

GRE Mathematics Practice Test 14

Bridge to Standard

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A bridge toward standard practice, with routine multi-step questions and a smaller number of challenging concepts. This is cycle 2 of 19, position 4 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Evaluate $\int_0^{\log 2} e^x(e^x + 3)^3 dx$.

- (A) $\frac{5^4 - 4^4}{4}$
- (B) $5^3 - 4^3$
- (C) $4 \left(\frac{5^4 - 4^4}{4} \right)$
- (D) $-\frac{5^4 - 4^4}{4}$
- (E) $\frac{5^4}{4}$

2. Let A be the 16×16 matrix whose diagonal entries are 13 and whose off-diagonal entries are 1. What is its smallest eigenvalue?

- (A) 14
- (B) 13
- (C) 10
- (D) 11
- (E) 12

3. Find the coefficient of x^3 in $e^{2x} \log(1 + 3x)$.

- (A) 36
- (B) -6
- (C) 12
- (D) 6
- (E) 9

Questions 4–6

4. X, Y are independent, with variances 6 and 7. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 3
- (B) 5
- (C) 2
- (D) 6
- (E) 4

5. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-3)^j}{j^{\frac{2}{3}} (\log j)^m}$ converge absolutely at $x = 2$? Give the least such m , or 0 if none exists.

- (A) 2
- (B) 1
- (C) 4
- (D) 0
- (E) 3

6. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 40$ and $x + y = 14$. Find xy .

- (A) 42
- (B) 41
- (C) 40
- (D) 39
- (E) 38

Questions 7–9

7. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{3t} \cos(3t)$, and $y = e^{3t} \sin(3t)$, find d^2f/dt^2 at $t = 1$.
- (A) 0
(B) 1
(C) 36
(D) 6
(E) -1
8. X has density cx^6 on $0 < x < 1$ and zero elsewhere. Find the normalizing constant c .
- (A) 9
(B) 7
(C) 5
(D) 3
(E) 11
9. Find the arc length of $r(t) = (3t, 3t)$ for $0 \leq t \leq 1$.
- (A) $-3\sqrt{2}$
(B) $\frac{3\sqrt{2}}{2}$
(C) $6\sqrt{2}$
(D) $3\sqrt{2}$
(E) $1 + 3\sqrt{2}$

Questions 10–12

10. A sequence satisfies $u_0 = 10$ and $u_{k+1} = 1u_k + 4$ for all $k \geq 0$. Find u_5 .

- (A) 30
- (B) 31
- (C) 28
- (D) 29
- (E) 32

11. Evaluate $\int_0^1 x^3(1-x)^2 dx$.

- (A) $\frac{1}{3} \left(\frac{3! 2!}{6!} \right)$
- (B) $5 \left(\frac{3! 2!}{6!} \right)$
- (C) $\frac{3! 2!}{6!}$
- (D) $6 \left(\frac{3! 2!}{6!} \right)$
- (E) $4 \left(\frac{3! 2!}{6!} \right)$

12. Independent exponential waiting times X, Y have rates 6 and 8. Find $P(X < Y)$.

- (A) $\frac{3}{7}$
- (B) $\frac{1}{7}$
- (C) $\frac{3}{14}$
- (D) $\frac{4}{7}$
- (E) $\frac{5}{7}$

Questions 13–15

13. For $f(x) = x^3 e^{3x}$, find $f^{(5)}(0)$.

- (A) $\frac{1}{20} \left(\frac{5! 3^2}{2} \right)$
- (B) $2 \left(\frac{5! 3^2}{2} \right)$
- (C) $\frac{1}{60} \left(\frac{5! 3^2}{2} \right)$
- (D) $\frac{5! 3^2}{2}$
- (E) $\frac{2}{3} \left(\frac{5! 3^2}{2} \right)$

14. A linear map $T : \mathbb{R}^{17} \rightarrow \mathbb{R}^{19}$ has rank 3. What is its nullity?

- (A) 15
- (B) 12
- (C) 14
- (D) 13
- (E) 16

15. The function $y(x)$ satisfies $y^3 + 3y = 2x$ and $y(0) = 0$. Find $y'(0)$.

- (A) 3
- (B) 0
- (C) $\frac{2}{3}$
- (D) $\frac{1}{3}$
- (E) $-\frac{2}{3}$

Questions 16–18

16. $X_1, \dots, X_6$ are independent with common mean and variance 8. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 40
- (B) 14
- (C) 27
- (D) 66
- (E) 53

17. For $f(x, y) = x^2 + 3xy + 3y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) $\sqrt{10}$
- (B) -1
- (C) 4
- (D) 10
- (E) -3

