

GRE Mathematics Practice Test 11

Advanced Challenge

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

The most demanding form in this five-test cycle. Emphasis on connected ideas, non-routine reasoning, and careful conceptual distinctions. This is cycle 2 of 19, position 1 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

zarimion.com/gre

100 practice tests, complete answer keys, and future updates.

Please retain the source attribution when sharing this material.



This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. If $y' = 6xy$ and $y(0) = 1$, find the coefficient of x^{12} in the Maclaurin series of y .
- (A) $\frac{81}{160}$
(B) $-\frac{81}{80}$
(C) $\frac{81}{80}$
(D) $\frac{81}{40}$
(E) $\frac{161}{80}$
2. Let V consist of real polynomials of degree at most 10 that have both value and first derivative zero at 7, 8, and 10. Find $\dim V$.
- (A) 5
(B) 3
(C) 7
(D) 6
(E) 4
3. For $f(x, y) = x^2 + 3xy + 6y^2$, what is its maximum directional derivative at $(1, -1)$?
- (A) -9
(B) 10
(C) 82
(D) $\sqrt{82}$
(E) -1

Questions 4–6

4. X, Y are independent, with variances 3 and 4. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 4
- (B) 0
- (C) 2
- (D) 3
- (E) 1

5. Let C be $x^2/36 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-y^3) dx + (\frac{x^3}{3}) dy$.

- (A) 117π
- (B) $\frac{117\pi}{4}$
- (C) $-\frac{117\pi}{2}$
- (D) $\frac{117\pi}{2}$
- (E) $1 + \frac{117\pi}{2}$

6. The roots r, s of $x^2 - 12x + 35 = 0$ are counted with multiplicity. Find $r^4 + s^4$.

- (A) $12^4 - 4(35)(12^2) + 2(35^2)$
- (B) $12^4 + 2(35^2)$
- (C) $12^4 - 4(35)(12^2) + 35^2$
- (D) $12^4 - 2(35)(12^2) + 2(35^2)$
- (E) $12^4 - 4(35)(12^2)$

Questions 7–9

7. Evaluate $\int_0^1 x^2(\log x)^5 dx$.

- (A) $\frac{(-1)^5 5!}{3^6}$
- (B) $\frac{1}{6} \left(\frac{(-1)^5 5!}{3^6} \right)$
- (C) $-\frac{(-1)^5 5!}{3^6}$
- (D) $3 \left(\frac{(-1)^5 5!}{3^6} \right)$
- (E) $\frac{81}{40} \left(\frac{(-1)^5 5!}{3^6} \right)$

8. X has density cx^3 on $0 < x < 1$ and zero elsewhere. Find $E[X^2]$.

- (A) $\frac{8}{9}$
- (B) $\frac{5}{6}$
- (C) $\frac{2}{3}$
- (D) $\frac{2}{9}$
- (E) $\frac{1}{3}$

9. Two solutions of $y'' + 3x^6y' + 6y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $\frac{1}{2e^{\frac{3}{7}}}$
- (B) $e^{-\frac{3}{7}}$
- (C) $\frac{2}{e^{\frac{3}{7}}}$
- (D) $e^{-\frac{3}{7}} + 1$
- (E) $-\frac{1}{e^{\frac{3}{7}}}$

Questions 10–12

10. For real t , let $M(t) = \begin{pmatrix} t-7 & 5 & 0 \\ 0 & t-7 & 1 \\ 0 & 0 & t-7 \end{pmatrix}$. At $t = 7$, what is $\dim \ker(M(t)^2) - \dim \ker M(t)$?

- (A) 2
- (B) 1
- (C) 0
- (D) 3
- (E) 4

11. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(3j)!}{(j!)^3} x^{6j}$?

- (A) 3^{-3}
- (B) 3^{-2}
- (C) $3^{-\frac{1}{2}}$
- (D) $\frac{1}{3}$
- (E) $3^{\frac{1}{2}}$

12. Independent exponential waiting times X, Y have rates 3 and 5. Find $E[\min(X, Y)]$.

- (A) $\frac{1}{24}$
- (B) $\frac{1}{16}$
- (C) $\frac{1}{8}$
- (D) $\frac{9}{16}$
- (E) $\frac{7}{8}$

Questions 13–15

13. Evaluate $\int_0^\infty x^6 e^{-3x} dx$.

- (A) $\frac{1}{6} \left(\frac{6!}{3^7} \right)$
- (B) $2187 \left(\frac{6!}{3^7} \right)$
- (C) $\frac{6!}{3^7}$
- (D) $3 \left(\frac{6!}{3^7} \right)$
- (E) $\frac{1}{3} \left(\frac{6!}{3^7} \right)$

