

GRE Mathematics Practice Test 08

Upper-Intermediate Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

Intermediate reinforcement, combining standard applications with selected more demanding arguments. This is cycle 1 of 19, position 3 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. The solution of $y' = 2x^4y^3$ has $y(0) = 1/3$. Find $y(1)$.

- (A) $\frac{\sqrt{205}}{41} + 1$
- (B) $\frac{\sqrt{205}}{82}$
- (C) $\frac{2\sqrt{205}}{41}$
- (D) $-\frac{\sqrt{205}}{41}$
- (E) $\frac{\sqrt{205}}{41}$

2. Subspaces U, V of $\mathbb{R}^{12}$ have dimensions 3 and 4. What is the smallest possible value of $\dim(U \cap V)$?

- (A) 3
- (B) 4
- (C) 2
- (D) 1
- (E) 0

3. Evaluate $\int_0^{\log 2} e^x(e^x + 2)^4 dx$.

- (A) $-\frac{4^5 - 3^5}{5}$
- (B) $\frac{4^5 - 3^5}{5}$
- (C) $\frac{4^5}{5}$
- (D) $5 \left(\frac{4^5 - 3^5}{5} \right)$
- (E) $4^4 - 3^4$

Questions 4–6

4. A box is chosen uniformly from two boxes. Box A contains 5 red and 7 blue balls; box B contains 7 red and 5 blue balls. Two balls are drawn without replacement and both are red. What is the conditional probability that A was chosen?

- (A) $\frac{21}{31}$
- (B) $\frac{5}{31}$
- (C) $\frac{10}{31}$
- (D) $\frac{10}{93}$
- (E) $\frac{41}{62}$

5. For $x > 0$, $x^2y'' + (-7)xy' + 12y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $-\frac{e^2}{4} + 1 + \frac{e^6}{4}$
- (B) $-\frac{(1 - e^4)e^2}{4}$
- (C) $\frac{(1 - e^4)e^2}{4}$
- (D) $-\frac{(1 - e^4)e^2}{2}$
- (E) $-\frac{(1 - e^4)e^2}{8}$

6. Let V consist of real polynomials of degree at most 7 that vanish at 4, 5, and 7. Find $\dim V$.

- (A) 4
- (B) 7
- (C) 5
- (D) 3
- (E) 6

Questions 7–9

7. Evaluate $\int_0^1 \int_x^1 2y^4 dy dx$.

- (A) $\frac{1}{15}$
- (B) $\frac{1}{1}$
- (C) $\frac{1}{3}$
- (D) $\frac{2}{5}$
- (E) $\frac{1}{2}$

8. Let $X_1, \dots, X_5$ be independent uniform random variables on $[0, 1]$. What is $E[(\max_i X_i)^2]$?

- (A) $\frac{5}{21}$
- (B) $\frac{2}{7}$
- (C) $\frac{5}{7}$
- (D) $\frac{6}{7}$
- (E) $\frac{5}{14}$

9. Evaluate $\int_0^{\pi/2} \sin^9 t \cos^5 t dt$.

- (A) $\frac{4! 2!}{2 \cdot 7!}$
- (B) $7 \left(\frac{4! 2!}{2 \cdot 7!} \right)$
- (C) $\frac{1}{2} \left(\frac{4! 2!}{2 \cdot 7!} \right)$
- (D) $2 \left(\frac{4! 2!}{2 \cdot 7!} \right)$
- (E) $15 \left(\frac{4! 2!}{2 \cdot 7!} \right)$

Questions 10–12

10. The roots r, s of $x^2 - 10x + 24 = 0$ are counted with multiplicity. Find $r^3 + s^3$.
- (A) 279
 - (B) 280
 - (C) 278
 - (D) 281
 - (E) 282
11. For $\mathbf{F} = (2x^4y, y^4z, xz^4)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.
- (A) 52
 - (B) 26
 - (C) 104
 - (D) -52
 - (E) 53
12. Independent trials have success probability $1/5$. Let T count trials up to and including the first success. Find $E[T]$.
- (A) 4
 - (B) 6
 - (C) 3
 - (D) 5
 - (E) 7

