

GRE Mathematics Practice Test 12

Advanced Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A demanding practice form with frequent multi-step arguments and a mix of computational and conceptual questions. This is cycle 2 of 19, position 2 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. For $x > 0$, $x^2 y'' + (1 - 8)xy' + 15y = 0$, $y(1) = 0$, $y'(1) = 3$. Find $y(e)$.

- (A) $-\frac{3(1 - e^2)e^3}{2}$
(B) $\frac{3(1 - e^2)e^3}{2}$
(C) $-e^5 + e^3$
(D) $e^3 + e^5$
(E) $\frac{e^3}{8}$

2. For $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 8 \\ 3 \\ 11 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find the minimum squared residual.

- (A) $\frac{163}{6}$
(B) $\frac{175}{6}$
(C) $\frac{169}{6}$
(D) $\frac{83}{3}$
(E) $\frac{86}{3}$

3. Let $f(x) = \log(1 + 3x)$. Find $f^{(5)}(0)$.

- (A) $-(-1)^4 4! 3^5$
(B) $5 \left((-1)^4 4! 3^5 \right)$
(C) $\frac{1}{5} \left((-1)^4 4! 3^5 \right)$
(D) $(-1)^4 4! 3^5$
(E) $\frac{1}{3} \left((-1)^4 4! 3^5 \right)$

Questions 4–6

4. X, Y are independent, with variances 4 and 5. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 0
- (B) 4
- (C) 1
- (D) 2
- (E) 3

5. Find the curvature of the helix $r(t) = (3 \cos t, 3 \sin t, 5t)$.

- (A) $-\frac{3}{34}$
- (B) $\frac{3}{34}$
- (C) $\frac{3}{68}$
- (D) $\frac{3}{17}$
- (E) $\frac{37}{34}$

6. What is the coefficient of x^7 in $(x + 2)^7(1 - x)^3$?

- (A) -69
- (B) -67
- (C) -68
- (D) -70
- (E) -71

Questions 7–9

7. Evaluate $\int_0^1 x^3(1-x)^4 dx$.

- (A) $\frac{1}{5} \left(\frac{3! 4!}{8!} \right)$
- (B) $4 \left(\frac{3! 4!}{8!} \right)$
- (C) $8 \left(\frac{3! 4!}{8!} \right)$
- (D) $\frac{3! 4!}{8!}$
- (E) $14 \left(\frac{3! 4!}{8!} \right)$

8. X has density cx^4 on $0 < x < 1$ and zero elsewhere. Find $E[X^2]$.

- (A) $\frac{2}{7}$
- (B) $\frac{5}{14}$
- (C) $\frac{5}{7}$
- (D) $\frac{5}{21}$
- (E) $\frac{6}{7}$

9. Let $F(x) = \int_0^{x^5} e^{3t} dt$. Find $F'(1)$.

- (A) $15e^3$
- (B) e^3
- (C) e^{15}
- (D) $5e^3$
- (E) $\frac{e^3}{5}$

Questions 10–12

10. A matrix A satisfies $(A - 9I)(A - 11I) = 0$, and its minimal polynomial has degree two. If $A^5 = uA + vI$, find u .

- (A) 51003
- (B) 51002
- (C) 51001
- (D) 50999
- (E) 51000

11. What is the maximum of x^5y^3 over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\left(\frac{5}{8}\right)^3 \left(\frac{3}{8}\right)^5$
- (B) $8 \left(\left(\frac{5}{8}\right)^5 \left(\frac{3}{8}\right)^3 \right)$
- (C) $\frac{15}{64}$
- (D) $\left(\frac{5}{8}\right)^5 \left(\frac{3}{8}\right)^3$
- (E) $\frac{1}{2^8}$

12. Independent exponential waiting times X, Y have rates 4 and 6. Find $P(X < Y)$.

- (A) $\frac{1}{5}$
- (B) $\frac{2}{15}$
- (C) $\frac{2}{5}$
- (D) $\frac{3}{5}$
- (E) $\frac{7}{10}$

Questions 13–15

13. Find the arc length of $r(t) = (3(t - t^3/3), 3t^2)$ for $0 \leq t \leq 1/5$.

- (A) $\frac{201}{125}$
- (B) $\frac{76}{125}$
- (C) $-\frac{76}{125}$
- (D) $\frac{38}{125}$
- (E) $\frac{152}{125}$