14. Let $u = (2, 6)^T$ and $v = (1, 0)^T$. Find the squared distance from $(2, 0)^T$ to $\text{span}(u)$.

- (A) $\frac{23}{5}$
- (B) $\frac{18}{5}$
- (C) $\frac{41}{10}$
- (D) $\frac{31}{10}$
- (E) $\frac{13}{5}$

15. Find the total outward flux of $\mathbf{F} = (3x^6y, y^6z, xz^6)$ through the boundary of $[0, 1]^3$.

- (A) 5
- (B) $-\frac{5}{2}$
- (C) $\frac{7}{2}$
- (D) $\frac{5}{2}$
- (E) $\frac{5}{4}$

Questions 16–18

16. $X_1, \dots, X_3$ are independent with common mean and variance 5. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 16
- (B) 10
- (C) 4
- (D) 7
- (E) 13

17. For $x > 0$, $x^2 y'' + (1 - 9)xy' + 18y = 0$, $y(1) = 0$, $y'(1) = 3$. Find $y(e)$.

- (A) $\frac{e^3}{9}$
- (B) $e^3 + e^6$
- (C) $-e^6 + e^3$
- (D) $-e^3 + e^6$
- (E) $-2(1 - e^3)e^3$

18. An idempotent 13×13 real matrix P has rank 3. What is $\det(I + P)$?

- (A) 9
- (B) 8
- (C) 6
- (D) 10
- (E) 7

Questions 19–21

19. Let $f(x) = \log(1 + 3x)$. Find $f^{(6)}(0)$.

- (A) $\frac{1}{3} \left((-1)^5 5! 3^6 \right)$
- (B) $6 \left((-1)^5 5! 3^6 \right)$
- (C) $\frac{1}{6} \left((-1)^5 5! 3^6 \right)$
- (D) $(-1)^5 5! 3^6$
- (E) $-(-1)^5 5! 3^6$

20. A permutation of 5 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z^2]$.

- (A) 2
- (B) 4
- (C) 3
- (D) 0
- (E) 1

21. Evaluate $\int_0^1 \int_x^1 3y^6 \, dy \, dx$.

- (A) $\frac{3}{2}$
- (B) $\frac{1}{2}$
- (C) $\frac{3}{56}$
- (D) $\frac{3}{8}$
- (E) $\frac{3}{7}$

Questions 22–24

22. For $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 7 \\ 5 \\ 12 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find the minimum squared residual.

- (A) $\frac{29}{2}$
- (B) $\frac{25}{2}$
- (C) 13
- (D) $\frac{27}{2}$
- (E) 14

23. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(3x) - \sum_{j=0}^4 (-1)^j (3x)^{2j+1} / (2j+1)!}{x^{11}}$.

- (A) $\frac{(-1)^5 3^{11}}{11!}$
- (B) $11 \left(\frac{(-1)^5 3^{11}}{11!} \right)$
- (C) $3 \left(\frac{(-1)^5 3^{11}}{11!} \right)$
- (D) $-\frac{(-1)^5 3^{11}}{11!}$
- (E) $\frac{1}{3} \left(\frac{(-1)^5 3^{11}}{11!} \right)$

24. Which subset of $\mathbb{R}$ is compact, with 3 fixed as displayed?

- (A) $3\mathbb{Z}$
- (B) $\{3/k : k \geq 1\}$
- (C) $\{0\} \cup \{3/k : k \geq 1\}$
- (D) $\mathbb{Q} \cap [0, 3]$
- (E) $(0, 3)$

Questions 25–27

25. Evaluate $\int_0^{\pi/2} \sin^{13} t \cos^7 t \, dt$.

- (A) $2 \left(\frac{6! 3!}{2 \cdot 10!} \right)$
- (B) $84 \left(\frac{6! 3!}{2 \cdot 10!} \right)$
- (C) $10 \left(\frac{6! 3!}{2 \cdot 10!} \right)$
- (D) $\frac{1}{2} \left(\frac{6! 3!}{2 \cdot 10!} \right)$
- (E) $\frac{6! 3!}{2 \cdot 10!}$

26. What is the coefficient of x^8 in $(x + 2)^8(1 - x)^3$?

- (A) -157
- (B) -158
- (C) -161
- (D) -160
- (E) -159

27. If $y' + 3y = 6x^6 e^{-3x}$ and $y(0) = 2$, find $y(1)$.

- (A) $\frac{40}{7e^3}$
- (B) $\frac{10}{7e^3}$
- (C) $\frac{20}{7e^3}$
- (D) $\frac{20}{7e^3} + 1$
- (E) $-\frac{20}{7e^3}$

