smallest possible value of $\text{tr}(A^2)$?

- (A) 12
- (B) 6
- (C) 18
- (D) 36
- (E) 9

36. Let

$$S = \left\{ \left(\frac{1}{n}, \frac{m}{n} \right) : n \in \mathbb{N}, m \in \{0, 1, \dots, n\} \right\} \subseteq \mathbb{R}^2.$$

Which set consists of all accumulation points of S in the usual topology?

- (A) $S \cup (\{0\} \times [0, 1])$
- (B) $\{0\} \times [0, 1]$
- (C) $[0, 1] \times [0, 1]$
- (D) $\{(0, 0)\}$
- (E) $\{(0, 0), (0, 1)\}$

Questions 37–39

37. Let y solve

$$y'' + y = \sin x, \quad y(0) = y'(0) = 0.$$

What is $y(\pi)$?

- (A) 1
- (B) $\frac{\pi}{2}$
- (C) π
- (D) $-\frac{\pi}{2}$
- (E) 0

38. The transformation $u = xe^y$, $v = ye^x$ has a smooth local inverse $(x, y) = (x(u, v), y(u, v))$ near the image of $(x, y) = (1, 0)$. What is $\frac{\partial x}{\partial v}$ at $(u, v) = (1, 0)$?

- (A) e^{-1}
- (B) $-e$
- (C) $-\frac{1}{e}$
- (D) -1
- (E) 0

39. Let G be a group of order 35, and let $\varphi : G \rightarrow S_4$ be a group homomorphism, where S_4 is the symmetric group on four letters. What is $|\ker \varphi|$?

- (A) 1
- (B) 7
- (C) 5
- (D) 24
- (E) 35

Questions 40–42

40. Random variables X, Y have joint density

$$f_{X,Y}(x, y) = \begin{cases} 2, & 0 < x < y < 1, \\ 0, & \text{otherwise.} \end{cases}$$

What is $\mathbb{E}[XY]$?

- (A) $\frac{1}{4}$
- (B) $\frac{1}{6}$
- (C) $\frac{1}{3}$
- (D) $\frac{1}{2}$
- (E) $\frac{2}{9}$

41. Let C be the counterclockwise boundary of the rectangle $0 \leq x \leq 2, 0 \leq y \leq 1$. What is

$$\oint_C (x^2 y \, dx + xy^2 \, dy) ?$$

- (A) -2
- (B) $-\frac{2}{3}$
- (C) 0
- (D) 2
- (E) $\frac{2}{3}$

42. Let f be twice continuously differentiable on $[0, 2]$, with $f(0) = f(2) = 0$ and $f''(x) > 0$ for $0 < x < 2$. Which statement must be true?

- (A) $f(1) > 0$.
- (B) $\int_0^2 f(x) \, dx = 0$.
- (C) $f'(1) = 0$.
- (D) There is exactly one $c \in (0, 2)$ for which $f'(c) = 0$.
- (E) $f'(x) > 0$ for every $x \in (0, 2)$.

Questions 43–45

43. How many distinct solutions in $[0, 2\pi)$ does

$$2 \cos^2 x + \sin x - 1 = 0$$

have?

- (A) 4
 - (B) 3
 - (C) 5
 - (D) 2
 - (E) 1
44. At exactly which points of $\mathbb{C}$ is the function $f(z) = |z|^2$ complex differentiable?
- (A) At every point of the imaginary axis, and nowhere else.
 - (B) At no point of $\mathbb{C}$.
 - (C) At every point of $\mathbb{C}$.
 - (D) Only at $z = 0$.
 - (E) At every point of the real axis, and nowhere else.
45. Let S be the triangular portion of the plane $x + y + z = 3$ in the first octant, oriented with a normal having positive coordinates. For $\mathbf{F}(x, y, z) = (x, y, z)$, what is

$$\iint_S \mathbf{F} \cdot \mathbf{n} \, dS?$$

- (A) 18
- (B) $\frac{9}{2}$
- (C) $\frac{27}{2}$
- (D) 27
- (E) 9

Questions 46–48

46. What is the area enclosed by one petal of the polar curve $r = 2 \sin(2\theta)$?

- (A) $\frac{\pi}{2}$
- (B) $\frac{\pi}{4}$
- (C) π
- (D) 4π
- (E) 2π

47. For exactly which real values of a is

$$x^2 + 2axy + (a + 2)y^2 > 0$$

for every nonzero $(x, y) \in \mathbb{R}^2$?

