

GRE Mathematics Practice Test 15

Standard-Level Target

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A mixed standard-level target, intended to approximate public GRE Mathematics practice in scope and overall demand. This is cycle 2 of 19, position 5 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Evaluate $\int_0^1 x^2(\log x)^1 dx$.

- (A) $\frac{1}{2} \left(\frac{(-1)^1 1!}{3^2} \right)$
- (B) $2 \left(\frac{(-1)^1 1!}{3^2} \right)$
- (C) $-\frac{(-1)^1 1!}{3^2}$
- (D) $\frac{(-1)^1 1!}{3^2}$
- (E) $3 \left(\frac{(-1)^1 1!}{3^2} \right)$

2. Let $A = \begin{bmatrix} 1 & 11 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 2)$ entry of A^3 ?

- (A) 35
- (B) 34
- (C) 32
- (D) 31
- (E) 33

3. Let $\mathbf{F} = (3y^2, z^2, x^2)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (0, 0, 1)$.

- (A) -12
- (B) 6
- (C) -6
- (D) -3
- (E) -5

Questions 4–6

4. X, Y are independent, with variances 7 and 8. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 5
- (B) 6
- (C) 3
- (D) 7
- (E) 4

5. Find the coefficient of x^2 in $e^{1x} \log(1 + 3x)$.

- (A) $\frac{3}{2}$
- (B) $-\frac{9}{2}$
- (C) -3
- (D) $-\frac{3}{2}$
- (E) $-\frac{3}{4}$

6. An invertible 17×17 matrix A has determinant 2. Find $\det(A^T A)$.

- (A) 3
- (B) 6
- (C) 5
- (D) 4
- (E) 2

Questions 7–9

7. The region $0 \leq x \leq 1$, $0 \leq y \leq 3x^2$ is revolved about the x -axis. Find its volume.
- (A) $\frac{18\pi}{5}$
(B) $1 + \frac{9\pi}{5}$
(C) $\frac{9\pi}{5}$
(D) $-\frac{9\pi}{5}$
(E) $\frac{9\pi}{10}$
8. X has density cx^7 on $0 < x < 1$ and zero elsewhere. Find the normalizing constant c .
- (A) 10
(B) 8
(C) 12
(D) 6
(E) 4
9. Evaluate $\lim_{x \rightarrow 0} (1 + 3x)^{1/(1x)}$.
- (A) e^4
(B) e^9
(C) $e^{\frac{1}{3}}$
(D) e^3
(E) 3

Questions 10–12

10. On the space P_{18} of real polynomials of degree at most 18, define $T(p) = p'$. What is the rank of T ?

- (A) 17
- (B) 16
- (C) 19
- (D) 18
- (E) 20

11. Evaluate $\int_0^\infty x^2 e^{-3x} dx$.

- (A) $3 \left(\frac{2!}{3^3} \right)$
- (B) $\frac{2!}{3^3}$
- (C) $27 \left(\frac{2!}{3^3} \right)$
- (D) $\frac{1}{2} \left(\frac{2!}{3^3} \right)$
- (E) $\frac{1}{3} \left(\frac{2!}{3^3} \right)$

12. Independent exponential waiting times X, Y have rates 7 and 9. Find $P(X < Y)$.

- (A) $\frac{9}{16}$
- (B) $\frac{48}{7}$
- (C) $\frac{23}{32}$
- (D) $\frac{7}{32}$
- (E) $\frac{7}{16}$

Questions 13–15

13. If $y'' + 4y = 0$, $y(0) = 3$, and $y'(0) = 6$, find $y(\pi/4)$.

- (A) $\frac{3}{2}$
- (B) -3
- (C) 4
- (D) 3
- (E) 6

14. Subspaces U, V of $\mathbb{R}^{19}$ have dimensions 3 and 4. What is the largest possible value of $\dim(U \cap V)$?

- (A) 1
- (B) 2
- (C) 4
- (D) 5
- (E) 3

15. Find the coefficient of x^2 in the Taylor series of $(1 - 3x)^{-2}$ at zero.

- (A) $\frac{1}{2} (3 \cdot 3^2)$
- (B) $2 (3 \cdot 3^2)$
- (C) $\frac{1}{3} (3 \cdot 3^2)$
- (D) $3 \cdot 3^2$
- (E) $\frac{2}{3} (3 \cdot 3^2)$