14. Let $V = \begin{bmatrix} 1 & 8 & 64 \\ 1 & 10 & 100 \\ 1 & 13 & 169 \end{bmatrix}$. Find $\det(V^T V)$.

- (A) 898
- (B) 900
- (C) 902
- (D) 901
- (E) 899

15. Find the coefficient of x^4 in $e^{4x} \log(1 + 3x)$.

- (A) $\frac{59}{4}$
- (B) $-\frac{47}{4}$
- (C) $\frac{47}{4}$
- (D) 282
- (E) $-\frac{81}{4}$

Questions 16–18

16. $X_1, \dots, X_4$ are independent with common mean and variance 6. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 18
- (B) 12
- (C) 24
- (D) 6
- (E) 30

17. For $\mathbf{F} = (3x^5y, y^5z, xz^5)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) 116
- (B) $\frac{115}{2}$
- (C) 115
- (D) -115
- (E) 230

18. Let $u = (1, 2, \dots, 14)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $v^T B^{-1}u$.

- (A) $\frac{79}{53}$
- (B) $\frac{211}{106}$
- (C) $\frac{132}{53}$
- (D) $\frac{105}{106}$
- (E) $\frac{26}{53}$

Questions 19–21

19. If $y' = 6xy$ and $y(0) = 1$, find the coefficient of x^{10} in the Maclaurin series of y .
- (A) $\frac{81}{40}$
(B) $\frac{81}{20}$
(C) $\frac{121}{40}$
(D) $\frac{81}{80}$
(E) $-\frac{81}{40}$
20. A permutation of 6 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z^2]$.
- (A) 1
(B) 2
(C) 0
(D) 3
(E) 4
21. Solve $y' + 3y = x^5e^{-3x}$, $y(0) = 0$, and find $y(1)$.
- (A) $\frac{e^3}{6}$
(B) $\frac{1}{5e^3}$
(C) $\frac{1}{2e^3}$
(D) $\frac{1}{6e^3}$
(E) e^{-3}

Questions 22–24

22. Let $D = \text{diag}(\underbrace{1, \dots, 1}_1, \underbrace{2, \dots, 2}_{13})$. Find the dimension of the space of real matrices X with $XD = DX$.

- (A) 172
- (B) 171
- (C) 170
- (D) 168
- (E) 169

23. Evaluate $\int_0^1 \int_x^1 3y^5 \, dy \, dx$.

- (A) $\frac{1}{14}$
- (B) $\frac{3}{5}$
- (C) $\frac{3}{2}$
- (D) $\frac{3}{7}$
- (E) $\frac{1}{2}$

24. Which subset of $\mathbb{R}$ is compact, with 4 fixed as displayed?

- (A) $(0, 4)$
- (B) $\mathbb{Q} \cap [0, 4]$
- (C) $\{0\} \cup \{4/k : k \geq 1\}$
- (D) $4\mathbb{Z}$
- (E) $\{4/k : k \geq 1\}$

Questions 25–27

25. Two solutions of $y'' + 3x^5y' + 5y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $-\frac{1}{e^{\frac{1}{2}}}$
- (B) $\frac{1}{2e^{\frac{1}{2}}}$
- (C) $\frac{2}{e^{\frac{1}{2}}}$
- (D) $e^{-\frac{1}{2}} + 1$
- (E) $e^{-\frac{1}{2}}$

26. When $x^3 + 1$ is divided by $(x - 9)(x - 10)$, the remainder is $ux + v$. Find u .

- (A) 273
- (B) 272
- (C) 271
- (D) 270
- (E) 269

27. Let $f(x, y) = x^2 + 6xy + (14)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 5
- (B) 56
- (C) 10
- (D) 14
- (E) 20

Questions 28–30

28. For $a_k = (-1)^k + 4/k$, find $\limsup_{k \rightarrow \infty} a_k - \liminf_{k \rightarrow \infty} a_k$.

- (A) 4
- (B) 3
- (C) 0
- (D) 1
- (E) 2

29. The solution of $y' = 3x^5y^3$ has $y(0) = 1/4$. Find $y(1)$.

- (A) $\frac{\sqrt{15}}{15}$
- (B) $\frac{\sqrt{15}}{30}$
- (C) $\frac{2\sqrt{15}}{15}$
- (D) $\frac{\sqrt{15}}{15} + 1$
- (E) $-\frac{\sqrt{15}}{15}$

30. Let $P = \text{diag}(\underbrace{1, \dots, 1}_2, \underbrace{0, \dots, 0}_{13})$. What is the dimension of the space of 15×15 real matrices X satisfying $PX = X$ and $XP = 0$?

