

GRE Mathematics Practice Test 06

Advanced Challenge

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

The most demanding form in this five-test cycle. Emphasis on connected ideas, non-routine reasoning, and careful conceptual distinctions. This is cycle 1 of 19, position 1 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Evaluate $\int_0^1 \int_0^x x^2 y^6 dy dx$.

- (A) $\frac{1}{63}$
- (B) $\frac{1}{80}$
- (C) $\frac{1}{70}$
- (D) $\frac{1}{10}$
- (E) $\frac{1}{21}$

2. A 3×3 matrix A has characteristic polynomial $(t - 2)(t + 1)(t - 2)$. What is $\text{tr}(A^3 - 2A + I)$?

- (A) 13
- (B) 10
- (C) 11
- (D) 14
- (E) 12

3. If $y' + 2y = 6x^6 e^{-2x}$ and $y(0) = 2$, find $y(1)$.

- (A) $\frac{40}{7e^2}$
- (B) $\frac{20}{7e^2} + 1$
- (C) $-\frac{20}{7e^2}$
- (D) $\frac{10}{7e^2}$
- (E) $\frac{20}{7e^2}$

Questions 4–6

4. A box is chosen uniformly from two boxes. Box A contains 3 red and 5 blue balls; box B contains 5 red and 3 blue balls. Two balls are drawn without replacement and both are red. What is the conditional probability that A was chosen?

- (A) $\frac{3}{26}$
- (B) $\frac{8}{13}$
- (C) $\frac{3}{13}$
- (D) $\frac{1}{13}$
- (E) $\frac{10}{13}$

5. Let $\mathbf{F} = (2y^6, z^6, x^6)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (1, 1, 2)/\sqrt{6}$.

- (A) $1 - 6\sqrt{6}$
- (B) $6\sqrt{6}$
- (C) $-6\sqrt{6}$
- (D) $-3\sqrt{6}$
- (E) $-12\sqrt{6}$

6. Let N be the 8×8 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\dim(\ker N^{14} / \ker N^7)$.

- (A) 1
- (B) 0
- (C) 3
- (D) 4
- (E) 2

Questions 7–9

7. The local inverse $y(x)$ of $x = 2y + y^3$ satisfies $y(0) = 0$. Find $y^{(7)}(0)$.

- (A) $\frac{60480}{2^{10}}$
- (B) $\frac{1}{2} \left(-\frac{60480}{2^{10}} \right)$
- (C) $\frac{1}{5040} \left(-\frac{60480}{2^{10}} \right)$
- (D) $2 \left(-\frac{60480}{2^{10}} \right)$
- (E) $-\frac{60480}{2^{10}}$

8. Let $X_1, \dots, X_3$ be independent uniform random variables on $[0, 1]$. What is $E[(\max_i X_i)^2]$?

- (A) $\frac{4}{5}$
- (B) $\frac{1}{5}$
- (C) $\frac{3}{5}$
- (D) $\frac{3}{10}$
- (E) $\frac{2}{5}$

9. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 2z^2 = x + 6y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) -4
- (B) -24
- (C) 24
- (D) -12
- (E) -6

Questions 10–12

10. Let A be the 8×8 matrix whose diagonal entries are 5 and whose off-diagonal entries are 1. What is its largest eigenvalue?

- (A) 13
- (B) 12
- (C) 14
- (D) 11
- (E) 10

11. Let $f(x) = \log(1 + 2x)$. Find $f^{(6)}(0)$.

- (A) $\frac{1}{6} \left((-1)^5 5! 2^6 \right)$
- (B) $6 \left((-1)^5 5! 2^6 \right)$
- (C) $-(-1)^5 5! 2^6$
- (D) $(-1)^5 5! 2^6$
- (E) $\frac{1}{2} \left((-1)^5 5! 2^6 \right)$

12. Independent trials have success probability $1/3$. Let T count trials up to and including the first success. Find $E[T^2]$.

- (A) 10
- (B) 25
- (C) 5
- (D) 20
- (E) 15

Questions 13–15

13. Let $F(x) = \int_0^x (x-t)^5 e^{2t} dt$. Find $F^{(6)}(0)$.

- (A) $\frac{1}{120}$
- (B) 120
- (C) 0
- (D) 720
- (E) 240

14. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 10$ and $x + y = 7$. Find $x^2 + y^2$.