Questions 16–18

16. $X_1, \dots, X_7$ are independent with common mean and variance 9. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 90
- (B) 36
- (C) 72
- (D) 18
- (E) 54

17. Evaluate $\int_0^1 \int_0^x x^3 y^2 \, dy \, dx$.

- (A) $\frac{1}{21}$
- (B) $\frac{1}{18}$
- (C) $\frac{1}{7}$
- (D) $\frac{1}{28}$
- (E) $\frac{1}{12}$

18. Let V consist of real polynomials of degree at most 6 that vanish at 11, 12, and 14. Find $\dim V$.

- (A) 6
- (B) 4
- (C) 2
- (D) 5
- (E) 3

Questions 19–21

19. Let $f(x, y) = e^{3x+2y}$. Find $\partial^2 f / \partial x^1 \partial y$ at $(0, 0)$.

- (A) $2 \cdot 3^1$
- (B) $\frac{3}{2} (2 \cdot 3^1)$
- (C) $\frac{1}{2} (2 \cdot 3^1)$
- (D) $\frac{2}{3} (2 \cdot 3^1)$
- (E) $2 (2 \cdot 3^1)$

20. A permutation of 9 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z]$.

- (A) 1
- (B) 3
- (C) 0
- (D) 2
- (E) 4

21. Let $F(x) = \int_0^{x^2} e^{3t} dt$. Find $F'(1)$.

- (A) $\frac{e^3}{2}$
- (B) $2e^3$
- (C) $6e^3$
- (D) e^6
- (E) e^3

Questions 22–24

22. The roots r, s of $x^2 - 13x + 22 = 0$ are counted with multiplicity. Find $r^2 + s^2$.

- (A) 126
- (B) 127
- (C) 125
- (D) 124
- (E) 123

23. The solution of $y' = 3x^1y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{\sqrt{6}}{3}$
- (B) $\frac{\sqrt{6}}{2}$
- (C) $\frac{\sqrt{3}}{3}$
- (D) $\frac{2}{3}$
- (E) $\frac{2\sqrt{6}}{3}$

24. Which subset of $\mathbb{R}$ is compact, with 7 fixed as displayed?

- (A) $\{7/k : k \geq 1\}$
- (B) $(0, 7)$
- (C) $7\mathbb{Z}$
- (D) $\mathbb{Q} \cap [0, 7]$
- (E) $\{0\} \cup \{7/k : k \geq 1\}$

Questions 25–27

25. Evaluate $\iiint_{x^2+y^2+z^2 \leq 3} (x^2 + y^2 + z^2)^1 dV$.

- (A) $\frac{4\pi 3^{\frac{5}{2}}}{5}$
- (B) $\frac{1}{2} \left(\frac{4\pi 3^{\frac{5}{2}}}{5} \right)$
- (C) $\frac{1}{3} \left(\frac{4\pi 3^{\frac{5}{2}}}{5} \right)$
- (D) $\frac{5}{9} \left(\frac{4\pi 3^{\frac{5}{2}}}{5} \right)$
- (E) $5 \left(\frac{4\pi 3^{\frac{5}{2}}}{5} \right)$

26. For real t , let $M(t) = \begin{pmatrix} t-11 & 2 & 0 \\ 0 & t-11 & 1 \\ 0 & 0 & t-11 \end{pmatrix}$. At $t = 11$, what is $\text{rank } M(t)$?

