

GRE Mathematics Practice Test 13

Upper-Intermediate Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

Intermediate reinforcement, combining standard applications with selected more demanding arguments. This is cycle 2 of 19, position 3 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

zarimion.com/gre

100 practice tests, complete answer keys, and future updates.

Please retain the source attribution when sharing this material.



This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Let $F(x) = \int_0^x (x-t)^3 e^{3t} dt$. Find $F^{(4)}(0)$.

- (A) 0
- (B) $\frac{1}{6}$
- (C) 24
- (D) 18
- (E) 6

2. Let $D = \text{diag}(\underbrace{1, \dots, 1}_1, \underbrace{2, \dots, 2}_{14})$. Find the dimension of the space of real matrices X with $XD = DX$.

- (A) 198
- (B) 195
- (C) 199
- (D) 196
- (E) 197

3. The solution of $y' = 3x^3y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{\sqrt{2} \cdot 3^{\frac{3}{4}}}{3}$
- (B) $\frac{\sqrt[4]{2} \cdot 3^{\frac{3}{4}}}{3}$
- (C) $\frac{4}{3}$
- (D) $\frac{3^{\frac{3}{4}}}{3}$
- (E) $\frac{\sqrt{2}\sqrt[4]{3}}{2}$

Questions 4–6

4. X, Y are independent, with variances 5 and 6. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 5
- (B) 1
- (C) 3
- (D) 2
- (E) 4

5. The local inverse $y(x)$ of $x = 3y + y^3$ satisfies $y(0) = 0$. Find $y^{(3)}(0)$.

- (A) $\frac{6}{3^4}$
- (B) $\frac{1}{6} \left(-\frac{6}{3^4} \right)$
- (C) $3 \left(-\frac{6}{3^4} \right)$
- (D) $\frac{1}{3} \left(-\frac{6}{3^4} \right)$
- (E) $-\frac{6}{3^4}$

6. When $x^3 + 1$ is divided by $(x - 10)(x - 11)$, the remainder is $ux + v$. Find u .

- (A) 333
- (B) 332
- (C) 331
- (D) 330
- (E) 329

Questions 7–9

7. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(3j)!}{(j!)^3} x^{4j}$?

- (A) $3^{-\frac{3}{4}}$
- (B) $\frac{1}{3}$
- (C) $3^{-\frac{4}{3}}$
- (D) 3^{-3}
- (E) $3^{\frac{3}{4}}$

8. X has density cx^5 on $0 < x < 1$ and zero elsewhere. Find the normalizing constant c .

- (A) 4
- (B) 6
- (C) 10
- (D) 8
- (E) 2

9. If $y'' - 6y' + 9y = 4e^{3x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $4e^3$
- (B) $1 + 2e^3$
- (C) $2e^3$
- (D) e^3
- (E) $-2e^3$

Questions 10–12

10. Let $P = \text{diag}(\underbrace{1, \dots, 1}_3, \underbrace{0, \dots, 0}_{13})$. What is the dimension of the space of 16×16 real matrices X satisfying $PX = X$ and $XP = 0$?

- (A) 38
- (B) 40
- (C) 37
- (D) 41
- (E) 39

11. Evaluate $\int_0^1 \frac{x^3}{1+3x^4} dx$.

- (A) $\frac{\log(2)}{6}$
- (B) $-\frac{\log(2)}{6}$
- (C) $\log(4)$
- (D) $\frac{\log(2)}{2}$
- (E) $\frac{2\log(2)}{3}$

12. Independent exponential waiting times X, Y have rates 5 and 7. Find $P(X < Y)$.

- (A) $\frac{7}{12}$
- (B) $\frac{17}{24}$
- (C) $\frac{5}{24}$
- (D) $\frac{5}{12}$
- (E) $\frac{5}{36}$

Questions 13–15

13. For $D = \{(x, y) : x^2 + y^2 \leq 3\}$, evaluate $\iint_D (x^2 + y^2)^3 dA$.

- (A) $\frac{27\pi}{4}$
- (B) 81π
- (C) $\frac{81\pi}{2}$
- (D) $\frac{81\pi}{4}$
- (E) $\frac{81\pi}{7}$