Questions 13–15

13. Evaluate $\iiint_{x^2+y^2+z^2 \leq 2} (x^2 + y^2 + z^2)^3 dV$.

(A) $\frac{1}{3} \left(\frac{4\pi 2^{\frac{9}{2}}}{9} \right)$

(B) $\frac{4\pi 2^{\frac{9}{2}}}{9}$

(C) $9 \left(\frac{4\pi 2^{\frac{9}{2}}}{9} \right)$

(D) $\frac{1}{2} \left(\frac{4\pi 2^{\frac{9}{2}}}{9} \right)$

(E) $\frac{9}{14} \left(\frac{4\pi 2^{\frac{9}{2}}}{9} \right)$

14. For real t , let $M(t) = \begin{pmatrix} t-4 & 6 & 0 \\ 0 & t-4 & 1 \\ 0 & 0 & t-4 \end{pmatrix}$. At $t = 4$, what is $\dim \ker(M(t)^2) - \dim \ker M(t)$?

(A) 4

(B) 3

(C) 2

(D) 1

(E) 0

15. Evaluate $\int_0^\infty x^4 e^{-2x} dx$.

(A) $\frac{1}{4} \left(\frac{4!}{2^5} \right)$

(B) $\frac{1}{2} \left(\frac{4!}{2^5} \right)$

(C) $32 \left(\frac{4!}{2^5} \right)$

(D) $2 \left(\frac{4!}{2^5} \right)$

(E) $\frac{4!}{2^5}$

Questions 16–18

16. A uniformly random subset of 5 objects is selected from a collection with 6 red and 7 blue objects. If X is the number of red objects selected, find $E[X]$.

- (A) $\frac{43}{13}$
- (B) $\frac{15}{13}$
- (C) $\frac{30}{13}$
- (D) $\frac{60}{13}$
- (E) $\frac{56}{13}$

17. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 2z^2 = x + 4y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) 16
- (B) -8
- (C) -16
- (D) -4
- (E) -32

18. Let $u = (1, 4)^T$ and $v = (1, 0)^T$. Find the squared distance from $(1, 0)^T$ to $\text{span}(u)$.

- (A) $\frac{49}{34}$
- (B) $\frac{15}{13}$
- (C) $\frac{34}{33}$
- (D) $\frac{17}{16}$
- (E) $\frac{17}{83}$

Questions 19–21

19. Evaluate $\int_0^1 \frac{x^3}{1+2x^4} dx$.

- (A) $\frac{\log(3)}{8}$
- (B) $\log(3)$
- (C) $\frac{\log(3)}{4}$
- (D) $-\frac{\log(3)}{8}$
- (E) $\frac{\log(3)}{2}$

20. X, Y are independent Poisson variables with means 5 and 6. Given $X+Y = 7$, find $\text{Var}(X \mid X+Y)$.

- (A) $\frac{331}{121}$
- (B) $\frac{420}{121}$
- (C) $\frac{452}{121}$
- (D) $\frac{210}{121}$
- (E) $\frac{105}{121}$

21. Find the coefficient of x^6 in $\cos(2x)/(1-3x^2)$.

- (A) $\frac{446}{45}$
- (B) $-\frac{4}{45}$
- (C) 7856
- (D) $\frac{491}{45}$
- (E) $-\frac{491}{45}$

Questions 22–24

22. An idempotent 10×10 real matrix P has rank 4. What is $\det(I + P)$?

- (A) 14
- (B) 15
- (C) 18
- (D) 17
- (E) 16

23. The local inverse $y(x)$ of $x = 2y + y^3$ satisfies $y(0) = 0$. Find $y^{(3)}(0)$.

- (A) $\frac{6}{2^4}$
- (B) $\frac{1}{6} \left(-\frac{6}{2^4} \right)$
- (C) $-\frac{6}{2^4}$
- (D) $2 \left(-\frac{6}{2^4} \right)$
- (E) $\frac{1}{2} \left(-\frac{6}{2^4} \right)$

