

GRE Mathematics Practice Test 16

Advanced Challenge

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

The most demanding form in this five-test cycle. Emphasis on connected ideas, non-routine reasoning, and careful conceptual distinctions. This is cycle 3 of 19, position 1 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. If $y'' - 8y' + 16y = 6e^{4x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $6e^4$
- (B) $\frac{3e^4}{2}$
- (C) $1 + 3e^4$
- (D) $-3e^4$
- (E) $3e^4$

2. When $x^3 + 1$ is divided by $(x - 13)(x - 14)$, the remainder is $ux + v$. Find u .

- (A) 549
- (B) 545
- (C) 548
- (D) 547
- (E) 546

3. For $f(x) = x^6 e^{4x}$, find $f^{(8)}(0)$.

- (A) $\frac{8! 4^2}{2}$
- (B) $\frac{1}{2} \left(\frac{8! 4^2}{2} \right)$
- (C) $2 \left(\frac{8! 4^2}{2} \right)$
- (D) $\frac{1}{20160} \left(\frac{8! 4^2}{2} \right)$
- (E) $\frac{1}{56} \left(\frac{8! 4^2}{2} \right)$

Questions 4–6

4. A coin success probability p has uniform prior on $[0, 1]$. After observing 3 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?

- (A) $\frac{4}{21}$
- (B) $\frac{4}{7}$
- (C) $\frac{3}{7}$
- (D) $\frac{2}{7}$
- (E) $\frac{11}{14}$

5. For $f(x, y) = x^2 + 4xy + 6y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) 68
- (B) -2
- (C) $2\sqrt{17}$
- (D) 10
- (E) -8

6. Let $P = \text{diag}(\underbrace{1, \dots, 1}_3, \underbrace{0, \dots, 0}_{16})$. What is the dimension of the space of 19×19 real matrices X satisfying $PX = X$ and $XP = 0$?

- (A) 50
- (B) 49
- (C) 46
- (D) 47
- (E) 48

Questions 7–9

7. Let $\mathbf{F} = (4y^6, z^6, x^6)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (1, 1, 2)/\sqrt{6}$.

- (A) $10\sqrt{6}$
- (B) $-20\sqrt{6}$
- (C) $-5\sqrt{6}$
- (D) $-10\sqrt{6}$
- (E) $1 - 10\sqrt{6}$

8. A random variable has mean 0 and variance 3. What upper bound does the Chebyshev inequality give for $P(|X| \geq 6)$?

- (A) $\frac{1}{12}$
- (B) $\frac{11}{12}$
- (C) $\frac{1}{24}$
- (D) $\frac{13}{24}$
- (E) $\frac{1}{36}$

9. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{4t} \cos(6t)$, and $y = e^{4t} \sin(6t)$, find d^2f/dt^2 at $t = 1$.

- (A) 12
- (B) 64
- (C) 104
- (D) 8
- (E) 0

Questions 10–12

10. A 3×3 matrix A has characteristic polynomial $(t - 12)(t + 1)(t - 2)$. What is $\text{tr}(A^3 - 2A + I)$?

- (A) 1711
- (B) 1710
- (C) 1713
- (D) 1712
- (E) 1714

11. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^4 y^5 dV$.

- (A) $\frac{4! 5!}{12!}$
- (B) $12 \left(\frac{4! 5!}{12!} \right)$
- (C) $\frac{1}{6} \left(\frac{4! 5!}{12!} \right)$
- (D) $\frac{1}{30}$
- (E) $6 \left(\frac{4! 5!}{12!} \right)$

12. A box is chosen uniformly from two boxes. Box A contains 8 red and 10 blue balls; box B contains 10 red and 8 blue balls. Two balls are drawn without replacement and both are red. What is the conditional probability that A was chosen?

