

GRE Mathematics Practice Test 09

Bridge to Standard

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A bridge toward standard practice, with routine multi-step questions and a smaller number of challenging concepts. This is cycle 1 of 19, position 4 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Evaluate $\int_0^1 x^1(\log x)^2 dx$.

- (A) $\frac{1}{2} \left(\frac{(-1)^2 2!}{2^3} \right)$
- (B) $-\frac{(-1)^2 2!}{2^3}$
- (C) $\frac{1}{3} \left(\frac{(-1)^2 2!}{2^3} \right)$
- (D) $\frac{(-1)^2 2!}{2^3}$
- (E) $2 \left(\frac{(-1)^2 2!}{2^3} \right)$

2. An idempotent 11×11 real matrix P has rank 5. What is $\text{tr } P$?

- (A) 4
- (B) 3
- (C) 7
- (D) 6
- (E) 5

3. Solve $y'' - 7y' + 10y = 0$ with $y(0) = 1$, $y'(0) = 0$. Find $y(1)$.

- (A) $\frac{(5 - 2e^3) e^2}{3}$
- (B) $\frac{2(5 - 2e^3) e^2}{3}$
- (C) $\frac{(-5 + 2e^3) e^2}{3}$
- (D) $\frac{(5 - 2e^3) e^2}{6}$
- (E) $\frac{(5 - 2e^3) e^2}{3} + 1$

Questions 4–6

4. A box is chosen uniformly from two boxes. Box A contains 6 red and 8 blue balls; box B contains 8 red and 6 blue balls. Two balls are drawn without replacement and both are red. What is the conditional probability that A was chosen?

- (A) $\frac{28}{43}$
- (B) $\frac{43}{15}$
- (C) $\frac{43}{29}$
- (D) $\frac{43}{15}$
- (E) $\frac{86}{5}$

5. Let $f(x) = \log(1 + 2x)$. Find $f^{(3)}(0)$.

- (A) $\frac{1}{2} \left((-1)^2 2! 2^3 \right)$
- (B) $(-1)^2 2! 2^3$
- (C) $-(-1)^2 2! 2^3$
- (D) $\frac{1}{3} \left((-1)^2 2! 2^3 \right)$
- (E) $3 \left((-1)^2 2! 2^3 \right)$

6. For $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $b = \begin{bmatrix} 5 \\ 4 \\ 9 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find x_2 .

- (A) 4
- (B) 2
- (C) 6
- (D) 5
- (E) 3

Questions 7–9

7. Find the upward flux of $\mathbf{F} = (0, 0, z)$ through $z = 2x^2 + 3y^2$, $0 \leq x, y \leq 1$.

- (A) $\frac{5}{6}$
- (B) $-\frac{5}{3}$
- (C) $\frac{10}{3}$
- (D) $\frac{5}{3}$
- (E) $\frac{8}{3}$

8. Let $X_1, \dots, X_6$ be independent uniform random variables on $[0, 1]$. What is $E[\max_i X_i]$?

- (A) $\frac{13}{14}$
- (B) $\frac{2}{7}$
- (C) $\frac{3}{7}$
- (D) $\frac{1}{7}$
- (E) $\frac{6}{7}$

9. Evaluate $\int_0^{\pi/2} \sin^7 t \cos^5 t \, dt$.

- (A) $\frac{1}{2} \left(\frac{3! 2!}{2 \cdot 6!} \right)$
- (B) $2 \left(\frac{3! 2!}{2 \cdot 6!} \right)$
- (C) $6 \left(\frac{3! 2!}{2 \cdot 6!} \right)$
- (D) $\frac{3! 2!}{2 \cdot 6!}$
- (E) $10 \left(\frac{3! 2!}{2 \cdot 6!} \right)$

Questions 10–12

10. What is the coefficient of x^3 in $(x + 1)^5$?

- (A) 11
- (B) 9
- (C) 12
- (D) 8
- (E) 10

11. Two solutions of $y'' + 2x^1y' + 3y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $\frac{1}{2e}$
- (B) $\frac{2}{e}$
- (C) $-\frac{1}{e}$
- (D) e^{-1}
- (E) $e^{-1} + 1$

12. Independent trials have success probability $1/6$. Let T count trials up to and including the first success. Find $E[T]$.

- (A) 10
- (B) 6
- (C) 2
- (D) 8
- (E) 4

Questions 13–15

13. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(2+3j)(5+3j)}$.

