

GRE Mathematics Practice Test 05

Standard-Level Simulation

66 questions 170 minutes A–E

Instructions

Choose the single best answer to each question. No calculator is permitted. Begin timing when you start Question 1. Use the blank space on the question pages for scratch work.

All angles are measured in radians unless stated otherwise. The notation $\ln$ denotes the natural logarithm, and $\mathbb{N} = \{1, 2, 3, \dots\}$. Norms are Euclidean unless specified. I_n is the identity matrix, $M_n(\mathbb{R})$ the real $n \times n$ matrices, and $\mathbb{F}_p$ the field with p elements.

Difficulty

A standard-level simulation combining routine questions with selected conceptual challenges.

Answer key

The complete answer key follows Question 66. Correct choices are provided without worked solutions.

Coverage

33 calculus questions, 17 algebra questions, and 16 questions on additional topics.

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Questions 1–3

1. Which is an antiderivative of $\frac{1}{1+4x^2}$? Here C is an arbitrary constant.

- (A) $\frac{1}{2} \arctan(2x) + C$
- (B) $\frac{1}{4} \ln(1+4x^2) + C$
- (C) $2 \arctan(2x) + C$
- (D) $\arctan(2x) + C$
- (E) $\arctan(4x) + C$

2. If a, b are distinct nonzero real numbers with $a + b \neq 0$, then

$$\frac{a^{-1} - b^{-1}}{a - b} =$$

- (A) $-\frac{1}{a+b}$
- (B) $\frac{1}{ab}$
- (C) $\frac{a+b}{ab}$
- (D) $-\frac{1}{ab}$
- (E) $\frac{1}{a+b}$

3. Two fair six-sided dice are rolled independently. Given that their sum is at least 10, what is the probability that both dice show the same number?

- (A) $\frac{1}{2}$
- (B) $\frac{1}{3}$
- (C) $\frac{1}{6}$
- (D) $\frac{1}{4}$
- (E) $\frac{2}{3}$

Questions 4–6

4. Let $f(x) = xe^x$ for $x \geq 0$, and let $g = f^{-1}$. What is $g'(e)$?

- (A) $\frac{1}{2e}$
- (B) $\frac{2}{e}$
- (C) $\frac{e}{2}$
- (D) e^{-1}
- (E) $2e$

5. What is the value of

$$\lim_{x \rightarrow 0} \frac{\tan(3x)}{\sin(2x)} ?$$

- (A) 1
- (B) 6
- (C) 3
- (D) $\frac{3}{2}$
- (E) $\frac{2}{3}$

6. Which set is a basis for the null space of

$$\begin{pmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ 1 & 1 & 0 \end{pmatrix} ?$$

- (A) $\{(1, -1, 1)\}$
- (B) $\{(1, 0, 1), (0, 1, 2)\}$
- (C) $\{(1, 1, 1)\}$
- (D) $\{(-1, 1, 1)\}$
- (E) $\{(0, 0, 1)\}$

Questions 7–9

7. What is the coefficient of x^4 in the Taylor series about 0 of

$$\frac{\cos x}{1-x}?$$

- (A) $\frac{1}{2}$
- (B) $\frac{11}{24}$
- (C) $\frac{13}{24}$
- (D) $\frac{1}{24}$
- (E) $\frac{25}{24}$

8. For $a_n = (-1)^n + 1/n$, what is $\limsup_{n \rightarrow \infty} a_n$?

- (A) -1
- (B) $\frac{1}{2}$
- (C) 1
- (D) 2
- (E) 0

9. Which equation describes the tangent plane to

$$x^2 + 2y^2 + z^2 = 4$$

at $(1, 1, 1)$?

- (A) $x + 2y + z = 4$
- (B) $x + 4y + z = 6$
- (C) $x + 2y + z = 0$
- (D) $2x + y + 2z = 5$
- (E) $x + y + z = 3$

Questions 10–12

10. Let y satisfy $y' = 3x^2y$ and $y(0) = 2$. What is $y(1)$?

- (A) $2e$
- (B) $6e$
- (C) e^3
- (D) e
- (E) $2e^3$

11. What is the order of the element 6 in the additive group $\mathbb{Z}_{20}$?

- (A) 4
- (B) 20
- (C) 5
- (D) 10
- (E) 2

12. Of ten distinct books, four are mathematics books and six are history books. How many five-book selections contain exactly two mathematics books?