- (A) 29
- (B) 28
- (C) 31
- (D) 27
- (E) 30

15. Evaluate $\lim_{x \rightarrow 0} \frac{e^{2x} - \sum_{j=0}^5 (2x)^j / j!}{x^6}$.

- (A) $6 \left(\frac{2^6}{6!} \right)$
- (B) $\frac{1}{2} \left(\frac{2^6}{6!} \right)$
- (C) $-\frac{2^6}{6!}$
- (D) $720 \left(\frac{2^6}{6!} \right)$
- (E) $\frac{2^6}{6!}$

Questions 16–18

16. A uniformly random subset of 3 objects is selected from a collection with 4 red and 5 blue objects. If X is the number of red objects selected, find $\text{Var}(X)$.

- (A) $\frac{5}{18}$
- (B) $\frac{4}{9}$
- (C) $\frac{5}{27}$
- (D) $\frac{7}{9}$
- (E) $\frac{5}{9}$

17. The solution of $y' = 2x^6y^3$ has $y(0) = 1/3$. Find $y(1)$.

- (A) $\frac{2\sqrt{413}}{59}$
- (B) $\frac{\sqrt{413}}{59}$
- (C) $-\frac{\sqrt{413}}{59}$
- (D) $\frac{\sqrt{413}}{118}$
- (E) $\frac{\sqrt{413}}{59} + 1$

18. A sequence satisfies $u_0 = 2$ and $u_{k+1} = 2u_k + 5$ for all $k \geq 0$. Find u_7 .

- (A) 893
- (B) 892
- (C) 891
- (D) 889
- (E) 890

Questions 19–21

19. Find the coefficient of x^6 in the Taylor series of $(1 - 2x)^{-2}$ at zero.

- (A) $2(7 \cdot 2^6)$
- (B) $\frac{1}{2}(7 \cdot 2^6)$
- (C) $\frac{1}{7}(7 \cdot 2^6)$
- (D) $7 \cdot 2^6$
- (E) $\frac{6}{7}(7 \cdot 2^6)$

20. X, Y are independent Poisson variables with means 3 and 4. Given $X+Y = 5$, find $\text{Var}(X \mid X+Y)$.

- (A) $\frac{120}{49}$
- (B) $\frac{30}{49}$
- (C) $\frac{60}{49}$
- (D) $\frac{109}{49}$
- (E) $\frac{158}{49}$

21. For $\mathbf{F} = (2x^6y, y^6z, xz^6)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) 222
- (B) 111
- (C) 444
- (D) -222
- (E) 223

Questions 22–24

22. Linear maps $T : \mathbb{R}^9 \rightarrow \mathbb{R}^9$ and $U : \mathbb{R}^9 \rightarrow \mathbb{R}^9$ have ranks 2 and 3. What is the largest possible rank of UT ?

- (A) 2
- (B) 1
- (C) 3
- (D) 0
- (E) 4

23. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 2x$ and has density $\rho(x, y) = x^4y$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{15}{8}$
- (B) $\frac{7}{8}$
- (C) $-\frac{7}{8}$
- (D) $\frac{7}{4}$
- (E) $\frac{7}{16}$

24. For which real a does $f_m(x) = m^a x^3(1-x)^m$ converge uniformly to zero on $[0, 1]$?

- (A) All real a
- (B) $a < 3$
- (C) $a < 0$
- (D) $a > 3$
- (E) $a \leq 3$

Questions 25–27

25. Evaluate $\int_0^{\log 2} e^x (e^x + 2)^6 dx$.

- (A) $-\frac{4^7 - 3^7}{7}$
- (B) $\frac{4^7 - 3^7}{7}$
- (C) $7 \left(\frac{4^7 - 3^7}{7} \right)$
- (D) $4^6 - 3^6$
- (E) $\frac{4^7}{7}$

26. Fix a nonzero vector $u \in \mathbb{R}^8$. What is the dimension of the space of skew-symmetric 8×8 matrices A satisfying $Au = 0$?