Questions 28–30

28. For $a_k = (-1)^k + 3/k$, find $\limsup_{k \rightarrow \infty} a_k - \liminf_{k \rightarrow \infty} a_k$.
- (A) 1
 - (B) 3
 - (C) 0
 - (D) 4
 - (E) 2
29. Find the upward flux of $\mathbf{F} = (x, y, z)$ through $z = 3x^2 + 6y^2$, $0 \leq x, y \leq 1$.
- (A) -3
 - (B) -2
 - (C) $-\frac{3}{2}$
 - (D) 3
 - (E) -6
30. A matrix A satisfies $(A - 8I)(A - 10I) = 0$, and its minimal polynomial has degree two. If $A^5 = uA + vI$, find u .
- (A) 33614
 - (B) 33615
 - (C) 33616
 - (D) 33618
 - (E) 33617

Questions 31–33

31. The region $0 \leq x \leq 1$, $0 \leq y \leq 3x^6$ is revolved about the line $x = -2$. Find its volume.

- (A) $\frac{69\pi}{28}$
- (B) $\frac{69\pi}{14}$
- (C) $-\frac{69\pi}{28}$
- (D) $1 + \frac{69\pi}{28}$
- (E) $\frac{69\pi}{56}$

32. For which real a is $f(x) = x^a$ in $L^3(0, 1)$?

- (A) $a \geq -1/3$
- (B) $a > -1/3$
- (C) All real a
- (D) $a > 0$
- (E) $a < 1/3$

33. Solve $y' + 3y = x^6 e^{-3x}$, $y(0) = 0$, and find $y(1)$.

- (A) e^{-3}
- (B) $\frac{e^3}{7}$
- (C) $\frac{1}{6e^3}$
- (D) $\frac{1}{7e^3}$
- (E) $\frac{3}{7e^3}$

Questions 34–36

34. Let $V = \begin{bmatrix} 1 & 7 & 49 \\ 1 & 9 & 81 \\ 1 & 12 & 144 \end{bmatrix}$. Find $\det(V^T V)$.

- (A) 900
- (B) 901
- (C) 902
- (D) 899
- (E) 898

35. Find the coefficient of x^4 in $e^{5x} \log(1 + 3x)$.

- (A) $-\frac{81}{4}$
- (B) 31
- (C) 744
- (D) 34
- (E) -31

36. $T : \mathbb{R} \rightarrow \mathbb{R}$ has Lipschitz constant at most $1/3$. If $x_1 = T(x_0)$ and $|x_1 - x_0| \leq 1$, what is the sharp upper bound on $|T^3(x_0) - x_*|$, where x_* is its fixed point?

- (A) $\frac{17}{18}$
- (B) $\frac{1}{54}$
- (C) $\frac{1}{36}$
- (D) $\frac{19}{36}$
- (E) $\frac{1}{18}$

Questions 37–39

37. For $D = \{(x, y) : x^2 + y^2 \leq 3\}$, evaluate $\iint_D (x^2 + y^2)^5 dA$.
- (A) 729π
(B) $\frac{81\pi}{2}$
(C) $\frac{729\pi}{11}$
(D) 243π
(E) $\frac{243\pi}{2}$
38. Let $u = (1, 2, \dots, 13)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $v^T B^{-1} u$.
- (A) $\frac{183}{92}$
(B) $\frac{137}{92}$
(C) $\frac{229}{92}$
(D) $\frac{91}{92}$
(E) $\frac{45}{92}$
39. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 3z^2 = x + 6y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.
- (A) -18
(B) -72
(C) 36
(D) -36
(E) -6

Questions 40–42

40. How many zeros, counted with multiplicity, does $z^3 + 6z + 1$ have in $|z| < 1$?

- (A) 1
- (B) 2
- (C) 0
- (D) 3
- (E) 4

41. Evaluate $\lim_{x \rightarrow 0} \frac{e^{3x} - \sum_{j=0}^5 (3x)^j / j!}{x^6}$.

- (A) $\frac{1}{3} \left(\frac{3^6}{6!} \right)$
- (B) $\frac{3^6}{6!}$
- (C) $6 \left(\frac{3^6}{6!} \right)$
- (D) $720 \left(\frac{3^6}{6!} \right)$
- (E) $-\frac{3^6}{6!}$