- (A) $\frac{28}{73}$
- (B) $\frac{14}{73}$
- (C) $\frac{101}{146}$
- (D) $\frac{28}{219}$
- (E) $\frac{45}{73}$

Questions 13–15

13. Evaluate $\int_0^\infty x^6 e^{-4x} dx$.

- (A) $4 \left(\frac{6!}{4^7} \right)$
- (B) $16384 \left(\frac{6!}{4^7} \right)$
- (C) $\frac{1}{6} \left(\frac{6!}{4^7} \right)$
- (D) $\frac{1}{4} \left(\frac{6!}{4^7} \right)$
- (E) $\frac{6!}{4^7}$

14. Let N be the 18×18 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\dim(\ker N^2 / \ker N^1)$.

- (A) 2
- (B) 1
- (C) 0
- (D) 3
- (E) 4

15. If $y' + 4y = 6x^6 e^{-4x}$ and $y(0) = 2$, find $y(1)$.

- (A) $\frac{20}{7e^4} + 1$
- (B) $\frac{40}{7e^4}$
- (C) $\frac{20}{7e^4}$
- (D) $\frac{10}{7e^4}$
- (E) $-\frac{20}{7e^4}$

Questions 16–18

16. Let $X_1, \dots, X_8$ be independent uniform random variables on $[0, 1]$. What is $E[(\max_i X_i)^2]$?

- (A) $\frac{2}{5}$
- (B) $\frac{4}{5}$
- (C) $\frac{9}{10}$
- (D) $\frac{1}{5}$
- (E) $\frac{4}{15}$

17. Evaluate $\int_0^1 \frac{x^5}{1 + 4x^6} dx$.

- (A) $\log(5)$
- (B) $\frac{\log(5)}{6}$
- (C) $\frac{\log(5)}{24}$
- (D) $\frac{\log(5)}{4}$
- (E) $-\frac{\log(5)}{24}$

18. Let A be the 18×18 matrix whose diagonal entries are 15 and whose off-diagonal entries are 1. What is its largest eigenvalue?

- (A) 31
- (B) 34
- (C) 30
- (D) 33
- (E) 32

Questions 19–21

19. Find the coefficient of x^4 in $e^{5x} \log(1 + 4x)$.

- (A) 30
- (B) 26
- (C) -26
- (D) -64
- (E) 624

20. Independent trials have success probability $1/8$. Let T count trials up to and including the first success. Find $E[T^2]$.

- (A) 40
- (B) 200
- (C) 160
- (D) 120
- (E) 80

21. For $x > 0$, $x^2 y'' + (1 - 10x)y' + 24y = 0$, $y(1) = 0$, $y'(1) = 4$. Find $y(e)$.

- (A) $\frac{e^4}{10}$
- (B) $-e^6 + e^4$
- (C) $-2(1 - e^2)e^4$
- (D) $2(1 - e^2)e^4$
- (E) $e^4 + e^6$

Questions 22–24

22. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 72$ and $x + y = 18$. Find $x^2 + y^2$.

- (A) 180
- (B) 182
- (C) 178
- (D) 181
- (E) 179

23. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-4)^j}{j^{\frac{5}{3}} (\log j)^m}$ converge absolutely at $x = 3$? Give the least such m , or 0 if none exists.

- (A) 0
- (B) 2
- (C) 4
- (D) 3
- (E) 1

24. A continuous $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(q) = q^3$ for every rational q . Find $f(2)$.

- (A) 2^5
- (B) 2^4
- (C) 2^2
- (D) 2^3
- (E) 2^6

Questions 25–27

25. Let C be the unit circle $x^2 + y^2 = 1$, $z = 6$, counterclockwise viewed from above. Evaluate $\int_C -4y(x^2 + y^2)^5 dx + (4x(x^2 + y^2)^5) dy + 2z dz$.