14. A 3×3 matrix A has characteristic polynomial $(t - 9)(t + 1)(t - 2)$. What is $\text{tr}(A^3)$?

- (A) 736
- (B) 734
- (C) 737
- (D) 738
- (E) 735

15. Let $\mathbf{F} = (3y^4, z^4, x^4)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (1, 1, 4)/\sqrt{18}$.

- (A) $-\frac{56\sqrt{2}}{3}$
- (B) $-\frac{14\sqrt{2}}{3}$
- (C) $-\frac{28\sqrt{2}}{3}$
- (D) $1 - \frac{28\sqrt{2}}{3}$
- (E) $\frac{28\sqrt{2}}{3}$

Questions 16–18

16. $X_1, \dots, X_5$ are independent with common mean and variance 7. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 46
- (B) 28
- (C) 37
- (D) 19
- (E) 10

17. Let $F(x) = \int_0^{x^4} e^{3t} dt$. Find $F'(1)$.

- (A) $12e^3$
- (B) e^{12}
- (C) e^3
- (D) $\frac{e^3}{4}$
- (E) $4e^3$

18. Let N be the 15×15 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\dim(\ker N^{30} / \ker N^{15})$.

- (A) 2
- (B) 0
- (C) 3
- (D) 1
- (E) 4

Questions 19–21

19. What is the maximum of x^4y^3 over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\left(\frac{4}{7}\right)^3 \left(\frac{3}{7}\right)^4$
- (B) $\left(\frac{4}{7}\right)^4 \left(\frac{3}{7}\right)^3$
- (C) $\frac{1}{2^7}$
- (D) $\frac{16807}{576} \left(\left(\frac{4}{7}\right)^4 \left(\frac{3}{7}\right)^3 \right)$
- (E) $7 \left(\left(\frac{4}{7}\right)^4 \left(\frac{3}{7}\right)^3 \right)$

20. A permutation of 7 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z^2]$.

- (A) 1
- (B) 0
- (C) 2
- (D) 3
- (E) 4

21. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 3x$ and has density $\rho(x, y) = x^2y$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{5}{3}$
- (B) $\frac{5}{6}$
- (C) $\frac{11}{6}$
- (D) $-\frac{5}{6}$
- (E) $\frac{5}{12}$

Questions 22–24

22. Let A be the 15×15 matrix whose diagonal entries are 12 and whose off-diagonal entries are 1. What is its largest eigenvalue?

- (A) 28
- (B) 24
- (C) 27
- (D) 26
- (E) 25

23. The solution of $y' = 3x^4y^3$ has $y(0) = 1/4$. Find $y(1)$.

- (A) $\frac{\sqrt{370}}{148}$
- (B) $\frac{\sqrt{370}}{74} + 1$
- (C) $-\frac{\sqrt{370}}{74}$
- (D) $\frac{\sqrt{370}}{74}$
- (E) $\frac{\sqrt{370}}{37}$

24. Which subset of $\mathbb{R}$ is compact, with 5 fixed as displayed?

- (A) $\{0\} \cup \{5/k : k \geq 1\}$
- (B) $5\mathbb{Z}$
- (C) $\mathbb{Q} \cap [0, 5]$
- (D) $\{5/k : k \geq 1\}$
- (E) $(0, 5)$

Questions 25–27

25. Find the coefficient of x^4 in $e^{3x} \log(1 + 3x)$.

- (A) 0
- (B) -1
- (C) 3
- (D) $-\frac{81}{4}$
- (E) 1

26. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 54$ and $x + y = 15$. Find $x^2 + y^2$.

- (A) 118
- (B) 115
- (C) 116
- (D) 117
- (E) 119

27. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-3)^j}{j^1 (\log j)^m}$ converge absolutely at $x = 2$? Give the least such m , or 0 if none exists.