- (A) 27
- (B) 26
- (C) 28
- (D) 24
- (E) 25

Questions 31–33

31. Let C be $x^2/25 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-y^3) dx + (\frac{x^3}{3}) dy$.

- (A) 35π
- (B) $\frac{35\pi}{2}$
- (C) 70π
- (D) -35π
- (E) $1 + 35\pi$

32. For which real a is $f(x) = x^a$ in $L^4(0, 1)$?

- (A) $a < 1/4$
- (B) $a > -1/4$
- (C) $a \geq -1/4$
- (D) All real a
- (E) $a > 0$

33. Evaluate $\int_0^1 \frac{x^4}{1 + 3x^5} dx$.

- (A) $-\frac{2 \log(2)}{15}$
- (B) $\log(4)$
- (C) $\frac{2 \log(2)}{3}$
- (D) $\frac{2 \log(2)}{5}$
- (E) $\frac{2 \log(2)}{15}$

Questions 34–36

34. A 3×3 matrix A has characteristic polynomial $(t - 8)(t + 1)(t - 2)$. What is $\text{tr}(A^3 - 2A + I)$?
- (A) 506
(B) 505
(C) 502
(D) 503
(E) 504
35. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(3x) - \sum_{j=0}^3 (-1)^j (3x)^{2j+1} / (2j+1)!}{x^9}$.
- (A) $\frac{1}{3} \left(\frac{(-1)^4 3^9}{9!} \right)$
(B) $-\frac{(-1)^4 3^9}{9!}$
(C) $9 \left(\frac{(-1)^4 3^9}{9!} \right)$
(D) $3 \left(\frac{(-1)^4 3^9}{9!} \right)$
(E) $\frac{(-1)^4 3^9}{9!}$
36. $T : \mathbb{R} \rightarrow \mathbb{R}$ has Lipschitz constant at most $1/4$. If $x_1 = T(x_0)$ and $|x_1 - x_0| \leq 1$, what is the sharp upper bound on $|T^3(x_0) - x_*|$, where x_* is its fixed point?
- (A) $\frac{1}{48}$
(B) $\frac{1}{49}$
(C) $\frac{1}{96}$
(D) $\frac{1}{144}$
(E) $\frac{1}{47}$

Questions 37–39

- 37.** Find the total outward flux of $\mathbf{F} = (3x^5y, y^5z, xz^5)$ through the boundary of $[0, 1]^3$.
- (A) $\frac{5}{4}$
 - (B) 5
 - (C) $\frac{7}{2}$
 - (D) $\frac{5}{2}$
 - (E) $-\frac{5}{2}$
- 38.** Let N be the 14×14 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\dim(\ker N^{28} / \ker N^{14})$.
- (A) 3
 - (B) 0
 - (C) 1
 - (D) 2
 - (E) 4
- 39.** Under $x = u + 3v$, $y = u - 5v$, what is the area of the image of $0 \leq u, v \leq 1$?
- (A) 15
 - (B) 8
 - (C) $\frac{1}{8}$
 - (D) 64
 - (E) 2

Questions 40–42

40. How many zeros, counted with multiplicity, does $z^4 + 7z + 1$ have in $|z| < 1$?

- (A) 4
- (B) 0
- (C) 3
- (D) 2
- (E) 1

41. The local inverse $y(x)$ of $x = 3y + y^3$ satisfies $y(0) = 0$. Find $y^{(5)}(0)$.

- (A) $\frac{360}{3^7}$
- (B) $3\left(\frac{360}{3^7}\right)$
- (C) $-\frac{360}{3^7}$
- (D) $\frac{1}{120}\left(\frac{360}{3^7}\right)$
- (E) $\frac{1}{3}\left(\frac{360}{3^7}\right)$

