

GRE Mathematics Practice Test 07

Advanced Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A demanding practice form with frequent multi-step arguments and a mix of computational and conceptual questions. This is cycle 1 of 19, position 2 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. For $f(x) = x^5 e^{2x}$, find $f^{(7)}(0)$.

- (A) $\frac{7! 2^2}{2}$
- (B) $\frac{1}{2520} \left(\frac{7! 2^2}{2} \right)$
- (C) $\frac{1}{42} \left(\frac{7! 2^2}{2} \right)$
- (D) $\frac{1}{2} \left(\frac{7! 2^2}{2} \right)$
- (E) $2 \left(\frac{7! 2^2}{2} \right)$

2. Linear maps $T : \mathbb{R}^{10} \rightarrow \mathbb{R}^{10}$ and $U : \mathbb{R}^{10} \rightarrow \mathbb{R}^{10}$ have ranks 3 and 3. What is the largest possible rank of UT ?

- (A) 1
- (B) 3
- (C) 4
- (D) 2
- (E) 5

3. Evaluate $\int_0^{\log 2} e^x (e^x + 2)^5 dx$.

- (A) $6 \left(\frac{4^6 - 3^6}{6} \right)$
- (B) $4^5 - 3^5$
- (C) $\frac{4^6}{6}$
- (D) $-\frac{4^6 - 3^6}{6}$
- (E) $\frac{4^6 - 3^6}{6}$

Questions 4–6

4. A box is chosen uniformly from two boxes. Box A contains 4 red and 6 blue balls; box B contains 6 red and 4 blue balls. Two balls are drawn without replacement and both are red. What is the conditional probability that A was chosen?

- (A) $\frac{9}{14}$
- (B) $\frac{5}{7}$
- (C) $\frac{2}{7}$
- (D) $\frac{1}{7}$
- (E) $\frac{2}{21}$

5. For $x > 0$, $x^2 y'' + (1 - 7)xy' + 10y = 0$, $y(1) = 0$, $y'(1) = 2$. Find $y(e)$.

- (A) $\frac{e^2}{7}$
- (B) $-e^5 + e^2$
- (C) $\frac{2(1 - e^3)e^2}{3}$
- (D) $-\frac{2(1 - e^3)e^2}{3}$
- (E) $e^2 + e^5$

6. Fix a nonzero vector $u \in \mathbb{R}^9$. What is the dimension of the space of skew-symmetric 9×9 matrices A satisfying $Au = 0$?

- (A) 29
- (B) 27
- (C) 28
- (D) 26
- (E) 30

Questions 7–9

7. For $D = \{(x, y) : x^2 + y^2 \leq 2\}$, evaluate $\iint_D (x^2 + y^2)^4 dA$.

- (A) 32π
- (B) $\frac{32\pi}{9}$
- (C) $\frac{16\pi}{5}$
- (D) $\frac{32\pi}{5}$
- (E) $\frac{64\pi}{5}$

8. Let $X_1, \dots, X_4$ be independent uniform random variables on $[0, 1]$. What is $E[(\max_i X_i)^2]$?

- (A) $\frac{2}{9}$
- (B) $\frac{1}{3}$
- (C) $\frac{8}{9}$
- (D) $\frac{5}{6}$
- (E) $\frac{2}{3}$

9. Let $f(x, y) = e^{2x+5y}$. Find $\partial^5 f / \partial x^4 \partial y$ at $(0, 0)$.

- (A) $\frac{125}{8} (5 \cdot 2^4)$
- (B) $\frac{2}{5} (5 \cdot 2^4)$
- (C) $\frac{625}{16} (5 \cdot 2^4)$
- (D) $5 \cdot 2^4$
- (E) $\frac{1}{5} (5 \cdot 2^4)$

Questions 10–12

10. Let $A = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 3)$ entry of A^4 ?

- (A) 52
- (B) 54
- (C) 53
- (D) 56
- (E) 55

11. Evaluate $\int_0^1 \frac{x^4}{1+2x^5} dx$.

- (A) $\frac{\log(3)}{2}$
- (B) $\frac{\log(3)}{10}$
- (C) $\log(3)$
- (D) $-\frac{\log(3)}{10}$
- (E) $\frac{\log(3)}{5}$

12. Independent trials have success probability $1/4$. Let T count trials up to and including the first success. Find $E[T^2]$.