- (A) 3
- (B) 1
- (C) 0
- (D) 4
- (E) 2

Questions 28–30

28. For $a_k = (-1)^k + 5/k$, find $\limsup_{k \rightarrow \infty} a_k - \liminf_{k \rightarrow \infty} a_k$.

- (A) 1
- (B) 4
- (C) 3
- (D) 2
- (E) 0

29. Evaluate $\int_0^\infty x^4 e^{-3x} dx$.

- (A) $\frac{1}{4} \left(\frac{4!}{3^5} \right)$
- (B) $\frac{4!}{3^5}$
- (C) $3 \left(\frac{4!}{3^5} \right)$
- (D) $\frac{1}{3} \left(\frac{4!}{3^5} \right)$
- (E) $243 \left(\frac{4!}{3^5} \right)$

30. A sequence satisfies $u_0 = 9$ and $u_{k+1} = 2u_k + 6$ for all $k \geq 0$. Find u_6 .

- (A) 955
- (B) 954
- (C) 952
- (D) 953
- (E) 956

Questions 31–33

31. Let C be $x^2/16 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-y^3) dx + (\frac{x^3}{3}) dy$.

- (A) -19π
- (B) 19π
- (C) 38π
- (D) $1 + 19\pi$
- (E) $\frac{19\pi}{2}$

32. For which real a is $f(x) = x^a$ in $L^5(0, 1)$?

- (A) $a < 1/5$
- (B) $a \geq -1/5$
- (C) $a > 0$
- (D) All real a
- (E) $a > -1/5$

33. Find the arc length of $r(t) = (3(t - t^3/3), 3t^2)$ for $0 \leq t \leq 1/4$.

- (A) $\frac{49}{128}$
- (B) $\frac{49}{32}$
- (C) $\frac{113}{64}$
- (D) $\frac{49}{64}$
- (E) $-\frac{49}{64}$

Questions 34–36

34. Linear maps $T : \mathbb{R}^{16} \rightarrow \mathbb{R}^{16}$ and $U : \mathbb{R}^{16} \rightarrow \mathbb{R}^{16}$ have ranks 4 and 3. What is the largest possible rank of UT ?

- (A) 1
- (B) 3
- (C) 2
- (D) 4
- (E) 5

35. Find the coefficient of x^6 in $\cos(3x)/(1 - 3x^2)$.

- (A) $\frac{351}{80}$
- (B) -3159
- (C) $-\frac{81}{80}$
- (D) $-\frac{351}{80}$
- (E) $-\frac{431}{80}$

36. $T : \mathbb{R} \rightarrow \mathbb{R}$ has Lipschitz constant at most $1/5$. If $x_1 = T(x_0)$ and $|x_1 - x_0| \leq 1$, what is the sharp upper bound on $|T^3(x_0) - x_*|$, where x_* is its fixed point?

- (A) $\frac{1}{100}$
- (B) $\frac{1}{200}$
- (C) $\frac{101}{200}$
- (D) $\frac{99}{100}$
- (E) $\frac{1}{300}$

Questions 37–39

37. Evaluate $\int_0^1 \int_x^1 3y^4 \, dy \, dx$.

- (A) $\frac{3}{4}$
- (B) $\frac{1}{2}$
- (C) $\frac{1}{10}$
- (D) $\frac{3}{2}$
- (E) $\frac{3}{5}$

38. Fix a nonzero vector $u \in \mathbb{R}^{15}$. What is the dimension of the space of skew-symmetric 15×15 matrices A satisfying $Au = 0$?

- (A) 90
- (B) 92
- (C) 93
- (D) 91
- (E) 89

39. Find the curvature of the helix $r(t) = (3 \cos t, 3 \sin t, 4t)$.

- (A) $\frac{3}{50}$
- (B) $-\frac{3}{25}$
- (C) $\frac{6}{25}$
- (D) $\frac{28}{25}$
- (E) $\frac{3}{25}$

Questions 40–42

40. How many zeros, counted with multiplicity, does $z^5 + 8z + 1$ have in $|z| < 1$?
- (A) 0
(B) 4
(C) 1
(D) 2
(E) 3
41. Solve $y' + 3y = x^4 e^{-3x}$, $y(0) = 0$, and find $y(1)$.
- (A) $\frac{3}{5e^3}$
(B) e^{-3}
(C) $\frac{1}{5e^3}$
(D) $\frac{e^3}{5}$
(E) $\frac{1}{4e^3}$
42. Integers x satisfy $x \equiv 0 \pmod{7}$ and $x \equiv 7 \pmod{9}$. Find the least nonnegative solution.
- (A) 6
(B) 7
(C) 8
(D) 5
(E) 9