- (A) $\frac{1}{3}$
- (B) $\frac{1}{2}$
- (C) $\frac{1}{6}$
- (D) $\frac{1}{5}$
- (E) $\frac{3}{2}$

14. A matrix A satisfies $(A - 6I)(A - 8I) = 0$, and its minimal polynomial has degree two. If $A^2 = uA + vI$, find u .

- (A) 15
- (B) 13
- (C) 16
- (D) 14
- (E) 12

15. The solution of $y' = 2x^3y^2$ has $y(0) = 1/3$. Find $y(1)$.

- (A) $\frac{4}{5}$
- (B) $\frac{1}{5}$
- (C) $\frac{2}{5}$
- (D) $-\frac{2}{5}$
- (E) $\frac{7}{5}$

Questions 16–18

16. A uniformly random subset of 6 objects is selected from a collection with 7 red and 8 blue objects. If X is the number of red objects selected, find $E[X]$.

- (A) $\frac{24}{5}$
- (B) $\frac{14}{5}$
- (C) $\frac{19}{5}$
- (D) $\frac{28}{5}$
- (E) $\frac{7}{5}$

17. For $x > 0$, $x^2y'' + (1 - 5)xy' + 6y = 0$, $y(1) = 0$, $y'(1) = 2$. Find $y(e)$.

- (A) $e^2 + e^3$
- (B) $2(1 - e)e^2$
- (C) $\frac{e^2}{5}$
- (D) $-e^3 + e^2$
- (E) $-2(1 - e)e^2$

18. Let $V = \begin{bmatrix} 1 & 5 & 25 \\ 1 & 7 & 49 \\ 1 & 10 & 100 \end{bmatrix}$. Find $\det V$.

- (A) 30
- (B) 31
- (C) 32
- (D) 28
- (E) 29

Questions 19–21

19. Find the directional derivative of $f(x, y, z) = x^2 + 2y^2 + 3z^2$ at $(1, 1, 1)$ in the direction $(1, 5, 0)$.

- (A) $\frac{11\sqrt{26}}{26}$
- (B) $\frac{11\sqrt{26}}{13}$
- (C) $1 + \frac{11\sqrt{26}}{13}$
- (D) $-\frac{11\sqrt{26}}{13}$
- (E) $\frac{22\sqrt{26}}{13}$

20. X, Y are independent Poisson variables with means 6 and 7. Given $X+Y = 8$, find $\text{Var}(X \mid X+Y)$.

- (A) $\frac{674}{169}$
- (B) $\frac{336}{169}$
- (C) $\frac{672}{169}$
- (D) $\frac{168}{169}$
- (E) $\frac{505}{169}$

21. Evaluate $\lim_{x \rightarrow 0} \frac{e^{2x} - \sum_{j=0}^2 (2x)^j / j!}{x^3}$.

- (A) $6 \left(\frac{2^3}{3!} \right)$
- (B) $\frac{1}{2} \left(\frac{2^3}{3!} \right)$
- (C) $-\frac{2^3}{3!}$
- (D) $\frac{2^3}{3!}$
- (E) $3 \left(\frac{2^3}{3!} \right)$

Questions 22–24

22. Let $u = (1, 2, \dots, 11)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $\det B$.

- (A) 67
- (B) 66
- (C) 65
- (D) 68
- (E) 69

23. Find the coefficient of x^3 in $e^{2x} \log(1 + 2x)$.

- (A) $-\frac{8}{3}$
- (B) $\frac{14}{3}$
- (C) $\frac{16}{3}$
- (D) 16
- (E) $\frac{8}{3}$

24. For which real a does $f_m(x) = x^{6m}/m^a$ converge uniformly to zero on $[0, 1]$?

- (A) $a > 0$
- (B) All real a
- (C) $a < 0$
- (D) $a > 1$
- (E) $a = 0$

Questions 25–27

25. For $y' = 2y(1 - y/3)$, what is the derivative of the right-hand side with respect to y at the positive equilibrium?

- (A) -3
- (B) -1
- (C) 2
- (D) -4
- (E) -2

26. Let D be a diagonal 11×11 real matrix with distinct diagonal entries. What is the dimension of the space of matrices X satisfying $XD = DX$?

- (A) 11
- (B) 10
- (C) 12
- (D) 13
- (E) 9

27. Evaluate $\int_0^1 \frac{x^2}{1 + 2x^3} dx$.

- (A) $\frac{\log(3)}{2}$
- (B) $\log(3)$
- (C) $-\frac{\log(3)}{6}$
- (D) $\frac{\log(3)}{3}$
- (E) $\frac{\log(3)}{6}$

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{6p}]$ converge?

- (A) $p > 1/6$
- (B) $p > 0$
- (C) $p \geq 1/6$
- (D) $p < 1/6$
- (E) All real p

29. Find the arc length of $r(t) = (2t, 3t)$ for $0 \leq t \leq 1$.

- (A) $2\sqrt{13}$
- (B) $\sqrt{13}$
- (C) $\frac{\sqrt{13}}{2}$
- (D) $-\sqrt{13}$
- (E) $1 + \sqrt{13}$

30. What is the remainder when $x^2 + 1$ is divided by $x - 6$?

- (A) 36
- (B) 39
- (C) 37
- (D) 38
- (E) 35

Questions 31–33

31. Let $\mathbf{F} = (2y^3, z^3, x^3)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (0, 0, 1)$.

- (A) -12
- (B) -5
- (C) -6
- (D) -3
- (E) 6

32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{6 + 1/k : k \geq 1\}$ have?