42. How many elements z of $\mathbb{F}_{7^3}$ satisfy $z^{49} = z$?

- (A) 7
- (B) 5
- (C) 8
- (D) 9
- (E) 6

Questions 43–45

43. If $y'' - 6y' + 9y = 5e^{3x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $\frac{5e^3}{4}$
- (B) $1 + \frac{5e^3}{2}$
- (C) $5e^3$
- (D) $\frac{5e^3}{2}$
- (E) $-\frac{5e^3}{2}$

44. A meromorphic function has 6 zeros and 2 poles inside a positively oriented simple closed contour, counted with multiplicity, and no zeros or poles on it. Find $\frac{1}{2\pi i} \int f'(z)/f(z) dz$.

- (A) 6
- (B) 4
- (C) 5
- (D) 3
- (E) 2

45. If $y'' + 25y = 3$ and $y(0) = y'(0) = 0$, find $y(\pi/5)$.

- (A) $\frac{12}{25}$
- (B) $\frac{6}{25}$
- (C) $-\frac{6}{25}$
- (D) $\frac{31}{25}$
- (E) $\frac{3}{25}$

Questions 46–48

46. How many generators does the cyclic group of order 243 have?

- (A) $3^4(3 - 1)$
- (B) $3^5(3 - 1)$
- (C) $3^7(3 - 1)$
- (D) $3^3(3 - 1)$
- (E) $3^6(3 - 1)$

47. For $x > 0$, $x^2y'' + (-10)xy' + 24y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $-\frac{2(1 - e^5)e^3}{5}$
- (B) $-\frac{(1 - e^5)e^3}{10}$
- (C) $-\frac{(1 - e^5)e^3}{5}$
- (D) $\frac{(1 - e^5)e^3}{5}$
- (E) $-\frac{e^3}{5} + 1 + \frac{e^8}{5}$

48. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 4 have no adjacent 1s?

- (A) F_7
- (B) F_5
- (C) F_8
- (D) F_6
- (E) F_4

Questions 49–51

49. Find the coefficient of x^6 in $\cos(3x)/(1 - 4x^2)$.

- (A) $-\frac{359}{80}$
- (B) $-\frac{81}{80}$
- (C) $\frac{359}{80}$
- (D) $\frac{3231}{80}$
- (E) $\frac{279}{80}$

50. Let $R = (\mathbb{Z}/40\mathbb{Z})/([5])$, where $([5])$ is the ideal generated by $[5]$. How many units does R have?

- (A) 5
- (B) 3
- (C) 2
- (D) 6
- (E) 4

51. Find the coefficient of x^5 in the Taylor series of $(1 - 3x)^{-2}$ at zero.

- (A) $\frac{5}{6} (6 \cdot 3^5)$
- (B) $-6 \cdot 3^5$
- (C) $\frac{1}{3} (6 \cdot 3^5)$
- (D) $\frac{1}{6} (6 \cdot 3^5)$
- (E) $6 \cdot 3^5$

Questions 52–54

52. For Newton iteration on $f(x) = x^2 - 16$ near the root $r = 4$, find $\lim_{k \rightarrow \infty} (x_{k+1} - r)/(x_k - r)^2$ for nonterminating convergent iterates.

- (A) $\frac{1}{24}$
- (B) $\frac{1}{8}$
- (C) $\frac{9}{16}$
- (D) $\frac{7}{8}$
- (E) $\frac{1}{16}$

53. Find the directional derivative of $f(x, y, z) = x^2y + 5xz^2 + 3y^2z$ at $(1, 1, 1)$ in the direction $(1, 3, 0)$.

- (A) $\frac{14\sqrt{10}}{5}$
- (B) $1 + \frac{14\sqrt{10}}{5}$
- (C) $-\frac{14\sqrt{10}}{5}$
- (D) $\frac{7\sqrt{10}}{5}$
- (E) $\frac{28\sqrt{10}}{5}$