- (A) 150
- (B) 120
- (C) 60
- (D) 90
- (E) 126

Questions 13–15

13. A differentiable function $y = y(x) > 0$ satisfies

$$xy + \ln y = 1, \quad y(1) = 1.$$

What is $y'(1)$?

- (A) $\frac{1}{2}$
- (B) 0
- (C) 1
- (D) -1
- (E) $-\frac{1}{2}$

14. The value of

$$\int_0^{\pi} x \cos x \, dx$$

is

- (A) -2
- (B) 2
- (C) $-\pi$
- (D) 0
- (E) π

15. What is the largest number of linearly independent eigenvectors of

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

over $\mathbb{R}$?

- (A) 2
- (B) 3
- (C) 0
- (D) 1
- (E) 4

Questions 16–18

16. The value of $(1 + i)^8$ is

- (A) -8
- (B) 16
- (C) -16
- (D) 1
- (E) 8

17. The value of

$$\int_0^1 \int_0^2 (x + y) \, dy \, dx$$

is

- (A) 6
- (B) 3
- (C) $\frac{3}{2}$
- (D) 4
- (E) 2

18. What is the maximum value of xe^{-x} for $x \geq 0$?

- (A) e
- (B) $\frac{1}{2e}$
- (C) 1
- (D) e^{-1}
- (E) $\frac{2}{e}$

Questions 19–21

19. A real 3×3 matrix A has determinant -2 . What is $\det(3A^{-1})$?

- (A) $\frac{27}{2}$
- (B) -6
- (C) $-\frac{3}{2}$
- (D) -54
- (E) $-\frac{27}{2}$

20. A continuous function $f : [0, 1] \rightarrow \mathbb{R}$ satisfies $f(x) > 0$ for every $x \in [0, 1]$. Which statement must be true?

- (A) f is increasing on $[0, 1]$.
- (B) f is differentiable at every point of $(0, 1)$.
- (C) The function $1/f$ is unbounded on $[0, 1]$.
- (D) $f(0) = f(1)$.
- (E) There is a real number $c > 0$ such that $f(x) \geq c$ for every $x \in [0, 1]$.

21. Positive differentiable functions $x = x(t)$ and $y = y(t)$ satisfy $xy = 12$. At an instant when $x = 3$ and $dx/dt = 1$, what is dy/dt ?

- (A) $-\frac{1}{3}$
- (B) $-\frac{4}{3}$
- (C) -4
- (D) $-\frac{3}{4}$
- (E) $\frac{4}{3}$

Questions 22–24

22. What is the value of

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \sqrt{1 - \frac{k^2}{n^2}}?$$

- (A) $\frac{2}{3}$
- (B) $\frac{\pi}{4}$
- (C) $\frac{1}{2}$
- (D) $\frac{\pi}{2}$
- (E) 1

23. What is the least positive integer x satisfying $7x \equiv 5 \pmod{11}$?

- (A) 9
- (B) 4
- (C) 7
- (D) 3
- (E) 5

24. A simple graph G has eight vertices, each of degree three. Its complement has the same vertices, with an edge between distinct vertices exactly when G does not have that edge. How many edges does the complement have?

- (A) 12
- (B) 16
- (C) 24
- (D) 28
- (E) 20

Questions 25–27

25. Let y solve

$$y'' - 4y = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

What is $y(\ln 2)$?

- (A) $\frac{9}{4}$
- (B) $\frac{5}{4}$
- (C) $\frac{15}{8}$
- (D) $\frac{17}{8}$
- (E) $\frac{7}{4}$

26. For the function

$$f(x, y) = x^2 - y^2 + 2x^2y^2,$$

how is the origin classified?

- (A) A strict local minimum
- (B) A point at which the gradient is nonzero
- (C) A saddle point
- (D) A strict local maximum
- (E) A global minimum

27. In the ring $R = \mathbb{Q}[x]/(x^2)$, let α denote the residue class of x . What is the multiplicative inverse of $1 + 2\alpha$?