- (A) 46
- (B) 28
- (C) 37
- (D) 10
- (E) 19

Questions 13–15

13. For $\mathbf{F} = (2x^5y, y^5z, xz^5)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) 210
- (B) $\frac{105}{2}$
- (C) 106
- (D) 105
- (E) -105

14. An invertible 9×9 matrix A has determinant 3. Find $\det(\text{adj } A)$.

- (A) 6559
- (B) 6562
- (C) 6561
- (D) 6560
- (E) 6563

15. Find the total outward flux of $\mathbf{F} = (2x^5y, y^5z, xz^5)$ through the boundary of $[0, 1]^3$.

- (A) 4
- (B) -2
- (C) 1
- (D) 2
- (E) 3

Questions 16–18

16. A uniformly random subset of 4 objects is selected from a collection with 5 red and 6 blue objects. If X is the number of red objects selected, find $E[X]$.

- (A) $\frac{40}{11}$
- (B) $\frac{10}{11}$
- (C) $\frac{20}{11}$
- (D) $\frac{31}{11}$
- (E) $\frac{42}{11}$

17. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 2x$ and has density $\rho(x, y) = x^3y$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{3}{7}$
- (B) $\frac{13}{7}$
- (C) $\frac{12}{7}$
- (D) $\frac{6}{7}$
- (E) $-\frac{6}{7}$

18. On P_{10} , the real polynomials of degree at most 10, define $T(p) = x^2p'' - 16xp' + 72p$. Find its rank.

- (A) 11
- (B) 7
- (C) 8
- (D) 10
- (E) 9

Questions 19–21

19. Evaluate $\int_0^1 x^2(1-x)^4 dx$.

- (A) $\frac{2!4!}{7!}$
- (B) $\frac{1}{5} \left(\frac{2!4!}{7!} \right)$
- (C) $2 \left(\frac{2!4!}{7!} \right)$
- (D) $7 \left(\frac{2!4!}{7!} \right)$
- (E) $3 \left(\frac{2!4!}{7!} \right)$

20. X, Y are independent Poisson variables with means 4 and 5. Given $X+Y = 6$, find $\text{Var}(X \mid X+Y)$.

- (A) $\frac{40}{27}$
- (B) $\frac{67}{27}$
- (C) $\frac{80}{27}$
- (D) $\frac{20}{27}$
- (E) $\frac{94}{27}$

21. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(2x) - \sum_{j=0}^3 (-1)^j (2x)^{2j+1} / (2j+1)!}{x^9}$.

- (A) $\frac{(-1)^4 2^9}{9!}$
- (B) $-\frac{(-1)^4 2^9}{9!}$
- (C) $2 \left(\frac{(-1)^4 2^9}{9!} \right)$
- (D) $9 \left(\frac{(-1)^4 2^9}{9!} \right)$
- (E) $\frac{1}{2} \left(\frac{(-1)^4 2^9}{9!} \right)$

Questions 22–24

22. Subspaces U, V of $\mathbb{R}^{11}$ have dimensions 4 and 5. What is the smallest possible value of $\dim(U \cap V)$?

- (A) 2
- (B) 4
- (C) 0
- (D) 1
- (E) 3

23. Find the upward flux of $\mathbf{F} = (x, y, z)$ through $z = 2x^2 + 5y^2$, $0 \leq x, y \leq 1$.

- (A) $-\frac{7}{3}$
- (B) $-\frac{4}{3}$
- (C) $-\frac{14}{3}$
- (D) $\frac{7}{3}$
- (E) $-\frac{7}{6}$

24. For which real a does $f_m(x) = m^a x^4(1-x)^m$ converge uniformly to zero on $[0, 1]$?

- (A) $a > 4$
- (B) $a < 4$
- (C) $a < 0$
- (D) $a \leq 4$
- (E) All real a

Questions 25–27

25. For $f(x, y) = x^2 + 2xy + 5y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) 8
- (B) 16
- (C) 64
- (D) -8
- (E) 0

26. Let V consist of real polynomials of degree at most 8 that have both value and first derivative zero at 3, 4, and 6. Find $\dim V$.