- (A) 0
- (B) 3
- (C) 2
- (D) 4
- (E) 1

27. Let $F(x) = \int_0^x (x-t)^1 e^{3t} dt$. Find $F^{(2)}(0)$.

- (A) $\frac{1}{2}$
- (B) 1
- (C) 0
- (D) 3
- (E) 2

Questions 28–30

28. For $a_k = (-1)^k + 7/k$, find $\limsup_{k \rightarrow \infty} a_k - \liminf_{k \rightarrow \infty} a_k$.

- (A) 1
- (B) 2
- (C) 0
- (D) 3
- (E) 4

29. For $\mathbf{F} = (3x^2, y^2, z^2)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) 13
- (B) 24
- (C) -12
- (D) 6
- (E) 12

30. Let $u = (2, 2)^T$ and $v = (1, 0)^T$. Find the squared norm of the orthogonal projection of v onto $\text{span}(u)$.

- (A) $\frac{3}{2}$
- (B) 1
- (C) 2
- (D) $\frac{1}{2}$
- (E) 0

Questions 31–33

31. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-3)^j}{j^{\frac{1}{3}} (\log j)^m}$ converge absolutely at $x = 2$? Give the least such m , or 0 if none exists.

- (A) 0
- (B) 2
- (C) 1
- (D) 3
- (E) 4

32. For which real a is $f(x) = x^a$ in $L^7(0, 1)$?

- (A) $a \geq -1/7$
- (B) $a < 1/7$
- (C) $a > -1/7$
- (D) $a > 0$
- (E) All real a

33. Evaluate $\int_C (2x + 3y) dx + (3x + 2y) dy$, where C is the line segment from $(0, 0)$ to $(6, 1)$.

- (A) 55
- (B) 110
- (C) -55
- (D) $\frac{55}{2}$
- (E) 56

Questions 34–36

34. An idempotent 17×17 real matrix P has rank 2. What is $\text{tr } P$?

- (A) 1
- (B) 2
- (C) 4
- (D) 3
- (E) 0

35. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^3 y^1 dV$.

- (A) $\frac{1}{8}$
- (B) $\frac{3! 1!}{7!}$
- (C) $6 \left(\frac{3! 1!}{7!} \right)$
- (D) $\frac{1}{6} \left(\frac{3! 1!}{7!} \right)$
- (E) $7 \left(\frac{3! 1!}{7!} \right)$

36. A map $T : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $|T(x) - T(y)| \leq |x - y|/7$. If $T(x_*) = x_*$ and $|x_0 - x_*| \leq 1$, what is the smallest guaranteed upper bound for $|T^3(x_0) - x_*|$?

- (A) $\frac{172}{343}$
- (B) $\frac{343}{342}$
- (C) $\frac{1}{1029}$
- (D) $\frac{1}{343}$
- (E) $\frac{1}{686}$

Questions 37–39

37. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 3z^2 = x + 2y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) -6
- (B) -2
- (C) -24
- (D) 12
- (E) -12

38. For $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $b = \begin{bmatrix} 11 \\ 2 \\ 13 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find x_2 .

- (A) 3
- (B) 4
- (C) 2
- (D) 1
- (E) 0

39. Solve $y'' - 8y' + 15y = 0$ with $y(0) = 1$, $y'(0) = 0$. Find $y(1)$.

- (A) $(5 - 3e^2)e^3$
- (B) $\frac{(-5 + 3e^2)e^3}{2}$
- (C) $\frac{(5 - 3e^2)e^3}{2} + 1$
- (D) $\frac{(5 - 3e^2)e^3}{4}$
- (E) $\frac{(5 - 3e^2)e^3}{2}$