24. For which real a does $f_m(x) = m^a x^5 (1 - x)^m$ converge uniformly to zero on $[0, 1]$?

- (A) $a < 0$
- (B) $a < 5$
- (C) $a > 5$
- (D) $a \leq 5$
- (E) All real a

Questions 25–27

25. Find the directional derivative of $f(x, y, z) = x^2y + 4xz^2 + 2y^2z$ at $(1, 1, 1)$ in the direction $(1, 4, 0)$.

- (A) $\frac{13\sqrt{17}}{17}$
- (B) $1 + \frac{26\sqrt{17}}{17}$
- (C) $\frac{26\sqrt{17}}{17}$
- (D) $-\frac{26\sqrt{17}}{17}$
- (E) $\frac{52\sqrt{17}}{17}$

26. For $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 4 \\ 6 \\ 10 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find x_2 .

- (A) 3
- (B) 4
- (C) 1
- (D) 5
- (E) 2

27. If $y' = y(1 - y/4)$ and $y(0) = 2$, find $y(-\log 2)$.

- (A) $\frac{2}{3}$
- (B) $\frac{4}{3}$
- (C) $\frac{7}{3}$
- (D) $-\frac{4}{3}$
- (E) $\frac{8}{3}$

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{5p}]$ converge?

- (A) All real p
- (B) $p < 1/5$
- (C) $p > 0$
- (D) $p \geq 1/5$
- (E) $p > 1/5$

29. Let C be $x^2/16 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-\frac{2y^3}{3}) dx + (\frac{x^3}{3}) dy$.

- (A) 9π
- (B) 36π
- (C) 18π
- (D) $1 + 18\pi$
- (E) -18π

30. What is the coefficient of x^7 in $(x+1)^6(1-x)^3$?

- (A) 1
- (B) 0
- (C) 2
- (D) 3
- (E) 4

Questions 31–33

31. Under $x = u + 2v$, $y = u - 4v$, what is the area of the image of $0 \leq u, v \leq 1$?
- (A) $\frac{1}{6}$
(B) 36
(C) 2
(D) 6
(E) 8
32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{5 + 1/k : k \geq 1\}$ have?
- (A) 2
(B) 4
(C) 0
(D) 3
(E) 1
33. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 2x)^{1/(3x)} - e^{2/3}}{x}$.
- (A) $\frac{2e^{\frac{2}{3}}}{3}$
(B) $-\frac{e^{\frac{2}{3}}}{3}$
(C) $-\frac{4e^{\frac{2}{3}}}{3}$
(D) $e^{\frac{2}{3}}$
(E) $-\frac{2e^{\frac{2}{3}}}{3}$

Questions 34–36

34. A matrix A satisfies $(A - 5I)(A - 7I) = 0$, and its minimal polynomial has degree two. If $A^4 = uA + vI$, find u .

- (A) 887
- (B) 886
- (C) 889
- (D) 890
- (E) 888

35. The solution of $y' = 2x^3y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\sqrt[4]{2}$
- (B) 1
- (C) 2
- (D) $2\sqrt[4]{2}$
- (E) $\frac{2^{\frac{3}{4}}}{2}$

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 5\}$ have?

- (A) 6
- (B) 2
- (C) 10
- (D) 4
- (E) 8

Questions 37–39

37. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{2t} \cos(4t)$, and $y = e^{2t} \sin(4t)$, find d^2f/dt^2 at $t = 1$.

- (A) 40
- (B) 8
- (C) 4
- (D) 0
- (E) 16

38. Let $V = \begin{bmatrix} 1 & 4 & 16 \\ 1 & 6 & 36 \\ 1 & 9 & 81 \end{bmatrix}$. Find $\det(V^T V)$.

- (A) 901
- (B) 902
- (C) 898
- (D) 900
- (E) 899

39. Evaluate $\int_0^1 \int_0^x x^2 y^4 dy dx$.

- (A) $\frac{1}{48}$
- (B) $\frac{1}{40}$
- (C) $\frac{1}{15}$
- (D) $\frac{1}{35}$
- (E) $\frac{1}{8}$