54. What is the largest integer k such that 3^k divides $17!$?

- (A) 5
- (B) 4
- (C) 6
- (D) 7
- (E) 8

Questions 55–57

55. Evaluate $\iiint_{x^2+y^2+z^2 \leq 3} (x^2 + y^2 + z^2)^4 dV$.

- (A) $\frac{4\pi 3^{\frac{11}{2}}}{11}$
- (B) $\frac{11}{27} \left(\frac{4\pi 3^{\frac{11}{2}}}{11} \right)$
- (C) $\frac{1}{3} \left(\frac{4\pi 3^{\frac{11}{2}}}{11} \right)$
- (D) $\frac{1}{2} \left(\frac{4\pi 3^{\frac{11}{2}}}{11} \right)$
- (E) $11 \left(\frac{4\pi 3^{\frac{11}{2}}}{11} \right)$

56. Bisection starts with an interval of length 2^4 . After 6 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{1}{16}$
- (B) $\frac{9}{16}$
- (C) $\frac{1}{8}$
- (D) $\frac{1}{24}$
- (E) $\frac{7}{8}$

57. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 3z^2 = x + 5y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) 30
- (B) -15
- (C) -6
- (D) -5
- (E) -30

Questions 58–60

58. In the ring $\mathbb{Z}/72\mathbb{Z}$, how many elements satisfy $x^2 = 1$?

- (A) 6
- (B) 7
- (C) 9
- (D) 8
- (E) 10

59. For $f(x) = x^5 e^{3x}$, find $f^{(7)}(0)$.

- (A) $\frac{2}{3} \left(\frac{7! 3^2}{2} \right)$
- (B) $\frac{1}{42} \left(\frac{7! 3^2}{2} \right)$
- (C) $\frac{7! 3^2}{2}$
- (D) $2 \left(\frac{7! 3^2}{2} \right)$
- (E) $\frac{1}{2520} \left(\frac{7! 3^2}{2} \right)$

60. What is the least positive multiplicative inverse of 2 modulo 9?

- (A) 3
- (B) 6
- (C) 4
- (D) 7
- (E) 5

Questions 61–63

61. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^3 y^4 dV$.

- (A) $6 \left(\frac{3! 4!}{10!} \right)$
- (B) $\frac{1}{20}$
- (C) $\frac{1}{6} \left(\frac{3! 4!}{10!} \right)$
- (D) $\frac{3! 4!}{10!}$
- (E) $10 \left(\frac{3! 4!}{10!} \right)$

62. A permutation consists of disjoint cycles of lengths 8 and 8, together with a disjoint transposition when the sum of these two lengths is odd. It fixes all other points. What is its order?

- (A) 9
- (B) 7
- (C) 6
- (D) 10
- (E) 8

63. Evaluate $\int_0^1 \int_0^x x^3 y^5 dy dx$.

- (A) $\frac{1}{10}$
- (B) $\frac{1}{60}$
- (C) $\frac{1}{54}$
- (D) $\frac{1}{24}$
- (E) $\frac{1}{70}$

Questions 64–66

64. How many truth assignments to $p, q, r_1, \dots, r_2$ make $p \Rightarrow q$ false?

- (A) 2^4
- (B) 2^2
- (C) 2^5
- (D) 2^1
- (E) 2^3

65. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(3j)!}{(j!)^3} x^{5j}$?

- (A) $3^{-\frac{3}{5}}$
- (B) $\frac{1}{3}$
- (C) $3^{\frac{3}{5}}$
- (D) 3^{-3}
- (E) $3^{-\frac{5}{3}}$

66. In $R = \mathbb{F}_7[x]/(x^3)$, how many elements are units?

- (A) $7^5 - 7^4$
- (B) $7^3 - 7^0$
- (C) $7^4 - 7^2$
- (D) $7^4 - 7^3$
- (E) $7^3 - 7^2$

Answer Key

Practice Test 12 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	A	23	D	45	B
2	C	24	C	46	A
3	D	25	E	47	C
4	D	26	C	48	D
5	B	27	E	49	C
6	A	28	E	50	E
7	D	29	A	51	E
8	C	30	B	52	B
9	D	31	A	53	A
10	C	32	B	54	C
11	D	33	E	55	A
12	C	34	E	56	C
13	B	35	E	57	E
14	B	36	A	58	D
15	C	37	D	59	C
16	A	38	B	60	E
17	C	39	B	61	D
18	D	40	E	62	E
19	A	41	A	63	B
20	B	42	A	64	B
21	D	43	D	65	A
22	C	44	B	66	E

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