- (A) 1
- (B) 5
- (C) 3
- (D) 2
- (E) 4

27. Evaluate $\iiint_{x^2+y^2+z^2 \leq 2} (x^2 + y^2 + z^2)^4 dV$.

- (A) $11 \left(\frac{4\pi 2^{\frac{11}{2}}}{11} \right)$
- (B) $\frac{11}{18} \left(\frac{4\pi 2^{\frac{11}{2}}}{11} \right)$
- (C) $\frac{1}{3} \left(\frac{4\pi 2^{\frac{11}{2}}}{11} \right)$
- (D) $\frac{4\pi 2^{\frac{11}{2}}}{11}$
- (E) $\frac{1}{2} \left(\frac{4\pi 2^{\frac{11}{2}}}{11} \right)$

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{4p}]$ converge?

- (A) All real p
- (B) $p > 0$
- (C) $p \geq 1/4$
- (D) $p > 1/4$
- (E) $p < 1/4$

29. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(2j)!}{(j!)^2} x^{5j}$?

- (A) 2^{-2}
- (B) $\frac{1}{2}$
- (C) $2^{-\frac{2}{5}}$
- (D) $2^{\frac{2}{5}}$
- (E) $2^{-\frac{5}{2}}$

30. The roots r, s of $x^2 - 6x + 9 = 0$ are counted with multiplicity. Find $r^3 + s^3$.

- (A) 54
- (B) 55
- (C) 56
- (D) 53
- (E) 52

Questions 31–33

31. The local inverse $y(x)$ of $x = 2y + y^3$ satisfies $y(0) = 0$. Find $y^{(5)}(0)$.

- (A) $\frac{1}{2} \left(\frac{360}{2^7} \right)$
- (B) $2 \left(\frac{360}{2^7} \right)$
- (C) $-\frac{360}{2^7}$
- (D) $\frac{360}{2^7}$
- (E) $\frac{1}{120} \left(\frac{360}{2^7} \right)$

32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{4 + 1/k : k \geq 1\}$ have?

- (A) 3
- (B) 2
- (C) 0
- (D) 1
- (E) 4

33. Find the directional derivative of $f(x, y, z) = x^2y + 5xz^2 + 2y^2z$ at $(1, 1, 1)$ in the direction $(1, 3, 0)$.

- (A) $\frac{11\sqrt{10}}{10}$
- (B) $\frac{11\sqrt{10}}{5}$
- (C) $-\frac{11\sqrt{10}}{5}$
- (D) $\frac{22\sqrt{10}}{5}$
- (E) $1 + \frac{11\sqrt{10}}{5}$

Questions 34–36

34. For real t , let $M(t) = \begin{pmatrix} t-3 & 3 & 0 \\ 0 & t-3 & 1 \\ 0 & 0 & t-3 \end{pmatrix}$. At $t = 3$, what is $\dim \ker(M(t)^2) - \dim \ker M(t)$?

- (A) 1
- (B) 3
- (C) 4
- (D) 0
- (E) 2

35. Evaluate $\int_0^1 \int_0^x x^2 y^5 dy dx$.

- (A) $\frac{1}{48}$
- (B) $\frac{1}{18}$
- (C) $\frac{1}{9}$
- (D) $\frac{1}{63}$
- (E) $\frac{1}{54}$

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 4\}$ have?

- (A) 3
- (B) 7
- (C) 6
- (D) 4
- (E) 5

Questions 37–39

37. For the curve $x = t^5$, $y = t^6 + 2t$, find d^2y/dx^2 at $t = 1$.

- (A) $\frac{8}{5}$
- (B) $-\frac{2}{25}$
- (C) $\frac{14}{25}$
- (D) $-\frac{4}{25}$
- (E) $-\frac{2}{5}$

38. Let $u = (1, 5)^T$ and $v = (1, 0)^T$. Find the squared distance from $(1, 0)^T$ to $\text{span}(u)$.

- (A) $\frac{32}{13}$
- (B) $\frac{25}{26}$
- (C) $\frac{19}{13}$
- (D) $\frac{51}{26}$
- (E) $\frac{6}{13}$

39. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{2t} \cos(5t)$, and $y = e^{2t} \sin(5t)$, find d^2f/dt^2 at $t = 1$.