- (A) $-2 < a < 1$
- (B) $|a| < 1$
- (C) $-1 \leq a \leq 2$
- (D) $-1 < a < 2$
- (E) $a > -2$

48. A connected simple graph is embedded in the plane. It has 8 vertices and 13 edges, and the boundary of every face, including the unbounded face, is either a triangle or a quadrilateral. How many faces are quadrilaterals?

- (A) 6
- (B) 3
- (C) 4
- (D) 7
- (E) 5

Questions 49–51

49. What is the sum of

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n(n+1)}?$$

- (A) $\frac{1}{2}$
- (B) $\ln(2)$
- (C) $2 - 2 \ln(2)$
- (D) $1 - \ln(2)$
- (E) $-1 + 2 \ln(2)$

50. Let

$$\Omega = \{(x, y, z) : x^2 + y^2 \leq 1, 0 \leq z \leq 2\}.$$

What is

$$\iiint_{\Omega} z(x^2 + y^2) dV?$$

- (A) 4π
- (B) $\frac{\pi}{2}$
- (C) 2π
- (D) π
- (E) $\frac{2\pi}{3}$

51. Let $F = \mathbb{F}_5[x]/(x^2 + 2)$, and let α be the residue class of x in F . Which element is the multiplicative inverse of $1 + \alpha$?

- (A) $4 + \alpha$
- (B) $2 + 2\alpha$
- (C) $3 + 2\alpha$
- (D) $2 + 3\alpha$
- (E) $1 + 4\alpha$

Questions 52–54

52. Newton's method is applied to $f(x) = x^2 - 3$, starting with $x_0 = 2$. What is x_2 ?

- (A) $\frac{7}{4}$
- (B) $\frac{13}{8}$
- (C) $\frac{95}{56}$
- (D) $\frac{97}{56}$
- (E) $\frac{24}{7}$

53. Let y satisfy

$$x^2 y'' - 2xy' + 2y = 0, \quad x > 0, \quad y(1) = 1, \quad y'(1) = 0.$$

What is $y(3)$?

- (A) 0
- (B) -3
- (C) 9
- (D) 3
- (E) -6

54. The value of

$$\int_{-2}^2 |x^2 - 1| dx$$

is

- (A) $\frac{4}{3}$
- (B) 8
- (C) $\frac{8}{3}$
- (D) $\frac{16}{3}$
- (E) 4

Questions 55–57

55. On the real vector space of polynomials of degree at most 3, define

$$(Tp)(x) = xp'(x) - 2p(x).$$

What is the rank of $T^2 - T$?

- (A) 0
- (B) 2
- (C) 1
- (D) 3
- (E) 4

56. On $X = (0, 1)$ define

$$d(x, y) = \left| \ln \frac{x}{1-x} - \ln \frac{y}{1-y} \right|.$$

Which statements about (X, d) are true?

- I. It is complete.
- II. It is compact.
- III. It is connected.