42. How many maximal proper subgroups does the cyclic group of order 132 have?

- (A) 5
- (B) 2
- (C) 1
- (D) 4
- (E) 3

Questions 43–45

43. Evaluate $\int_0^1 x^3(1-x)^5 dx$.

- (A) $4 \left(\frac{3! 5!}{9!} \right)$
- (B) $\frac{1}{6} \left(\frac{3! 5!}{9!} \right)$
- (C) $\frac{3! 5!}{9!}$
- (D) $21 \left(\frac{3! 5!}{9!} \right)$
- (E) $9 \left(\frac{3! 5!}{9!} \right)$

44. A meromorphic function has 5 zeros and 2 poles inside a positively oriented simple closed contour, counted with multiplicity, and no zeros or poles on it. Find $\frac{1}{2\pi i} \int f'(z)/f(z) dz$.

- (A) 2
- (B) 4
- (C) 5
- (D) 3
- (E) 1

45. Find the directional derivative of $f(x, y, z) = x^2y + 6xz^2 + 3y^2z$ at $(1, 1, 1)$ in the direction $(1, 2, 0)$.

- (A) $-\frac{22\sqrt{5}}{5}$
- (B) $\frac{44\sqrt{5}}{5}$
- (C) $\frac{11\sqrt{5}}{5}$
- (D) $1 + \frac{22\sqrt{5}}{5}$
- (E) $\frac{22\sqrt{5}}{5}$

Questions 46–48

46. How many residues x modulo 36 satisfy $6x \equiv 36 \pmod{36}$ and $2x \equiv 0 \pmod{36}$?

- (A) 0
- (B) 4
- (C) 1
- (D) 2
- (E) 3

47. For $x > 0$, $x^2y'' + (-11)xy' + 27y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $-\frac{(1 - e^6)e^3}{3}$
- (B) $\frac{(1 - e^6)e^3}{6}$
- (C) $-\frac{(1 - e^6)e^3}{6}$
- (D) $-\frac{e^3}{6} + 1 + \frac{e^9}{6}$
- (E) $-\frac{(1 - e^6)e^3}{12}$

48. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 3 have no adjacent 1s?

- (A) F_7
- (B) F_5
- (C) F_3
- (D) F_4
- (E) F_6

Questions 49–51

49. If $y'' + 36y = 3$ and $y(0) = y'(0) = 0$, find $y(\pi/6)$.

- (A) $-\frac{1}{6}$
- (B) $\frac{7}{6}$
- (C) $\frac{1}{6}$
- (D) $\frac{1}{3}$
- (E) $\frac{1}{12}$

50. The field K has 11^3 elements. Find the largest possible multiplicative order of a nonzero element of K .

- (A) $11^3 - 1$
- (B) 11^3
- (C) $11^2 - 1$
- (D) $11^3 + 1$
- (E) $11^4 - 1$

51. Let $F(x) = \int_0^x e^{3t} dt$. Find $F'(1)$.

- (A) e^{18}
- (B) $\frac{e^3}{6}$
- (C) $6e^3$
- (D) $18e^3$
- (E) e^3

Questions 52–54

52. For Newton iteration on $f(x) = x^2 - 9$ near the root $r = 3$, find $\lim_{k \rightarrow \infty} (x_{k+1} - r)/(x_k - r)^2$ for nonterminating convergent iterates.

- (A) $\frac{1}{6}$
- (B) $\frac{7}{12}$
- (C) $\frac{1}{12}$
- (D) $\frac{5}{6}$
- (E) $\frac{1}{18}$

53. Let $f(x, y) = e^{3x+6y}$. Find $\partial^6 f / \partial x^5 \partial y$ at $(0, 0)$.

- (A) $16(6 \cdot 3^5)$
- (B) $\frac{1}{6}(6 \cdot 3^5)$
- (C) $\frac{1}{2}(6 \cdot 3^5)$
- (D) $32(6 \cdot 3^5)$
- (E) $6 \cdot 3^5$

54. Let $G = \mathbb{Z}/72\mathbb{Z} \times \mathbb{Z}/52\mathbb{Z}$. Find the order of G/H , where $H = \{g \in G : 4g = 0\}$.