- (A) $1 - 2\alpha$
- (B) $1 + 4\alpha$
- (C) $1 + 2\alpha$
- (D) $2 - \alpha$
- (E) $1 - \alpha$

Questions 28–30

28. A random variable X has the binomial distribution with parameters $n = 6$ and $p = 1/3$. What is $\mathbb{E}[X^2]$?

- (A) $\frac{16}{3}$
- (B) 6
- (C) $\frac{20}{3}$
- (D) $\frac{4}{3}$
- (E) 4

29. The value of

$$\int_0^{\pi} \sin^2 x \, dx$$

is

- (A) π
- (B) $\frac{1}{2}$
- (C) 2
- (D) 2π
- (E) $\frac{\pi}{2}$

30. The triangular region bounded by $y = x$, $y = 2x$, and $x = 1$ is revolved about the y -axis. What is the volume of the resulting solid?

- (A) π
- (B) $\frac{4\pi}{3}$
- (C) $\frac{2\pi}{3}$
- (D) 2π
- (E) $\frac{\pi}{3}$

Questions 31–33

31. What is the squared Euclidean distance from $(1, 2, 3)$ to the line spanned by $(1, 1, 1)$?

- (A) 14
- (B) 1
- (C) 6
- (D) 3
- (E) 2

32. For propositions P, Q, R , which expression is logically equivalent to $\neg(P \wedge (Q \vee R))$?

- (A) $P \vee (\neg Q \wedge \neg R)$
- (B) $\neg P \vee (\neg Q \wedge \neg R)$
- (C) $\neg P \wedge \neg Q \wedge \neg R$
- (D) $\neg P \vee \neg Q \vee \neg R$
- (E) $\neg P \wedge (\neg Q \vee \neg R)$

33. What is the value of

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^2}{x^2 + y^2} ?$$

- (A) -1
- (B) $\frac{1}{2}$
- (C) 1
- (D) The limit does not exist.
- (E) 0

Questions 34–36

34. If

$$F(x) = \int_{x^2}^x (1 + t^2) dt,$$

what is $F'(1)$?

- (A) 4
- (B) 2
- (C) 0
- (D) -4
- (E) -2

35. A real symmetric matrix A has eigenvectors u, v satisfying

$$Au = 2u, \quad Av = -v, \quad \|u\| = 1, \quad \|v\| = 2.$$

What is $\|u + v\|$?

- (A) 3
- (B) 5
- (C) $\sqrt{3}$
- (D) 1
- (E) $\sqrt{5}$

36. In the usual topology on $\mathbb{R}$, let

$$S = (\mathbb{Q} \cap [0, 1]) \cup (1, 2).$$

What is the boundary of S ?

- (A) $[0, 2]$
- (B) $[0, 1] \cup \{2\}$
- (C) $[0, 1]$
- (D) $\{0, 1, 2\}$
- (E) $\{0, 2\}$

Questions 37–39

37. If $f(x) = e^x \sin x$, what is $f^{(3)}(0)$?

- (A) 6
- (B) 0
- (C) 1
- (D) 3
- (E) 2

38. For exactly which real values of x does

$$\sum_{n=1}^{\infty} n!x^n$$

converge?

- (A) $|x| \leq 1$
- (B) All real values of x
- (C) $x \leq 0$
- (D) $x = 0$ only
- (E) $|x| < 1$

39. A group G , with identity e , satisfies $g^2 = e$ for every $g \in G$. Which statement must be true?

- (A) G has no proper nontrivial subgroup.
- (B) G is finite.
- (C) G is cyclic.
- (D) G has exactly two elements.
- (E) G is abelian.

Questions 40–42

40. Independent random variables X_1, X_2, X_3 each have variance $\sigma^2 > 0$. What is

$$\frac{1}{\sigma^2} \operatorname{Var}\left(\frac{X_1 + X_2 + 2X_3}{4}\right)?$$

- (A) $\frac{1}{4}$
- (B) $\frac{3}{8}$
- (C) $\frac{1}{2}$
- (D) $\frac{1}{8}$
- (E) $\frac{3}{4}$

41. Let C be the ellipse $x^2/9 + y^2/4 = 1$, oriented counterclockwise. What is

$$\oint_C (-y \, dx + x \, dy)?$$

- (A) 8π
- (B) 24π
- (C) 12π
- (D) 10π
- (E) 6π

42. A differentiable function $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f'(x) > 1$ for every real x . Which statement must be true about the equation $f(x) = x$?