- (A) 23
- (B) 20
- (C) 22
- (D) 21
- (E) 19

27. Let C be the unit circle $x^2 + y^2 = 1$, $z = 6$, counterclockwise viewed from above. Evaluate $\int_C -2y(x^2 + y^2)^5 dx + (2x(x^2 + y^2)^5) dy + 2z dz$.

- (A) -4π
- (B) $1 + 4\pi$
- (C) 4π
- (D) 2π
- (E) 8π

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{3p}]$ converge?

- (A) $p < 1/3$
- (B) $p > 0$
- (C) $p \geq 1/3$
- (D) All real p
- (E) $p > 1/3$

29. For $f(x) = x^6 e^{2x}$, find $f^{(8)}(0)$.

- (A) $\frac{8! 2^2}{2}$
- (B) $\frac{1}{2} \left(\frac{8! 2^2}{2} \right)$
- (C) $2 \left(\frac{8! 2^2}{2} \right)$
- (D) $\frac{1}{20160} \left(\frac{8! 2^2}{2} \right)$
- (E) $\frac{1}{56} \left(\frac{8! 2^2}{2} \right)$

30. Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 3)$ entry of A^3 ?

- (A) 30
- (B) 29
- (C) 32
- (D) 31
- (E) 28

Questions 31–33

31. Find the curvature of the helix $r(t) = (2 \cos t, 2 \sin t, 6t)$.

- (A) $-\frac{1}{20}$
- (B) $\frac{1}{20}$
- (C) $\frac{1}{10}$
- (D) $\frac{21}{20}$
- (E) $\frac{1}{40}$

32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{3 + 1/k : k \geq 1\}$ have?

- (A) 1
- (B) 3
- (C) 0
- (D) 4
- (E) 2

33. For the curve $x = t^6$, $y = t^7 + 2t$, find d^2y/dx^2 at $t = 1$.

- (A) $-\frac{1}{2}$
- (B) $-\frac{1}{12}$
- (C) $\frac{17}{36}$
- (D) $\frac{3}{2}$
- (E) $-\frac{1}{3}$

Questions 34–36

34. An invertible 8×8 matrix A has determinant 2. Find $\det(\operatorname{adj} A)$.

- (A) 128
- (B) 129
- (C) 126
- (D) 130
- (E) 127

35. Evaluate $\int_0^1 \int_x^1 2y^6 \, dy \, dx$.

- (A) $\frac{1}{3}$
- (B) $\frac{1}{28}$
- (C) $\frac{2}{7}$
- (D) $\frac{1}{4}$
- (E) 1

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 3\}$ have?

- (A) 3
- (B) 5
- (C) 2
- (D) 6
- (E) 4

Questions 37–39

37. Let C be $x^2/36 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C \left(-\frac{2y^3}{3}\right) dx + \left(\frac{x^3}{3}\right) dy$.

- (A) 57π
- (B) $1 + 57\pi$
- (C) -57π
- (D) $\frac{57\pi}{2}$
- (E) 114π

38. On P_9 , the real polynomials of degree at most 9, define $T(p) = x^2p'' - 14xp' + 56p$. Find its rank.

- (A) 9
- (B) 6
- (C) 10
- (D) 8
- (E) 7

39. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^2 y^5 dV$.

- (A) $10 \left(\frac{2! 5!}{10!} \right)$
- (B) $\frac{2! 5!}{10!}$
- (C) $\frac{1}{6} \left(\frac{2! 5!}{10!} \right)$
- (D) $\frac{1}{18}$
- (E) $6 \left(\frac{2! 5!}{10!} \right)$

Questions 40–42

40. The circle $|z| = 2$ is traversed counterclockwise. What is $\frac{1}{2\pi i} \int_{|z|=2} \frac{e^z}{z^4} dz$?
- (A) $1/6!$
(B) $3!$
(C) $1/2!$
(D) $1/3!$
(E) $1/4!$
41. Under $x = u + 2v$, $y = u - 6v$, what is the area of the image of $0 \leq u, v \leq 1$?
- (A) $\frac{1}{8}$
(B) 64
(C) 8
(D) 4
(E) 12
42. In the additive group $\mathbb{Z}/24\mathbb{Z}$, find the index of the subgroup generated by $[2]$.
- (A) 0
(B) 3
(C) 1
(D) 4
(E) 2

Questions 43–45

43. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-2)^j}{j^{\frac{5}{3}} (\log j)^m}$ converge absolutely at $x = 1$? Give the least such m , or 0 if none exists.