- (A) 10
- (B) 58
- (C) 0
- (D) 16
- (E) 4

Questions 40–42

40. The circle $|z| = 2$ is traversed counterclockwise. What is $\frac{1}{2\pi i} \int_{|z|=2} \frac{e^z}{z^5} dz$?

- (A) $1/8!$
- (B) $1/3!$
- (C) $4!$
- (D) $1/5!$
- (E) $1/4!$

41. Find the coefficient of x^6 in $\cos(2x)/(1 - 4x^2)$.

- (A) $\frac{1511}{45}$
- (B) $-\frac{1556}{45}$
- (C) $\frac{1556}{45}$
- (D) 24896
- (E) $-\frac{4}{45}$

42. How many group homomorphisms $\mathbb{Z}/48\mathbb{Z} \rightarrow \mathbb{Z}/36\mathbb{Z}$ have image of order 12?

- (A) 4
- (B) 5
- (C) 3
- (D) 6
- (E) 2

Questions 43–45

43. Two solutions of $y'' + 2x^5y' + 5y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $e^{-\frac{1}{3}}$
- (B) $\frac{1}{2e^{\frac{1}{3}}}$
- (C) $-\frac{1}{e^{\frac{1}{3}}}$
- (D) $\frac{2}{e^{\frac{1}{3}}}$
- (E) $e^{-\frac{1}{3}} + 1$

44. Find the residue of $z^4/(z - 1)^3$ at $z = 1$.

- (A) 10
- (B) 6
- (C) 8
- (D) 2
- (E) 4

45. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 2x)^{1/(4x)} - e^{2/4}}{x}$.

- (A) $\frac{e^{\frac{1}{2}}}{2}$
- (B) $-e^{\frac{1}{2}}$
- (C) $e^{\frac{1}{2}}$
- (D) $-\frac{e^{\frac{1}{2}}}{4}$
- (E) $-\frac{e^{\frac{1}{2}}}{2}$

Questions 46–48

46. How many 2-dimensional subspaces does $\mathbb{F}_2^6$ have?

- (A) $\frac{(2^9 - 1)(2^9 - 2)}{(2^2 - 1)(2^2 - 2)}$
- (B) $\frac{(2^8 - 1)(2^8 - 2)}{(2^2 - 1)(2^2 - 2)}$
- (C) $\frac{(2^6 - 1)(2^6 - 2)}{(2^2 - 1)(2^2 - 2)}$
- (D) $\frac{(2^7 - 1)(2^7 - 2)}{(2^2 - 1)(2^2 - 2)}$
- (E) $\frac{(2^5 - 1)(2^5 - 2)}{(2^2 - 1)(2^2 - 2)}$

47. If $y' = 4xy$ and $y(0) = 1$, find the coefficient of x^{10} in the Maclaurin series of y .

- (A) $\frac{8}{15}$
- (B) $\frac{19}{15}$
- (C) $\frac{4}{15}$
- (D) $-\frac{4}{15}$
- (E) $\frac{2}{15}$

48. How many functions from a set of 4 elements onto a set of 3 elements exist?

- (A) $3^4 - 3(2^4) - 3$
- (B) 3^4
- (C) $3^4 - 3(2^4)$
- (D) $3^4 - 3(2^4) + 3$
- (E) $3^4 - 2^4$

Questions 49–51

49. If $y'' - 4y' + 4y = 5e^{2x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $-\frac{5e^2}{2}$
- (B) $5e^2$
- (C) $1 + \frac{5e^2}{2}$
- (D) $\frac{5e^2}{4}$
- (E) $\frac{5e^2}{2}$

50. How many monic irreducible polynomials of degree 4 are there over $\mathbb{F}_2$?

- (A) $2(2^4 - 2^2)$
- (B) $\frac{2^4 - 2^2}{4}$
- (C) $2^4 - 2^2$
- (D) $\frac{2(2^4 - 2^2)}{4}$
- (E) $\frac{3(2^4 - 2^2)}{4}$

51. Evaluate $\int_0^1 x^1(\log x)^4 dx$.

- (A) $\frac{1}{5} \left(\frac{(-1)^4 4!}{2^5} \right)$
- (B) $-\frac{(-1)^4 4!}{2^5}$
- (C) $\frac{(-1)^4 4!}{2^5}$
- (D) $\frac{2}{3} \left(\frac{(-1)^4 4!}{2^5} \right)$
- (E) $2 \left(\frac{(-1)^4 4!}{2^5} \right)$

Questions 52–54

52. How many nonnegative integer triples sum to 8, if each coordinate is at most 4?

- (A) 25
- (B) 10
- (C) 20
- (D) 15
- (E) 5

53. The region $0 \leq x \leq 1$, $0 \leq y \leq 2x^5$ is revolved about the line $x = -3$. Find its volume.

- (A) $1 + \frac{18\pi}{7}$
- (B) $-\frac{18\pi}{7}$
- (C) $\frac{18\pi}{7}$
- (D) $\frac{9\pi}{7}$
- (E) $\frac{36\pi}{7}$