Questions 40–42

40. The circle $|z| = 2$ is traversed counterclockwise. What is $\frac{1}{2\pi i} \int_{|z|=2} \frac{e^z}{z^6} dz$?

- (A) $5!$
- (B) $1/10!$
- (C) $1/4!$
- (D) $1/5!$
- (E) $1/6!$

41. Evaluate $\int_C (4x^3 + 2y) dx + (2x + 4y^3) dy$, where C is $r(t) = (t, 4t^2)$, $0 \leq t \leq 1$.

- (A) -265
- (B) 530
- (C) $\frac{265}{2}$
- (D) 265
- (E) 266

42. In the ring $\mathbb{Z}/48\mathbb{Z}$, how many elements satisfy $x^2 = 1$?

- (A) 9
- (B) 8
- (C) 10
- (D) 6
- (E) 7

Questions 43–45

43. Let C be the unit circle $x^2 + y^2 = 1$, $z = 4$, counterclockwise viewed from above. Evaluate $\int_C -2y(x^2 + y^2)^3 dx + (2x(x^2 + y^2)^3) dy + 4z dz$.

- (A) -4π
- (B) 2π
- (C) $1 + 4\pi$
- (D) 4π
- (E) 8π

44. Find the residue of $z^5/(z - 1)^3$ at $z = 1$.

- (A) 13
- (B) 10
- (C) 4
- (D) 16
- (E) 7

45. Find the coefficient of x^4 in $e^{3x} \log(1 + 2x)$.

- (A) 8
- (B) -4
- (C) 4
- (D) 6
- (E) 96

Questions 46–48

46. A permutation consists of disjoint cycles of lengths 3 and 11, together with a disjoint transposition when the sum of these two lengths is odd. It fixes all other points. What is its order?

- (A) 31
- (B) 33
- (C) 34
- (D) 32
- (E) 35

47. Let $F(x) = \int_0^x (x-t)^3 e^{2t} dt$. Find $F^{(4)}(0)$.

- (A) 12
- (B) 0
- (C) 6
- (D) $\frac{1}{6}$
- (E) 24

48. How many functions from a set of 5 elements onto a set of 3 elements exist?

- (A) $3^5 - 3(2^5) + 3$
- (B) 3^5
- (C) $3^5 - 2^5$
- (D) $3^5 - 3(2^5)$
- (E) $3^5 - 3(2^5) - 3$

Questions 49–51

49. Find the curvature of the helix $r(t) = (2 \cos t, 2 \sin t, 4t)$.

- (A) $-\frac{1}{10}$
- (B) $\frac{1}{5}$
- (C) $\frac{11}{10}$
- (D) $\frac{1}{10}$
- (E) $\frac{1}{20}$

50. In $R = \mathbb{F}_5[x]/(x^2)$, how many elements are units?

- (A) $5^2 - 5^0$
- (B) $5^3 - 5^1$
- (C) $5^4 - 5^3$
- (D) $5^3 - 5^2$
- (E) $5^2 - 5^1$

51. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(2x) - \sum_{j=0}^2 (-1)^j (2x)^{2j+1} / (2j+1)!}{x^7}$.

- (A) $-\frac{(-1)^3 2^7}{7!}$
- (B) $\frac{1}{2} \left(\frac{(-1)^3 2^7}{7!} \right)$
- (C) $\frac{(-1)^3 2^7}{7!}$
- (D) $2 \left(\frac{(-1)^3 2^7}{7!} \right)$
- (E) $7 \left(\frac{(-1)^3 2^7}{7!} \right)$

Questions 52–54

52. How many nonnegative integer triples sum to 10, if each coordinate is at most 5?

- (A) 35
- (B) 28
- (C) 21
- (D) 14
- (E) 7

53. Evaluate $\iiint_B 2z^2(x^2 + y^2 + z^2)^4 dV$, where B is the unit ball.

- (A) $\frac{16\pi}{39}$
- (B) $\frac{4\pi}{39}$
- (C) $-\frac{8\pi}{39}$
- (D) $\frac{8\pi}{39} + 1$
- (E) $\frac{8\pi}{39}$