- (A) 4
- (B) 1
- (C) 3
- (D) 2
- (E) 0

44. Find the residue of $z^3/(z-1)^3$ at $z = 1$.

- (A) 3
- (B) 5
- (C) 4
- (D) 1
- (E) 2

45. The solution of $y' = 2x^5 y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\sqrt[6]{3}$
- (B) 3
- (C) $\frac{2^{\frac{5}{6}}}{2}$
- (D) $\frac{3^{\frac{5}{6}}}{3}$
- (E) $\frac{2^{\frac{5}{6}} \sqrt[6]{3}}{2}$

Questions 46–48

46. How many invertible 4×4 matrices over $\mathbb{F}_2$ are there whose first column is $(1, 0, 0, 0)^T$?

- (A) $\frac{(2^4 - 2^0)(2^4 - 2^0)(2^4 - 2^0)(2^4 - 2^0)}{2^4 - 1}$
(B) $\frac{(2^4 - 2^1)(2^4 - 2^2)(2^4 - 2^3)(2^4 - 2^4)}{2^4 - 1}$
(C) $\frac{(2^4 - 2^0)(2^4 - 2^1)(2^4 - 2^2)(2^4 - 2^3)}{2^4 - 1}$
(D) $\frac{2^4 2^4 2^4 2^4}{2^4 - 1}$
(E) $\frac{(2^4 - 2^0)(2^4 - 2^1)(2^4 - 2^2)}{2^4 - 1}$

47. For $x > 0$, $x^2 y'' + (-9)xy' + 16y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $-\frac{(1 - e^6)e^2}{3}$
(B) $-\frac{e^2}{6} + 1 + \frac{e^8}{6}$
(C) $\frac{(1 - e^6)e^2}{6}$
(D) $-\frac{(1 - e^6)e^2}{6}$
(E) $-\frac{(1 - e^6)e^2}{12}$

48. How many functions from a set of 3 elements onto a set of 3 elements exist?

- (A) $3^3 - 2^3$
(B) $3^3 - 3(2^3) + 3$
(C) $3^3 - 3(2^3)$
(D) 3^3
(E) $3^3 - 3(2^3) - 3$

Questions 49–51

49. Find the arc length of $r(t) = (2(t - t^3/3), 2t^2)$ for $0 \leq t \leq 1/6$.

- (A) $\frac{433}{324}$
- (B) $\frac{109}{162}$
- (C) $\frac{648}{109}$
- (D) $-\frac{109}{324}$
- (E) $\frac{109}{324}$

50. How many permutations in S_8 consist of one 4-cycle and 4 fixed points?

- (A) 421
- (B) 422
- (C) 419
- (D) 420
- (E) 418

51. Evaluate $\int_0^\infty x^6 e^{-2x} dx$.

- (A) $\frac{1}{2} \left(\frac{6!}{2^7} \right)$
- (B) $\frac{6!}{2^7}$
- (C) $\frac{1}{6} \left(\frac{6!}{2^7} \right)$
- (D) $128 \left(\frac{6!}{2^7} \right)$
- (E) $2 \left(\frac{6!}{2^7} \right)$

Questions 52–54

52. How many nonnegative integer triples sum to 6, if each coordinate is at most 3?

- (A) 13
- (B) 16
- (C) 7
- (D) 4
- (E) 10

53. Solve $y' + 2y = x^6 e^{-2x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{1}{6e^2}$
- (B) $\frac{e^2}{7}$
- (C) $\frac{1}{7e^2}$
- (D) e^{-2}
- (E) $\frac{2}{7e^2}$