54. How many elements z of $\mathbb{F}_{7^2}$ satisfy $z^{49} = z$?

- (A) 51
- (B) 50
- (C) 48
- (D) 47
- (E) 49

Questions 55–57

55. Let $f(x) = \log(1 + 2x)$. Find $f^{(5)}(0)$.

- (A) $5 \left((-1)^4 4! 2^5 \right)$
- (B) $(-1)^4 4! 2^5$
- (C) $\frac{1}{5} \left((-1)^4 4! 2^5 \right)$
- (D) $\frac{1}{2} \left((-1)^4 4! 2^5 \right)$
- (E) $-(-1)^4 4! 2^5$

56. A tree has 4 vertices of degree 3; every other vertex has degree 1. How many leaves does it have?

- (A) 10
- (B) 8
- (C) 2
- (D) 6
- (E) 4

57. For $x > 0$, $x^2 y'' + (-8)xy' + 14y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $-\frac{(1 - e^5) e^2}{5}$
- (B) $\frac{(1 - e^5) e^2}{5}$
- (C) $-\frac{2(1 - e^5) e^2}{5}$
- (D) $-\frac{e^2}{5} + 1 + \frac{e^7}{5}$
- (E) $-\frac{(1 - e^5) e^2}{10}$

Questions 58–60

58. How many generators does the cyclic group of order 32 have?

- (A) $2^4(2 - 1)$
- (B) $2^7(2 - 1)$
- (C) $2^6(2 - 1)$
- (D) $2^5(2 - 1)$
- (E) $2^3(2 - 1)$

59. If $y'' + 25y = 2$ and $y(0) = y'(0) = 0$, find $y(\pi/5)$.

- (A) $\frac{8}{25}$
- (B) $\frac{29}{25}$
- (C) $\frac{2}{25}$
- (D) $-\frac{4}{25}$
- (E) $\frac{4}{25}$

60. A connected planar graph has 7 vertices and 9 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 4
- (B) 2
- (C) 6
- (D) 5
- (E) 3

Questions 61–63

61. Evaluate $\iiint_B 2z^2(x^2 + y^2 + z^2)^5 dV$, where B is the unit ball.

- (A) $\frac{8\pi}{45}$
- (B) $\frac{8\pi}{45} + 1$
- (C) $\frac{4\pi}{45}$
- (D) $-\frac{8\pi}{45}$
- (E) $\frac{16\pi}{45}$

62. Let $R = (\mathbb{Z}/8\mathbb{Z})/([4])$, where $([4])$ is the ideal generated by $[4]$. How many units does R have?

- (A) 1
- (B) 0
- (C) 4
- (D) 2
- (E) 3

63. Let C be $x^2/25 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-\frac{2y^3}{3}) dx + (\frac{x^3}{3}) dy$.

- (A) $-\frac{135\pi}{4}$
- (B) $1 + \frac{135\pi}{4}$
- (C) $\frac{135\pi}{8}$
- (D) $\frac{135\pi}{2}$
- (E) $\frac{135\pi}{4}$

Questions 64–66

64. How many integers in $\{1, \dots, 24\}$ are divisible by 2 or 3?

- (A) 26
- (B) 11
- (C) 21
- (D) 6
- (E) 16

65. Let $F(x) = \int_0^x (x-t)^4 e^{2t} dt$. Find $F^{(5)}(0)$.

- (A) 48
- (B) 24
- (C) $\frac{1}{24}$
- (D) 120
- (E) 0

66. What is the largest integer k such that 3^k divides $12!$?

- (A) 7
- (B) 6
- (C) 4
- (D) 3
- (E) 5

Answer Key

Practice Test 07 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	A	23	A	45	E
2	B	24	B	46	C
3	E	25	A	47	C
4	C	26	C	48	D
5	D	27	D	49	E
6	C	28	D	50	B
7	D	29	C	51	C
8	E	30	A	52	D
9	D	31	D	53	C
10	B	32	B	54	E
11	B	33	B	55	B
12	B	34	A	56	D
13	D	35	E	57	A
14	C	36	E	58	A
15	D	37	B	59	E
16	C	38	B	60	A
17	D	39	C	61	A
18	E	40	E	62	D
19	A	41	C	63	E
20	A	42	A	64	E
21	A	43	A	65	B
22	C	44	B	66	E

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