Questions 43–45

43. For $\mathbf{F} = (3x^4y, y^4z, xz^4)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) 60
- (B) 120
- (C) 61
- (D) 30
- (E) -60

44. A meromorphic function has 7 zeros and 2 poles inside a positively oriented simple closed contour, counted with multiplicity, and no zeros or poles on it. Find $\frac{1}{2\pi i} \int f'(z)/f(z) dz$.

- (A) 7
- (B) 5
- (C) 3
- (D) 4
- (E) 6

45. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{3t} \cos(4t)$, and $y = e^{3t} \sin(4t)$, find d^2f/dt^2 at $t = 1$.

- (A) 36
- (B) 6
- (C) 50
- (D) 0
- (E) 8

Questions 46–48

46. The symmetric group S_{15} acts on the two-element subsets of $\{1, \dots, 15\}$. What is the index of the setwise stabilizer of $\{1, 2\}$?

- (A) 104
- (B) 105
- (C) 103
- (D) 106
- (E) 107

47. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^3 y^3 dV$.

- (A) $6 \left(\frac{3! 3!}{9!} \right)$
- (B) $9 \left(\frac{3! 3!}{9!} \right)$
- (C) $\frac{3! 3!}{9!}$
- (D) $\frac{1}{6} \left(\frac{3! 3!}{9!} \right)$
- (E) $\frac{1}{16}$

48. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 5 have no adjacent 1s?

- (A) F_6
- (B) F_5
- (C) F_9
- (D) F_8
- (E) F_7

Questions 49–51

49. Find the total outward flux of $\mathbf{F} = (3x^4y, y^4z, xz^4)$ through the boundary of $[0, 1]^3$.

- (A) $\frac{7}{2}$
- (B) 5
- (C) $\frac{5}{2}$
- (D) $-\frac{5}{2}$
- (E) $\frac{5}{4}$

50. In $\mathbb{Z}/258\mathbb{Z}$, how many residues are not idempotent ($x^2 \neq x$)?

- (A) 251
- (B) 252
- (C) 249
- (D) 248
- (E) 250

51. Evaluate $\int_0^1 \int_0^x x^3 y^4 dy dx$.

- (A) $\frac{1}{20}$
- (B) $\frac{1}{9}$
- (C) $\frac{1}{54}$
- (D) $\frac{1}{40}$
- (E) $\frac{1}{45}$

Questions 52–54

52. Apply one Newton iteration to solve $x^2 - 5 = 0$, starting at $x_0 = 1$. What is x_1 ?

- (A) 3
- (B) 5
- (C) 4
- (D) 1
- (E) 2

53. For $x > 0$, $x^2 y'' + (1 - 7)xy' + 12y = 0$, $y(1) = 0$, $y'(1) = 3$. Find $y(e)$.

- (A) $-3(1 - e)e^3$
- (B) $3(1 - e)e^3$
- (C) $\frac{e^3}{7}$
- (D) $e^3 + e^4$
- (E) $-e^4 + e^3$

54. In the additive group $\mathbb{Z}/24\mathbb{Z}$, find the index of the subgroup generated by $[6]$.

- (A) 4
- (B) 8
- (C) 6
- (D) 7
- (E) 5

Questions 55–57

55. Evaluate $\int_0^1 x^3(1-x)^3 dx$.

- (A) $\frac{3! 3!}{7!}$
- (B) $\frac{1}{4} \left(\frac{3! 3!}{7!} \right)$
- (C) $4 \left(\frac{3! 3!}{7!} \right)$
- (D) $7 \left(\frac{3! 3!}{7!} \right)$
- (E) $\frac{35}{4} \left(\frac{3! 3!}{7!} \right)$

56. Bisection starts with an interval of length 2^5 . After 7 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{9}{16}$
- (B) $\frac{1}{16}$
- (C) $\frac{7}{8}$
- (D) $\frac{1}{8}$
- (E) $\frac{1}{24}$