54. How many maximal proper subgroups does the cyclic group of order 72 have?

- (A) 4
- (B) 1
- (C) 2
- (D) 3
- (E) 0

Questions 55–57

55. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(2j)!}{(j!)^2} x^{6j}$?
- (A) $2^{-\frac{1}{3}}$
(B) 2^{-3}
(C) $\frac{1}{2}$
(D) $2^{\frac{1}{3}}$
(E) 2^{-2}
56. A tree has 3 vertices of degree 3; every other vertex has degree 1. How many leaves does it have?
- (A) 4
(B) 7
(C) 6
(D) 5
(E) 3
57. The region $0 \leq x \leq 1$, $0 \leq y \leq 2x^6$ is revolved about the line $x = -2$. Find its volume.
- (A) $1 + \frac{23\pi}{14}$
(B) $-\frac{23\pi}{14}$
(C) $\frac{23\pi}{28}$
(D) $\frac{23\pi}{7}$
(E) $\frac{23\pi}{14}$

Questions 58–60

58. How many residues x modulo 24 satisfy $8x \equiv 56 \pmod{24}$ and $2x \equiv 0 \pmod{24}$?

- (A) 0
- (B) 1
- (C) 4
- (D) 2
- (E) 3

59. Evaluate $\int_0^1 x^2(1-x)^5 dx$.

- (A) $\frac{28}{3} \left(\frac{2! 5!}{8!} \right)$
- (B) $\frac{1}{6} \left(\frac{2! 5!}{8!} \right)$
- (C) $8 \left(\frac{2! 5!}{8!} \right)$
- (D) $\frac{2! 5!}{8!}$
- (E) $3 \left(\frac{2! 5!}{8!} \right)$

60. A connected planar graph has 6 vertices and 7 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 5
- (B) 4
- (C) 3
- (D) 2
- (E) 1

Questions 61–63

61. If $y'' + 36y = 2$ and $y(0) = y'(0) = 0$, find $y(\pi/6)$.

- (A) $\frac{1}{9}$
- (B) $-\frac{1}{9}$
- (C) $\frac{1}{18}$
- (D) $\frac{2}{9}$
- (E) $\frac{10}{9}$

62. The field K has 11^2 elements. Find the largest possible multiplicative order of a nonzero element of K .

- (A) $11^2 + 1$
- (B) 11^2
- (C) $11^3 - 1$
- (D) $11^2 - 1$
- (E) $11^1 - 1$

63. If $y'' - 4y' + 4y = 6e^{2x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $6e^2$
- (B) $-3e^2$
- (C) $1 + 3e^2$
- (D) $3e^2$
- (E) $\frac{3e^2}{2}$

Questions 64–66

64. How many integers in $\{1, \dots, 18\}$ are divisible by 2 or 3?

- (A) 8
- (B) 12
- (C) 4
- (D) 20
- (E) 16

65. Let $f(x, y) = e^{2x+6y}$. Find $\partial^6 f / \partial x^5 \partial y$ at $(0, 0)$.

- (A) $\frac{1}{3} (6 \cdot 2^5)$
- (B) $\frac{1}{6} (6 \cdot 2^5)$
- (C) $81 (6 \cdot 2^5)$
- (D) $6 \cdot 2^5$
- (E) $243 (6 \cdot 2^5)$

66. Let $G = \mathbb{Z}/42\mathbb{Z} \times \mathbb{Z}/32\mathbb{Z}$. Find the order of G/H , where $H = \{g \in G : 2g = 0\}$.

- (A) 335
- (B) 334
- (C) 336
- (D) 337
- (E) 338

Answer Key

Practice Test 06 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	C	23	B	45	A
2	E	24	B	46	C
3	E	25	B	47	D
4	C	26	D	48	B
5	C	27	C	49	E
6	A	28	E	50	D
7	E	29	A	51	B
8	C	30	A	52	E
9	B	31	B	53	C
10	B	32	E	54	C
11	D	33	B	55	A
12	E	34	A	56	D
13	B	35	D	57	E
14	A	36	E	58	A
15	E	37	A	59	D
16	E	38	D	60	C
17	B	39	B	61	A
18	C	40	D	62	D
19	D	41	C	63	D
20	C	42	E	64	B
21	A	43	B	65	D
22	A	44	A	66	C

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