18. What is the dimension of the real vector space of 16×16 symmetric matrices?

- (A) 134
- (B) 136
- (C) 137
- (D) 135
- (E) 138

Questions 19–21

19. Solve $y' + 3y = x^3e^{-3x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{1}{3e^3}$
- (B) e^{-3}
- (C) $\frac{1}{4e^3}$
- (D) $\frac{e^3}{4}$
- (E) $\frac{3}{4e^3}$

20. A permutation of 8 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z]$.

- (A) 0
- (B) 4
- (C) 2
- (D) 1
- (E) 3

21. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 3x$ and has density $\rho(x, y) = x$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{3}{8}$
- (B) $\frac{3}{4}$
- (C) $-\frac{3}{4}$
- (D) $\frac{3}{2}$
- (E) $\frac{7}{4}$

Questions 22–24

22. Let $A = \begin{bmatrix} 1 & 10 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 2)$ entry of A^2 ?

- (A) 20
- (B) 19
- (C) 18
- (D) 22
- (E) 21

23. For $y' = 3y(1 - y/3)$, what is the derivative of the right-hand side with respect to y at the positive equilibrium?

- (A) -2
- (B) $-\frac{3}{2}$
- (C) -3
- (D) 3
- (E) -6

24. Which subset of $\mathbb{R}$ is compact, with 6 fixed as displayed?

- (A) $(0, 6)$
- (B) $\{6/k : k \geq 1\}$
- (C) $\mathbb{Q} \cap [0, 6]$
- (D) $\{0\} \cup \{6/k : k \geq 1\}$
- (E) $6\mathbb{Z}$

Questions 25–27

25. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^3 y^2 dV$.

- (A) $\frac{3! 2!}{8!}$
- (B) $6 \left(\frac{3! 2!}{8!} \right)$
- (C) $\frac{1}{6} \left(\frac{3! 2!}{8!} \right)$
- (D) $\frac{1}{12}$
- (E) $8 \left(\frac{3! 2!}{8!} \right)$

26. An invertible 16×16 matrix A has determinant 4. Find $\det(A^T A)$.

- (A) 17
- (B) 18
- (C) 16
- (D) 15
- (E) 14

27. The solution of $y' = 3x^3 y^2$ has $y(0) = 1/4$. Find $y(1)$.

- (A) $\frac{2}{13}$
- (B) $\frac{8}{13}$
- (C) $\frac{4}{13}$
- (D) $\frac{17}{13}$
- (E) $-\frac{4}{13}$

Questions 28–30

- 28.** For $a_k = (-1)^k + 6/k$, find $\limsup_{k \rightarrow \infty} a_k - \liminf_{k \rightarrow \infty} a_k$.
- (A) 1
(B) 0
(C) 4
(D) 3
(E) 2
- 29.** The solution of $y' = 3x^2y^2$, $y(0) = 1$, has its first positive singularity at which x ?
- (A) 2
(B) 1
(C) $\frac{3^{\frac{2}{3}}}{3}$
(D) $\frac{2^{\frac{2}{3}}}{2}$
(E) $\frac{1}{2}$
- 30.** On the space P_{17} of real polynomials of degree at most 17, define $T(p) = p'$. What is the rank of T ?
- (A) 18
(B) 16
(C) 19
(D) 17
(E) 15

Questions 31–33

31. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(3j)!}{(j!)^3} x^{3j}$?

- (A) 3^{-3}
- (B) $\frac{1}{2} (3^{-1})$
- (C) 3^1
- (D) $2 (3^{-1})$
- (E) 3^{-1}

32. For which real a is $f(x) = x^a$ in $L^6(0, 1)$?

- (A) All real a
- (B) $a > -1/6$
- (C) $a \geq -1/6$
- (D) $a > 0$
- (E) $a < 1/6$

33. The region $0 \leq x \leq 1$, $0 \leq y \leq 3x^3$ is revolved about the x -axis. Find its volume.

- (A) $\frac{18\pi}{7}$
- (B) $\frac{9\pi}{14}$
- (C) $1 + \frac{9\pi}{7}$
- (D) $-\frac{9\pi}{7}$
- (E) $\frac{9\pi}{7}$

Questions 34–36

34. Subspaces U, V of $\mathbb{R}^{18}$ have dimensions 4 and 5. What is the largest possible value of $\dim(U \cap V)$?

- (A) 4
- (B) 5
- (C) 3
- (D) 6
- (E) 2

35. Under $x = u + 3v$, $y = u - 3v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) $\frac{1}{6}$
- (B) 0
- (C) 36
- (D) 6
- (E) 9

36. $T : \mathbb{R} \rightarrow \mathbb{R}$ has Lipschitz constant at most $1/6$. If $x_1 = T(x_0)$ and $|x_1 - x_0| \leq 1$, what is the sharp upper bound on $|T^3(x_0) - x_*|$, where x_* is its fixed point?