- (A) 235
- (B) 236
- (C) 234
- (D) 232
- (E) 233

Questions 55–57

55. Under $x = u + 3v$, $y = u - 6v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) 9
- (B) 81
- (C) 18
- (D) $\frac{1}{9}$
- (E) 3

56. Bisection starts with an interval of length 2^3 . After 5 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{1}{16}$
- (B) $\frac{7}{8}$
- (C) $\frac{9}{16}$
- (D) $\frac{1}{24}$
- (E) $\frac{1}{8}$

57. Find the coefficient of x^6 in the Taylor series of $(1 - 3x)^{-2}$ at zero.

- (A) $\frac{1}{3} (7 \cdot 3^6)$
- (B) $7 \cdot 3^6$
- (C) $2 (7 \cdot 3^6)$
- (D) $\frac{6}{7} (7 \cdot 3^6)$
- (E) $\frac{1}{7} (7 \cdot 3^6)$

Questions 58–60

58. How many group homomorphisms $\mathbb{Z}/72\mathbb{Z} \rightarrow \mathbb{Z}/52\mathbb{Z}$ have image of order 4?

- (A) 0
- (B) 1
- (C) 3
- (D) 4
- (E) 2

59. Evaluate $\int_0^{\log 2} e^x(e^x + 3)^6 dx$.

- (A) $\frac{5^7}{7}$
- (B) $7 \left(\frac{5^7 - 4^7}{7} \right)$
- (C) $-\frac{5^7 - 4^7}{7}$
- (D) $\frac{5^7 - 4^7}{7}$
- (E) $5^6 - 4^6$

60. What is the least positive multiplicative inverse of 2 modulo 7?

- (A) 3
- (B) 4
- (C) 2
- (D) 5
- (E) 6

Questions 61–63

61. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 3x)^{1/(5x)} - e^{3/5}}{x}$.
- (A) $\frac{9e^{\frac{3}{5}}}{10}$
(B) $-\frac{9e^{\frac{3}{5}}}{10}$
(C) $-\frac{9e^{\frac{3}{5}}}{5}$
(D) $e^{\frac{3}{5}}$
(E) $-\frac{3e^{\frac{3}{5}}}{10}$
62. How many 2-dimensional subspaces does $\mathbb{F}_3^7$ have?
- (A) $\frac{(3^{10} - 1)(3^{10} - 3)}{(3^2 - 1)(3^2 - 3)}$
(B) $\frac{(3^8 - 1)(3^8 - 3)}{(3^2 - 1)(3^2 - 3)}$
(C) $\frac{(3^9 - 1)(3^9 - 3)}{(3^2 - 1)(3^2 - 3)}$
(D) $\frac{(3^6 - 1)(3^6 - 3)}{(3^2 - 1)(3^2 - 3)}$
(E) $\frac{(3^7 - 1)(3^7 - 3)}{(3^2 - 1)(3^2 - 3)}$
63. If $y'' - 6y' + 9y = 6e^{3x}$ and $y(0) = y'(0) = 0$, find $y(1)$.
- (A) $-3e^3$
(B) $3e^3$
(C) $\frac{3e^3}{2}$
(D) $1 + 3e^3$
(E) $6e^3$

Questions 64–66

64. How many truth assignments to $p, q, r_1, \dots, r_1$ make $p \Rightarrow q$ false?

- (A) 2^3
- (B) 2^4
- (C) 2^2
- (D) 2^1
- (E) 2^0

65. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-3)^j}{j^{\frac{5}{3}} (\log j)^m}$ converge absolutely at $x = 2$? Give the least such m , or 0 if none exists.

- (A) 1
- (B) 4
- (C) 2
- (D) 3
- (E) 0

66. How many monic irreducible polynomials of degree 5 are there over $\mathbb{F}_3$?

- (A) $\frac{3(3^5 - 3)}{5}$
- (B) $2(3^5 - 3)$
- (C) $\frac{2(3^5 - 3)}{5}$
- (D) $3^5 - 3$
- (E) $\frac{3^5 - 3}{5}$

Answer Key

Practice Test 11 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	C	23	A	45	E
2	A	24	C	46	D
3	D	25	E	47	C
4	E	26	E	48	B
5	D	27	C	49	C
6	A	28	E	50	A
7	A	29	A	51	C
8	C	30	C	52	A
9	B	31	A	53	E
10	B	32	B	54	C
11	C	33	D	55	A
12	C	34	A	56	E
13	C	35	B	57	B
14	B	36	E	58	E
15	D	37	E	59	D
16	B	38	D	60	B
17	D	39	D	61	B
18	B	40	A	62	E
19	D	41	B	63	B
20	A	42	E	64	D
21	D	43	C	65	A
22	D	44	D	66	E

zarimion.com/gre

100 practice tests, complete answer keys, and future updates.
Please retain the source attribution when sharing this material.