- (A) It has infinitely many real solutions.
- (B) It has at least two real solutions.
- (C) It has exactly one real solution.
- (D) It has at most one real solution.
- (E) It has no real solution.

Questions 43–45

43. What is the maximum value of $2 \sin x - \cos x$ as x ranges over $\mathbb{R}$?

- (A) 2
- (B) 3
- (C) 5
- (D) $\sqrt{5}$
- (E) 1

44. For which real value of a is

$$f(x + iy) = 3x^2 - 3y^2 + i axy$$

holomorphic on all of $\mathbb{C}$?

- (A) 3
- (B) -6
- (C) 2
- (D) 0
- (E) 6

45. Let S be the boundary of the cube $0 \leq x, y, z \leq 1$, oriented outward. For $\mathbf{F}(x, y, z) = (x, y, 2z)$, what is

$$\iint_S \mathbf{F} \cdot \mathbf{n} \, dS?$$

- (A) 4
- (B) 3
- (C) 2
- (D) 8
- (E) 6

Questions 46–48

46. Which equation describes the tangent line at $t = 1$ to the parametric curve

$$x = t^2 + 1, \quad y = t^3 - t?$$

- (A) $y = x + 2$
- (B) $y = x - 2$
- (C) $x = 2$
- (D) $y = 2x - 4$
- (E) $y = -x + 2$

47. A field F has 81 elements. What is its dimension as a vector space over its prime subfield?

- (A) 81
- (B) 4
- (C) 3
- (D) 9
- (E) 27

48. How many surjective functions are there from a four-element set to a three-element set?

- (A) 18
- (B) 81
- (C) 24
- (D) 48
- (E) 36

Questions 49–51

49. For exactly which real values of p does

$$\int_e^{\infty} \frac{dx}{x(\ln x)^p}$$

converge?

- (A) $p > 1$
- (B) $p > 0$
- (C) $p \neq 1$
- (D) $p \geq 1$
- (E) $p < 1$

50. Let $B = \{(x, y, z) : x^2 + y^2 + z^2 \leq 1\}$. What is

$$\iiint_B (x + 2y + 3z + 4) \, dV ?$$

- (A) 8π
- (B) 4π
- (C) $\frac{16\pi}{3}$
- (D) $\frac{4\pi}{3}$
- (E) $\frac{8\pi}{3}$

51. Which is the minimal polynomial over $\mathbb{Q}$ of $1 + \sqrt{3}$?

- (A) $t^2 - 3$
- (B) $t^2 - 4$
- (C) $t^2 - 2t - 2$
- (D) $t^2 - 2t + 2$
- (E) $t^2 + 2t - 2$

Questions 52–54

52. The bisection method starts with an interval of length 1. What is the smallest number of bisections after which the remaining interval is guaranteed to have length at most 10^{-3} ?
- (A) 9
(B) 8
(C) 12
(D) 10
(E) 11
53. Which is the general real solution of $y'' + 2y' + 5y = 0$? Here C_1, C_2 are arbitrary real constants.
- (A) $e^{-x}(C_1 \cos 2x + C_2 \sin 2x)$
(B) $e^{-2x}(C_1 \cos x + C_2 \sin x)$
(C) $C_1 e^{-x} + C_2 e^{-5x}$
(D) $C_1 \cos(\sqrt{5}x) + C_2 \sin(\sqrt{5}x)$
(E) $e^x(C_1 \cos 2x + C_2 \sin 2x)$
54. What is the minimum distance from the point $(0, 1)$ to the parabola $y = x^2$?
- (A) 1
(B) $\frac{\sqrt{3}}{2}$
(C) $\frac{1}{2}$
(D) $\frac{\sqrt{2}}{2}$
(E) $\sqrt{2}$

Questions 55–57

55. What is the remainder when $x^{10} + x^5 + 1$ is divided by $x^2 - 1$?

- (A) $x + 2$
- (B) $2x + 2$
- (C) $2x + 1$
- (D) 3
- (E) $x + 1$

56. Let X be an infinite set with the discrete topology. Which statements are true?

- I. Every subset of X is open.
- II. Every compact subset of X is finite.
- III. X is connected.