- (A) $\frac{179}{180}$
- (B) $\frac{1}{180}$
- (C) $\frac{1}{360}$
- (D) $\frac{181}{360}$
- (E) $\frac{1}{540}$

Questions 37–39

37. Find the curvature at $t = 0$ of $r(t) = (3t, 3t^2)$.

- (A) $\frac{1}{3}$
- (B) $\frac{5}{3}$
- (C) $\frac{2}{3}$
- (D) $\frac{4}{3}$
- (E) $-\frac{2}{3}$

38. Let V consist of real polynomials of degree at most 7 that vanish at 10, 11, and 13. Find $\dim V$.

- (A) 6
- (B) 3
- (C) 4
- (D) 7
- (E) 5

39. Find the coefficient of x^6 in $\cos(3x)/(1 - 2x^2)$.

- (A) $-\frac{81}{80}$
- (B) $-\frac{421}{80}$
- (C) $\frac{341}{80}$
- (D) $-\frac{341}{80}$
- (E) -3069

Questions 40–42

40. How many zeros, counted with multiplicity, does $z^6 + 9z + 1$ have in $|z| < 1$?

- (A) 4
- (B) 2
- (C) 1
- (D) 0
- (E) 3

41. Evaluate $\lim_{x \rightarrow 0} (1 + 3x)^{1/(2x)}$.

- (A) $e^{\frac{3}{2}}$
- (B) e^9
- (C) $e^{\frac{2}{3}}$
- (D) e^4
- (E) 3

42. The roots r, s of $x^2 - 14x + 40 = 0$ are counted with multiplicity. Find $r^2 + s^2$.

- (A) 118
- (B) 116
- (C) 115
- (D) 117
- (E) 114

Questions 43–45

43. Find the total outward flux of $\mathbf{F} = (3x^3, y^3, z^3)$ through the boundary of $[0, 1]^3$.

- (A) 6
- (B) $\frac{5}{2}$
- (C) 10
- (D) 5
- (E) -5

44. A meromorphic function has 8 zeros and 2 poles inside a positively oriented simple closed contour, counted with multiplicity, and no zeros or poles on it. Find $\frac{1}{2\pi i} \int f'(z)/f(z) dz$.

- (A) 8
- (B) 6
- (C) 2
- (D) 4
- (E) 10

45. Evaluate $\int_0^1 \int_0^x x^3 y^3 dy dx$.

- (A) $\frac{1}{28}$
- (B) $\frac{1}{8}$
- (C) $\frac{1}{40}$
- (D) $\frac{1}{16}$
- (E) $\frac{1}{32}$

Questions 46–48

46. How many residues x modulo 30 satisfy $9x \equiv 18 \pmod{30}$?

- (A) 2
- (B) 4
- (C) 1
- (D) 5
- (E) 3

47. Evaluate $\int_0^{\pi/2} \sin^7 t \cos^7 t \, dt$.

- (A) $\frac{3!3!}{2 \cdot 7!}$
- (B) $\frac{1}{2} \left(\frac{3!3!}{2 \cdot 7!} \right)$
- (C) $2 \left(\frac{3!3!}{2 \cdot 7!} \right)$
- (D) $20 \left(\frac{3!3!}{2 \cdot 7!} \right)$
- (E) $7 \left(\frac{3!3!}{2 \cdot 7!} \right)$

48. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 6 have no adjacent 1s?

- (A) F_6
- (B) F_8
- (C) F_9
- (D) F_{10}
- (E) F_7

Questions 49–51

49. Let $F(x) = \int_0^{x^3} e^{3t} dt$. Find $F'(1)$.

- (A) $\frac{e^3}{3}$
- (B) $3e^3$
- (C) e^3
- (D) e^9
- (E) $9e^3$

50. The field K has 3^3 elements. Find $[K : \mathbb{F}_3]$.

- (A) 4
- (B) 1
- (C) 2
- (D) 3
- (E) 5

51. Evaluate $\lim_{x \rightarrow 0} \frac{e^{3x} - \sum_{j=0}^2 (3x)^j / j!}{x^3}$.

- (A) $-\frac{3^3}{3!}$
- (B) $3 \left(\frac{3^3}{3!} \right)$
- (C) $\frac{3^3}{3!}$
- (D) $6 \left(\frac{3^3}{3!} \right)$
- (E) $\frac{1}{3} \left(\frac{3^3}{3!} \right)$

Questions 52–54

52. Apply one Newton iteration to solve $x^2 - 6 = 0$, starting at $x_0 = 1$. What is x_1 ?

- (A) $\frac{11}{2}$
- (B) $\frac{7}{4}$
- (C) $\frac{7}{2}$
- (D) $\frac{9}{2}$
- (E) 7