- (A) 1
- (B) 0
- (C) 4
- (D) 3
- (E) 2

33. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 2x$ and has density $\rho(x, y) = x$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{3}{2}$
- (B) $\frac{3}{8}$
- (C) $\frac{7}{4}$
- (D) $\frac{3}{4}$
- (E) $-\frac{3}{4}$

Questions 34–36

34. Let $P = \text{diag}(\underbrace{1, \dots, 1}_2, \underbrace{0, \dots, 0}_{10})$. What is the dimension of the space of 12×12 real matrices X satisfying $PX = X$?

- (A) 26
- (B) 22
- (C) 24
- (D) 23
- (E) 25

35. Under $x = u + 2v$, $y = u - 3v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) 5
- (B) 25
- (C) 1
- (D) $\frac{1}{5}$
- (E) 6

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 6\}$ have?

- (A) 3
- (B) 11
- (C) 7
- (D) 9
- (E) 5

Questions 37–39

37. Evaluate $\int_0^\infty x^3 e^{-2x} dx$.

- (A) $16 \left(\frac{3!}{2^4} \right)$
- (B) $\frac{1}{3} \left(\frac{3!}{2^4} \right)$
- (C) $\frac{3!}{2^4}$
- (D) $\frac{1}{2} \left(\frac{3!}{2^4} \right)$
- (E) $2 \left(\frac{3!}{2^4} \right)$

38. A 3×3 matrix A has characteristic polynomial $(t - 5)(t + 1)(t - 2)$. What is $\text{tr}(A^2)$?

- (A) 28
- (B) 30
- (C) 31
- (D) 32
- (E) 29

39. Let C be $x^2/9 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-y) dx + (2x) dy$.

- (A) 9π
- (B) $\frac{9\pi}{2}$
- (C) -9π
- (D) $1 + 9\pi$
- (E) 18π

Questions 40–42

40. The circle $|z| = 2$ is traversed counterclockwise. What is $\frac{1}{2\pi i} \int_{|z|=2} \frac{e^z}{z^7} dz$?

- (A) $6!$
- (B) $1/5!$
- (C) $1/6!$
- (D) $1/12!$
- (E) $1/7!$

41. For $D = \{(x, y) : x^2 + y^2 \leq 2\}$, evaluate $\iint_D (x^2 + y^2)^2 dA$.

- (A) 8π
- (B) $\frac{8\pi}{3}$
- (C) $\frac{4\pi}{3}$
- (D) $\frac{16\pi}{3}$
- (E) $\frac{8\pi}{5}$

42. Let N be the 11×11 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\text{rank } N^{10}$.

- (A) 4
- (B) 3
- (C) 2
- (D) 0
- (E) 1

Questions 43–45

43. Solve $y' + 2y = x^3e^{-2x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{1}{2e^2}$
- (B) $\frac{e^2}{4}$
- (C) $\frac{1}{3e^2}$
- (D) $\frac{1}{4e^2}$
- (E) e^{-2}

44. Find the residue of $z^6/(z-1)^2$ at $z = 1$.

- (A) 2
- (B) 10
- (C) 8
- (D) 4
- (E) 6

45. Evaluate $\int_C (3x^2 + 2y) dx + (2x + 3y^2) dy$, where C is the line segment from $(0, 0)$ to $(5, 1)$.

- (A) 272
- (B) 68
- (C) 137
- (D) -136
- (E) 136

Questions 46–48

46. How many invertible 2×2 matrices over $\mathbb{F}_2$ are there with determinant 1?

- (A) $2^2 2^2$
- (B) $(2^2 - 2^1)(2^2 - 2^2)$
- (C) $(2^2 - 2^0)$
- (D) $(2^2 - 2^0)(2^2 - 2^1)$
- (E) $(2^2 - 2^0)(2^2 - 2^0)$

47. What is the maximum of $x^3 y^2$ over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $5 \left(\left(\frac{3}{5} \right)^3 \left(\frac{2}{5} \right)^2 \right)$
- (B) $\frac{1}{2^5}$
- (C) $\left(\frac{3}{5} \right)^3 \left(\frac{2}{5} \right)^2$
- (D) $\left(\frac{3}{5} \right)^2 \left(\frac{2}{5} \right)^3$
- (E) $\frac{125}{18} \left(\left(\frac{3}{5} \right)^3 \left(\frac{2}{5} \right)^2 \right)$

48. How many functions from a set of 6 elements onto a set of 3 elements exist?

- (A) 3^6
- (B) $3^6 - 3(2^6) + 3$
- (C) $3^6 - 3(2^6)$
- (D) $3^6 - 2^6$
- (E) $3^6 - 3(2^6) - 3$

Questions 49–51

49. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(2j)!}{(j!)^2} x^{3j}$?