- (A) I only
- (B) I, II, and III
- (C) II and III only
- (D) III only
- (E) I and III only

57. Among all continuous real-valued functions f on $[0, 1]$ satisfying

$$\int_0^1 f(x)^2 dx = 1,$$

what is the largest possible value of

$$\int_0^1 (2x - 1)f(x) dx?$$

- (A) $\frac{\sqrt{6}}{3}$
- (B) $\frac{1}{2}$
- (C) $\frac{1}{3}$
- (D) 1
- (E) $\frac{\sqrt{3}}{3}$

Questions 58–60

58. Let C be any piecewise smooth path from $(0, 0)$ to $(\pi, 1)$. What is

$$\int_C ((2xy + \cos x) dx + (x^2 + 2y) dy) ?$$

- (A) π^2
- (B) $1 + 2\pi$
- (C) $-1 + \pi^2$
- (D) $1 + \pi^2$
- (E) 1

59. A nonzero complex number z satisfies $z + z^{-1} = 1$. What is $z^{2026} + z^{-2026}$?

- (A) 1
- (B) 2
- (C) -1
- (D) -2
- (E) 0

60. Random variables X, Y satisfy

$$\text{Var}(X) = 4, \quad \text{Var}(Y) = 9, \quad \text{Cov}(X, Y) = 3.$$

As t ranges over $\mathbb{R}$, what is the minimum value of $\text{Var}(X - tY)$?

- (A) 3
- (B) 1
- (C) $\frac{13}{4}$
- (D) 2
- (E) 4

Questions 61–63

61. A differentiable function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ satisfies

$$f(tx, ty) = t^3 f(x, y) \quad \text{for all } t > 0.$$

If $f(1, 2) = 4$ and $f_x(1, 2) = 2$, what is $f_y(1, 2)$?

- (A) 4
- (B) 6
- (C) 2
- (D) 10
- (E) 5

62. For exactly which real values of a does

$$\int_0^\infty \frac{x^{a-1}}{(1+x)^2} dx$$

converge?

- (A) $0 \leq a \leq 2$
- (B) $0 < a < 2$
- (C) $a > 0$
- (D) $a < 2$
- (E) $-1 < a < 1$

63. What is the sum of all positive real solutions $x \neq 1$ of

$$\log_2 x + \log_x 2 = \frac{5}{2}?$$

- (A) 6
- (B) $\sqrt{2} + 2$
- (C) $4\sqrt{2}$
- (D) $\sqrt{2} + 4$
- (E) $\frac{5}{2}$

Questions 64–66

64. Let $P(m)$ be a statement about an integer m . Which statement is the logical negation of the following assertion?

For every integer n , there is an integer $m > n$ such that $P(m)$ holds.

- (A) There is an integer n such that $P(m)$ holds for every integer $m > n$.
- (B) There is an integer n such that $P(m)$ is false for every integer $m > n$.
- (C) For every integer m , the statement $P(m)$ is false.
- (D) For every integer n , there is an integer $m > n$ such that $P(m)$ is false.
- (E) There is an integer m such that $P(m)$ is false.

65. What is the curvature of the graph $y = \ln(\cos x)$ at $x = \pi/4$?

- (A) $\frac{\sqrt{2}}{2}$
- (B) 2
- (C) $\sqrt{2}$
- (D) $\frac{1}{2}$
- (E) 1

66. A real 3×3 matrix A satisfies

$$\operatorname{tr} A = 0, \quad \det A = 2.$$

What is $\operatorname{tr}(A^3)$?

- (A) -6
- (B) 8
- (C) 0
- (D) 6
- (E) 2

Answer Key

Practice Test 04 — correct choices only.

Question	Answer	Question	Answer	Question	Answer
1	C	23	B	45	C
2	A	24	D	46	A
3	A	25	B	47	D
4	B	26	E	48	E
5	C	27	A	49	E
6	B	28	C	50	D
7	E	29	B	51	D
8	E	30	C	52	D
9	C	31	C	53	B
10	E	32	D	54	E
11	A	33	D	55	B
12	C	34	C	56	E
13	E	35	A	57	E
14	E	36	B	58	D
15	E	37	B	59	C
16	C	38	C	60	A
17	A	39	E	61	E
18	C	40	A	62	B
19	A	41	A	63	D
20	A	42	D	64	B
21	D	43	B	65	A
22	B	44	D	66	D

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