Questions 40–42

40. How many zeros, counted with multiplicity, does $z^7 + 10z + 1$ have in $|z| < 1$?

- (A) 1
- (B) 4
- (C) 3
- (D) 0
- (E) 2

41. Evaluate $\lim_{x \rightarrow 0} \frac{e^{3x} - \sum_{j=0}^1 (3x)^j / j!}{x^2}$.

- (A) $\frac{3^2}{2!}$
- (B) $\frac{1}{3} \left(\frac{3^2}{2!} \right)$
- (C) $\frac{1}{2} \left(\frac{3^2}{2!} \right)$
- (D) $-\frac{3^2}{2!}$
- (E) $2 \left(\frac{3^2}{2!} \right)$

42. What is the coefficient of x^2 in $(x + 2)^4$?

- (A) 25
- (B) 23
- (C) 24
- (D) 22
- (E) 26

Questions 43–45

43. The function $y(x)$ satisfies $y^3 + 3y = 1x$ and $y(0) = 0$. Find $y'(0)$.

- (A) $\frac{1}{6}$
- (B) 0
- (C) $-\frac{1}{3}$
- (D) $\frac{1}{3}$
- (E) 3

44. A meromorphic function has 9 zeros and 2 poles inside a positively oriented simple closed contour, counted with multiplicity, and no zeros or poles on it. Find $\frac{1}{2\pi i} \int f'(z)/f(z) dz$.

- (A) 3
- (B) 5
- (C) 11
- (D) 9
- (E) 7

45. Find the arc length of $r(t) = (3t, 2t)$ for $0 \leq t \leq 1$.

- (A) $\frac{\sqrt{13}}{2}$
- (B) $2\sqrt{13}$
- (C) $-\sqrt{13}$
- (D) $1 + \sqrt{13}$
- (E) $\sqrt{13}$

Questions 46–48

46. A matrix A satisfies $(A - 12I)(A - 14I) = 0$, and its minimal polynomial has degree two. If $A^2 = uA + vI$, find u .

- (A) 25
- (B) 27
- (C) 26
- (D) 28
- (E) 24

47. Find the curvature at $t = 0$ of $r(t) = (2t, 3t^2)$.

- (A) $\frac{5}{2}$
- (B) 3
- (C) $\frac{3}{2}$
- (D) $-\frac{3}{2}$
- (E) $\frac{3}{4}$

48. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 7 have no adjacent 1s?

- (A) F_{10}
- (B) F_{11}
- (C) F_7
- (D) F_8
- (E) F_9

Questions 49–51

49. Under $x = u + 3v$, $y = u - 2v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) 5
- (B) $\frac{1}{5}$
- (C) 6
- (D) 1
- (E) 25

50. How many generators does the cyclic group of order 3 have?

- (A) 2
- (B) 4
- (C) 3
- (D) 0
- (E) 1

51. Let $f(x) = \log(1 + 3x)$. Find $f^{(2)}(0)$.

- (A) $2 \left((-1)^1 1! 3^2 \right)$
- (B) $(-1)^1 1! 3^2$
- (C) $\frac{1}{2} \left((-1)^1 1! 3^2 \right)$
- (D) $-(-1)^1 1! 3^2$
- (E) $\frac{1}{3} \left((-1)^1 1! 3^2 \right)$

Questions 52–54

52. Apply one Newton iteration to solve $x^2 - 7 = 0$, starting at $x_0 = 1$. What is x_1 ?

- (A) 5
- (B) 3
- (C) 4
- (D) 2
- (E) 6

53. Solve $y' + 3y = x^2e^{-3x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{e^3}{3}$
- (B) $\frac{1}{2e^3}$
- (C) e^{-3}
- (D) $\frac{1}{3e^3}$
- (E) $\frac{2}{3e^3}$

54. Let $R = (\mathbb{Z}/12\mathbb{Z})/([3])$, where $([3])$ is the ideal generated by $[3]$. How many elements does R have?