- (A) $2^{-\frac{3}{2}}$
- (B) $2^{-\frac{2}{3}}$
- (C) 2^{-2}
- (D) $2^{\frac{2}{3}}$
- (E) $\frac{1}{2}$

50. How many permutations in S_{11} consist of one 3-cycle and 8 fixed points?

- (A) 329
- (B) 331
- (C) 332
- (D) 330
- (E) 328

51. Evaluate $\lim_{x \rightarrow 0} (1 + 2x)^{1/(2x)}$.

- (A) $2e$
- (B) e
- (C) e^3
- (D) 2
- (E) e^4

Questions 52–54

52. How many nonnegative integer triples (a, b, c) satisfy $a + b + c = 6$?

- (A) 37
- (B) 19
- (C) 46
- (D) 10
- (E) 28

53. Let $f(x, y) = x^2 + 4xy + (7)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 28
- (B) 7
- (C) 3
- (D) 6
- (E) 12

54. How many subgroups does the cyclic group of order 108 have?

- (A) 12
- (B) 11
- (C) 14
- (D) 13
- (E) 10

Questions 55–57

55. Let C be the unit circle $x^2 + y^2 = 1$, $z = 3$, counterclockwise viewed from above. Evaluate $\int_C -2y(x^2 + y^2)^2 dx + (2x(x^2 + y^2)^2) dy$.

- (A) -4π
- (B) 2π
- (C) 4π
- (D) 8π
- (E) $1 + 4\pi$

56. A tree has 10 vertices. What is the sum of its vertex degrees?

- (A) 30
- (B) 6
- (C) 12
- (D) 18
- (E) 24

57. Let $F(x) = \int_0^{x^3} e^{2t} dt$. Find $F'(1)$.

- (A) $3e^2$
- (B) $6e^2$
- (C) e^2
- (D) $\frac{e^2}{3}$
- (E) e^6

Questions 58–60

58. How many residues x modulo 18 satisfy $3x \equiv 9 \pmod{18}$?

- (A) 2
- (B) 3
- (C) 1
- (D) 4
- (E) 5

59. Find the coefficient of x^3 in the Taylor series of $(1 - 2x)^{-2}$ at zero.

- (A) $\frac{3}{4} (4 \cdot 2^3)$
- (B) $\frac{1}{2} (4 \cdot 2^3)$
- (C) $-4 \cdot 2^3$
- (D) $4 \cdot 2^3$
- (E) $\frac{1}{4} (4 \cdot 2^3)$

60. A connected planar graph has 9 vertices and 13 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 8
- (B) 10
- (C) 6
- (D) 4
- (E) 2

Questions 61–63

61. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 2z^2 = x + 3y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) -4
- (B) -3
- (C) -12
- (D) 12
- (E) -6

62. The field K has 3^2 elements. Find $[K : \mathbb{F}_3]$.

- (A) 4
- (B) 3
- (C) 1
- (D) 2
- (E) 0

63. For the curve $x = t^3$, $y = t^4 + 2t$, find d^2y/dx^2 at $t = 1$.

- (A) $\frac{8}{9}$
- (B) $-\frac{2}{3}$
- (C) 2
- (D) 1
- (E) 0

Questions 64–66

- 64.** How many integers in $\{1, \dots, 36\}$ are divisible by 2 or 3?
- (A) 24
 - (B) 8
 - (C) 40
 - (D) 16
 - (E) 32
- 65.** For $f(x, y) = x^2 + 2xy + 3y^2$, what is its maximum directional derivative at $(1, -1)$?
- (A) 4
 - (B) -4
 - (C) 0
 - (D) 16
 - (E) 8
- 66.** Let $G = \mathbb{Z}/60\mathbb{Z} \times \mathbb{Z}/44\mathbb{Z}$. Find the order of $(1, 1)$.
- (A) 661
 - (B) 662
 - (C) 660
 - (D) 659
 - (E) 658

Answer Key

Practice Test 09 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	D	23	E	45	E
2	E	24	A	46	D
3	A	25	E	47	C
4	B	26	A	48	B
5	B	27	E	49	B
6	A	28	A	50	D
7	D	29	B	51	B
8	E	30	C	52	E
9	D	31	C	53	E
10	E	32	E	54	A
11	D	33	D	55	C
12	B	34	C	56	D
13	C	35	A	57	A
14	D	36	C	58	B
15	C	37	C	59	D
16	B	38	B	60	C
17	E	39	A	61	C
18	A	40	C	62	D
19	B	41	B	63	E
20	B	42	E	64	A
21	D	43	D	65	A
22	A	44	E	66	C

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