54. Integers x satisfy $x \equiv 8 \pmod{11}$ and $x \equiv 10 \pmod{16}$. Find the least nonnegative solution.

- (A) 74
- (B) 73
- (C) 72
- (D) 76
- (E) 75

Questions 55–57

55. Find the arc length of $r(t) = (2(t - t^3/3), 2t^2)$ for $0 \leq t \leq 1/4$.

- (A) $\frac{49}{48}$
- (B) $\frac{49}{96}$
- (C) $\frac{145}{96}$
- (D) $\frac{49}{192}$
- (E) $-\frac{49}{96}$

56. A tree has 9 vertices. What is the sum of its vertex degrees?

- (A) 26
- (B) 6
- (C) 16
- (D) 21
- (E) 11

57. Let $f(x, y) = e^{2x+4y}$. Find $\partial^4 f / \partial x^3 \partial y$ at $(0, 0)$.

- (A) $4 \cdot 2^3$
- (B) $4(4 \cdot 2^3)$
- (C) $\frac{1}{4}(4 \cdot 2^3)$
- (D) $\frac{1}{2}(4 \cdot 2^3)$
- (E) $8(4 \cdot 2^3)$

Questions 58–60

58. The symmetric group S_{10} acts on the two-element subsets of $\{1, \dots, 10\}$. What is the index of the setwise stabilizer of $\{1, 2\}$?

- (A) 45
- (B) 46
- (C) 47
- (D) 43
- (E) 44

59. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 2x$ and has density $\rho(x, y) = x^2y$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{5}{6}$
- (B) $\frac{11}{6}$
- (C) $\frac{5}{12}$
- (D) $-\frac{5}{6}$
- (E) $\frac{5}{3}$

60. A connected planar graph has 8 vertices and 11 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 7
- (B) 4
- (C) 3
- (D) 6
- (E) 5

Questions 61–63

61. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(2+4j)(6+4j)}$.

- (A) $\frac{1}{2}$
- (B) $\frac{1}{4}$
- (C) 2
- (D) $\frac{1}{8}$
- (E) $\frac{1}{6}$

62. In $\mathbb{Z}/138\mathbb{Z}$, how many residues are not idempotent ($x^2 \neq x$)?

- (A) 128
- (B) 132
- (C) 129
- (D) 131
- (E) 130

63. Two solutions of $y'' + 2x^4y' + 4y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $-\frac{1}{e^{\frac{2}{5}}}$
- (B) $\frac{1}{2e^{\frac{2}{5}}}$
- (C) $e^{-\frac{2}{5}} + 1$
- (D) $\frac{2}{e^{\frac{2}{5}}}$
- (E) $e^{-\frac{2}{5}}$

Questions 64–66

64. How many integers in $\{1, \dots, 30\}$ are divisible by 2 or 3?

- (A) 26
- (B) 20
- (C) 32
- (D) 14
- (E) 8

65. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^2 y^3 dV$.

- (A) $6 \left(\frac{2! 3!}{8!} \right)$
- (B) $\frac{1}{6} \left(\frac{2! 3!}{8!} \right)$
- (C) $\frac{1}{12}$
- (D) $8 \left(\frac{2! 3!}{8!} \right)$
- (E) $\frac{2! 3!}{8!}$

66. In the additive group $\mathbb{Z}/12\mathbb{Z}$, find the index of the subgroup generated by $[6]$.

- (A) 6
- (B) 4
- (C) 7
- (D) 5
- (E) 8

Answer Key

Practice Test 08 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	E	23	C	45	C
2	E	24	B	46	B
3	B	25	C	47	C
4	C	26	A	48	A
5	B	27	B	49	D
6	C	28	E	50	E
7	C	29	C	51	C
8	C	30	B	52	C
9	A	31	D	53	E
10	B	32	A	54	A
11	A	33	E	55	B
12	D	34	E	56	C
13	B	35	A	57	A
14	D	36	A	58	A
15	E	37	D	59	A
16	C	38	D	60	E
17	C	39	B	61	D
18	D	40	D	62	E
19	A	41	D	63	E
20	D	42	B	64	B
21	D	43	D	65	E
22	E	44	B	66	A

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