53. Find the coefficient of x^3 in the Taylor series of $(1 - 3x)^{-2}$ at zero.

- (A) $\frac{3}{4} (4 \cdot 3^3)$
- (B) $4 \cdot 3^3$
- (C) $\frac{1}{3} (4 \cdot 3^3)$
- (D) $\frac{1}{4} (4 \cdot 3^3)$
- (E) $-4 \cdot 3^3$

54. Let $G = \mathbb{Z}/90\mathbb{Z} \times \mathbb{Z}/64\mathbb{Z}$. Find the order of $(1, 1)$.

- (A) 2880
- (B) 2879
- (C) 2878
- (D) 2882
- (E) 2881

Questions 55–57

55. Evaluate $\int_C (3x^2 + 3y) dx + (3x + 3y^2) dy$, where C is the line segment from $(0, 0)$ to $(5, 1)$.

- (A) 141
- (B) 282
- (C) 142
- (D) -141
- (E) $\frac{141}{2}$

56. Bisection starts with an interval of length 2^6 . After 8 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{1}{16}$
- (B) $\frac{7}{8}$
- (C) $\frac{9}{16}$
- (D) $\frac{1}{24}$
- (E) $\frac{1}{8}$

57. If $y'' + 9y = 0$, $y(0) = 3$, and $y'(0) = 5$, find $y(\pi/6)$.

- (A) $-\frac{5}{3}$
- (B) $\frac{5}{6}$
- (C) $\frac{8}{3}$
- (D) $\frac{5}{3}$
- (E) $\frac{10}{3}$

Questions 58–60

58. How many group homomorphisms $\mathbb{Z}/90\mathbb{Z} \rightarrow \mathbb{Z}/64\mathbb{Z}$ are there?

- (A) 3
- (B) 2
- (C) 4
- (D) 0
- (E) 1

59. Let C be $x^2/9 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-y) dx + (3x) dy$.

- (A) 24π
- (B) 6π
- (C) -12π
- (D) 12π
- (E) $1 + 12\pi$

60. What is the least positive multiplicative inverse of 2 modulo 13?

- (A) 5
- (B) 9
- (C) 7
- (D) 11
- (E) 3

Questions 61–63

61. Evaluate $\int_0^1 \int_x^1 3y^3 \, dy \, dx$.

- (A) $\frac{3}{20}$
- (B) $\frac{3}{4}$
- (C) $\frac{3}{2}$
- (D) 1
- (E) $\frac{3}{5}$

62. How many 1-dimensional subspaces does $\mathbb{F}_3^4$ have?

- (A) $\frac{3^7 - 1}{3 - 1}$
- (B) $\frac{3^4 - 1}{3 - 1}$
- (C) $\frac{3^6 - 1}{3 - 1}$
- (D) $\frac{3^5 - 1}{3 - 1}$
- (E) $\frac{3^3 - 1}{3 - 1}$

63. Evaluate $\iiint_B 3(x^2 + y^2 + z^2)^{3/2} \, dV$, where B is the unit ball.

- (A) 2π
- (B) 4π
- (C) -2π
- (D) π
- (E) $1 + 2\pi$

Questions 64–66

64. How many truth assignments to $p, q, r_1, \dots, r_4$ make $p \Rightarrow q$ false?

- (A) 2^5
- (B) 2^6
- (C) 2^7
- (D) 2^3
- (E) 2^4

65. Find the directional derivative of $f(x, y, z) = x^2 + 3y^2 + 3z^2$ at $(1, 1, 1)$ in the direction $(1, 5, 0)$.

- (A) $1 + \frac{16\sqrt{26}}{13}$
- (B) $-\frac{16\sqrt{26}}{13}$
- (C) $\frac{8\sqrt{26}}{13}$
- (D) $\frac{32\sqrt{26}}{13}$
- (E) $\frac{16\sqrt{26}}{13}$

66. How many monic irreducible polynomials of degree 2 are there over $\mathbb{F}_3$?

- (A) 1
- (B) 5
- (C) 2
- (D) 4
- (E) 3

Answer Key

Practice Test 14 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	A	23	C	45	E
2	E	24	D	46	E
3	D	25	A	47	A
4	E	26	C	48	B
5	D	27	C	49	B
6	C	28	E	50	D
7	A	29	B	51	C
8	B	30	D	52	C
9	D	31	E	53	B
10	A	32	B	54	A
11	C	33	E	55	A
12	A	34	A	56	E
13	D	35	D	57	D
14	C	36	B	58	B
15	C	37	C	59	D
16	A	38	E	60	C
17	A	39	D	61	E
18	B	40	C	62	B
19	C	41	A	63	A
20	D	42	B	64	E
21	B	43	D	65	E
22	A	44	B	66	E

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