57. Evaluate $\int_0^{\log 2} e^x(e^x + 3)^4 dx$.

- (A) $\frac{5^5}{5}$
- (B) $5 \left(\frac{5^5 - 4^5}{5} \right)$
- (C) $-\frac{5^5 - 4^5}{5}$
- (D) $5^4 - 4^4$
- (E) $\frac{5^5 - 4^5}{5}$

Questions 58–60

58. How many invertible 3×3 matrices over $\mathbb{F}_3$ are there whose first column is $(1, 0, 0)^T$?

- (A) $\frac{(3^3 - 3^1)(3^3 - 3^2)(3^3 - 3^3)}{3^3 - 1}$
(B) $\frac{(3^3 - 3^0)(3^3 - 3^1)}{3^3 - 1}$
(C) $\frac{(3^3 - 3^0)(3^3 - 3^1)(3^3 - 3^2)}{3^3 - 1}$
(D) $\frac{(3^3 - 3^0)(3^3 - 3^0)(3^3 - 3^0)}{3^3 - 1}$
(E) $\frac{3^3 3^3 3^3}{3^3 - 1}$

59. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 3x)^{1/(3x)} - e^{3/3}}{x}$.

- (A) $-\frac{3e}{2}$
(B) $-3e$
(C) e
(D) $-\frac{e}{2}$
(E) $\frac{3e}{2}$

60. What is the least positive multiplicative inverse of 2 modulo 11?

- (A) 2
(B) 8
(C) 6
(D) 4
(E) 10

Questions 61–63

61. If $y'' + 16y = 3$ and $y(0) = y'(0) = 0$, find $y(\pi/4)$.

- (A) $\frac{3}{4}$
- (B) $-\frac{3}{8}$
- (C) $\frac{3}{8}$
- (D) $\frac{11}{8}$
- (E) $\frac{3}{16}$

62. How many permutations in S_{15} consist of one 5-cycle and 10 fixed points?

- (A) 72070
- (B) 72071
- (C) 72074
- (D) 72073
- (E) 72072

63. Find the upward flux of $\mathbf{F} = (x, y, z)$ through $z = 3x^2 + 4y^2$, $0 \leq x, y \leq 1$.

- (A) $-\frac{7}{3}$
- (B) $-\frac{14}{3}$
- (C) $\frac{7}{3}$
- (D) $-\frac{4}{3}$
- (E) $-\frac{7}{6}$

Questions 64–66

64. How many truth assignments to $p, q, r_1, \dots, r_3$ make $p \Rightarrow q$ false?

- (A) 2^5
- (B) 2^2
- (C) 2^6
- (D) 2^4
- (E) 2^3

65. Let $f(x, y) = e^{3x+4y}$. Find $\partial^4 f / \partial x^3 \partial y$ at $(0, 0)$.

- (A) $\frac{64}{27} (4 \cdot 3^3)$
- (B) $\frac{1}{4} (4 \cdot 3^3)$
- (C) $\frac{3}{4} (4 \cdot 3^3)$
- (D) $4 \cdot 3^3$
- (E) $\frac{16}{9} (4 \cdot 3^3)$

66. How many maximal proper subgroups does the cyclic group of order 156 have?

- (A) 5
- (B) 2
- (C) 1
- (D) 3
- (E) 4

Answer Key

Practice Test 13 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	E	23	D	45	D
2	E	24	A	46	B
3	A	25	A	47	C
4	C	26	D	48	E
5	E	27	E	49	C
6	C	28	D	50	E
7	A	29	B	51	E
8	B	30	B	52	A
9	C	31	B	53	A
10	E	32	E	54	C
11	A	33	D	55	A
12	D	34	B	56	D
13	D	35	D	57	E
14	A	36	A	58	C
15	C	37	B	59	A
16	B	38	D	60	C
17	E	39	E	61	C
18	B	40	C	62	E
19	B	41	C	63	A
20	C	42	B	64	E
21	B	43	A	65	D
22	D	44	B	66	D

zarimion.com/gre

100 practice tests, complete answer keys, and future updates.
Please retain the source attribution when sharing this material.