- (A) II and III only
- (B) II only
- (C) I, II, and III
- (D) I and II only
- (E) I only

57. What is the value of

$$\lim_{x \rightarrow +\infty} x(\sqrt{x^2 + 1} - x) ?$$

- (A) 0
- (B) 1
- (C) $\frac{1}{2}$
- (D) $\frac{1}{4}$
- (E) 2

Questions 58–60

58. Let C be any piecewise smooth path from $(1, 0)$ to $(1, \ln 2)$. What is

$$\int_C (ye^{xy} dx + xe^{xy} dy) ?$$

- (A) 0
- (B) 2
- (C) $\ln(2)$
- (D) 1
- (E) $2 \ln(2)$

59. Let V be the real vector space of polynomials of degree at most 4, and define $T : V \rightarrow V$ by $T(p) = p''$. What is $\text{rank } T$?

- (A) 4
- (B) 5
- (C) 3
- (D) 2
- (E) 1

60. Independent random variables U, V are uniformly distributed on $[0, 1]$. What is $\mathbb{P}(UV \leq 1/2)$?

- (A) $\frac{\ln(2)}{2} + \frac{1}{2}$
- (B) $\frac{1}{2}$
- (C) $\frac{3}{4}$
- (D) $1 - \frac{\ln(2)}{2}$
- (E) $\ln(2)$

Questions 61–63

61. Let $D = \{(x, y) : 0 \leq x + y \leq 2, 0 \leq x - y \leq 1\}$. What is

$$\iint_D x \, dx \, dy?$$

- (A) $\frac{1}{4}$
- (B) $\frac{3}{2}$
- (C) $\frac{3}{4}$
- (D) $\frac{1}{2}$
- (E) 1

62. The value of

$$\int_0^1 \max\{x^2, 1 - x\} \, dx$$

is

- (A) $\frac{2}{3}$
- (B) $-\frac{1}{12} + \frac{5\sqrt{5}}{12}$
- (C) $-\frac{1}{4} + \frac{5\sqrt{5}}{12}$
- (D) $-\frac{7}{12} + \frac{5\sqrt{5}}{12}$
- (E) $\frac{1}{6} + \frac{\sqrt{5}}{6}$

63. What is the smallest positive integer n for which $n!$ is divisible by 10^6 ?

- (A) 24
- (B) 30
- (C) 25
- (D) 26
- (E) 20

Questions 64–66

64. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be continuous, with

$$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow +\infty} f(x) = 0.$$

Which statements must be true?

- I. f is bounded on $\mathbb{R}$.
- II. f is uniformly continuous on $\mathbb{R}$.
- III. f attains a maximum value on $\mathbb{R}$.

- (A) I, II, and III
- (B) II only
- (C) I only
- (D) I and III only
- (E) I and II only

65. What is the total area enclosed by the cardioid $r = 2 + 2 \cos \theta$?

- (A) 6π
- (B) 4π
- (C) 3π
- (D) 12π
- (E) 8π

66. Real 3×3 matrices A, B satisfy $AB = 0$, and $A + B$ is invertible. What is $\text{rank } A + \text{rank } B$?

- (A) 4
- (B) 6
- (C) 1
- (D) 3
- (E) 2

Answer Key

Practice Test 05 — correct choices only.

Question	Answer	Question	Answer	Question	Answer
1	A	23	C	45	A
2	D	24	B	46	B
3	B	25	D	47	B
4	A	26	C	48	E
5	D	27	A	49	A
6	D	28	A	50	C
7	C	29	E	51	C
8	C	30	C	52	D
9	A	31	E	53	A
10	A	32	B	54	B
11	D	33	E	55	A
12	B	34	E	56	D
13	E	35	E	57	C
14	A	36	B	58	D
15	A	37	E	59	C
16	B	38	D	60	A
17	B	39	E	61	C
18	D	40	B	62	C
19	E	41	C	63	C
20	E	42	D	64	E
21	B	43	D	65	A
22	B	44	E	66	D

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