- (A) 1
- (B) 3
- (C) 5
- (D) 2
- (E) 4

Questions 55–57

55. Let $f(x, y) = x^2 + 6xy + (11)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 11
- (B) 8
- (C) 4
- (D) 44
- (E) 2

56. Bisection starts with an interval of length 2^7 . After 9 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{1}{24}$
- (B) $\frac{9}{16}$
- (C) $\frac{1}{8}$
- (D) $\frac{7}{8}$
- (E) $\frac{1}{16}$

57. Find the directional derivative of $f(x, y, z) = x^2 + 3y^2 + 2z^2$ at $(1, 1, 1)$ in the direction $(1, 6, 0)$.

- (A) $\frac{76\sqrt{37}}{37}$
- (B) $\frac{38\sqrt{37}}{37}$
- (C) $-\frac{38\sqrt{37}}{37}$
- (D) $1 + \frac{38\sqrt{37}}{37}$
- (E) $\frac{19\sqrt{37}}{37}$

Questions 58–60

58. What is the largest integer k such that 2^k divides $20!$?

- (A) 16
- (B) 18
- (C) 17
- (D) 20
- (E) 19

59. Find the upward flux of $\mathbf{F} = (0, 0, z)$ through $z = 3x^2 + 2y^2$, $0 \leq x, y \leq 1$.

- (A) $-\frac{5}{3}$
- (B) $\frac{10}{3}$
- (C) $\frac{8}{3}$
- (D) $\frac{5}{6}$
- (E) $\frac{5}{3}$

60. What is the least positive multiplicative inverse of 2 modulo 15?

- (A) 6
- (B) 4
- (C) 12
- (D) 8
- (E) 10

Questions 61–63

61. For $x > 0$, $xy' = 3y$ and $y(1) = 1$. Find $y(e^2)$.

- (A) $1 + e^6$
- (B) $\frac{e^6}{2}$
- (C) $2e^6$
- (D) e^6
- (E) $-e^6$

62. In the ring $\mathbb{Z}/90\mathbb{Z}$, how many elements are units?

- (A) 22
- (B) 24
- (C) 26
- (D) 23
- (E) 25

63. Evaluate $\int_0^{\pi/2} \sin^5 t \cos^7 t \, dt$.

- (A) $\frac{1}{2} \left(\frac{2! 3!}{2 \cdot 6!} \right)$
- (B) $6 \left(\frac{2! 3!}{2 \cdot 6!} \right)$
- (C) $2 \left(\frac{2! 3!}{2 \cdot 6!} \right)$
- (D) $10 \left(\frac{2! 3!}{2 \cdot 6!} \right)$
- (E) $\frac{2! 3!}{2 \cdot 6!}$

Questions 64–66

64. How many truth assignments to $p, q, r_1, \dots, r_5$ make $p \Rightarrow q$ false?

- (A) 2^4
- (B) 2^8
- (C) 2^5
- (D) 2^6
- (E) 2^7

65. Let C be the unit circle $x^2 + y^2 = 1$, $z = 2$, counterclockwise viewed from above. Evaluate $\int_C -3y(x^2 + y^2) dx + (3x(x^2 + y^2)) dy$.

- (A) 6π
- (B) $1 + 6\pi$
- (C) -6π
- (D) 3π
- (E) 12π

66. A permutation consists of disjoint cycles of lengths 4 and 11 and fixes all other points. What is its order?

- (A) 43
- (B) 46
- (C) 44
- (D) 45
- (E) 42

Answer Key

Practice Test 15 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	D	23	A	45	E
2	E	24	E	46	C
3	C	25	A	47	C
4	A	26	C	48	E
5	D	27	B	49	A
6	D	28	B	50	A
7	C	29	E	51	B
8	B	30	D	52	C
9	D	31	A	53	D
10	D	32	C	54	B
11	B	33	A	55	B
12	E	34	B	56	C
13	D	35	B	57	B
14	E	36	D	58	B
15	D	37	E	59	E
16	E	38	C	60	D
17	A	39	E	61	D
18	B	40	A	62	B
19	A	41	A	63	E
20	A	42	C	64	C
21	B	43	D	65	A
22	C	44	E	66	C

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