- (A) -8π
- (B) 4π
- (C) 8π
- (D) $1 + 8\pi$
- (E) 16π

26. A sequence satisfies $u_0 = 12$ and $u_{k+1} = 2u_k + 6$ for all $k \geq 0$. Find u_9 .

- (A) 9211
- (B) 9210
- (C) 9212
- (D) 9209
- (E) 9208

27. The solution of $y' = 4x^6y^3$ has $y(0) = 1/5$. Find $y(1)$.

- (A) $\frac{\sqrt{1169}}{167} + 1$
- (B) $\frac{\sqrt{1169}}{167}$
- (C) $-\frac{\sqrt{1169}}{167}$
- (D) $\frac{2\sqrt{1169}}{167}$
- (E) $\frac{\sqrt{1169}}{334}$

Questions 28–30

28. Find the radius of convergence of $\sum_{k \geq 1} k^2 z^{2k} / 3^{2k} + \sum_{k \geq 0} z^{2k+1} / (4)^{2k+1}$.

- (A) 1
- (B) 5
- (C) 4
- (D) 3
- (E) 2

29. For $D = \{(x, y) : x^2 + y^2 \leq 4\}$, evaluate $\iint_D (x^2 + y^2)^5 dA$.

- (A) 4096π
- (B) $\frac{4096\pi}{11}$
- (C) $\frac{512\pi}{3}$
- (D) $\frac{2048\pi}{3}$
- (E) $\frac{4096\pi}{3}$

30. Linear maps $T : \mathbb{R}^{19} \rightarrow \mathbb{R}^{19}$ and $U : \mathbb{R}^{19} \rightarrow \mathbb{R}^{19}$ have ranks 4 and 3. What is the largest possible rank of UT ?

- (A) 3
- (B) 1
- (C) 5
- (D) 2
- (E) 4

Questions 31–33

31. Find the arc length of $r(t) = (4(t - t^3/3), 4t^2)$ for $0 \leq t \leq 1/6$.

- (A) $\frac{109}{81}$
- (B) $-\frac{109}{162}$
- (C) $\frac{109}{324}$
- (D) $\frac{109}{162}$
- (E) $\frac{271}{162}$

32. On $[0, 1]$, f is 3 on rationals and $1/3$ on irrationals. Find its upper Darboux integral minus its lower Darboux integral.

- (A) $\frac{16}{3}$
- (B) $\frac{4}{3}$
- (C) $\frac{14}{3}$
- (D) $\frac{8}{3}$
- (E) $\frac{11}{3}$

33. Evaluate $\iiint_B 4z^2(x^2 + y^2 + z^2)^6 dV$, where B is the unit ball.

- (A) $-\frac{16\pi}{51}$
- (B) $\frac{16\pi}{51}$
- (C) $\frac{8\pi}{51}$
- (D) $\frac{32\pi}{51}$
- (E) $\frac{16\pi}{51} + 1$

Questions 34–36

34. Fix a nonzero vector $u \in \mathbb{R}^{18}$. What is the dimension of the space of skew-symmetric 18×18 matrices A satisfying $Au = 0$?

- (A) 137
- (B) 134
- (C) 135
- (D) 136
- (E) 138

35. The region $0 \leq x \leq 1$, $0 \leq y \leq 4x^6$ is revolved about the line $x = -2$. Find its volume.

- (A) $\frac{23\pi}{14}$
- (B) $-\frac{23\pi}{7}$
- (C) $\frac{23\pi}{7}$
- (D) $\frac{46\pi}{7}$
- (E) $1 + \frac{23\pi}{7}$

36. A set with 3 elements has the discrete topology. How many of its subsets are simultaneously open and closed?

- (A) 2^4
- (B) 2^6
- (C) 2^5
- (D) 2^2
- (E) 2^3

Questions 37–39

37. Evaluate $\int_0^{\pi/2} \sin^{13} t \cos^9 t \, dt$.

- (A) $\frac{1}{2} \left(\frac{6! 4!}{2 \cdot 11!} \right)$
- (B) $\frac{6! 4!}{2 \cdot 11!}$
- (C) $11 \left(\frac{6! 4!}{2 \cdot 11!} \right)$
- (D) $210 \left(\frac{6! 4!}{2 \cdot 11!} \right)$
- (E) $2 \left(\frac{6! 4!}{2 \cdot 11!} \right)$

38. Let $A = \begin{bmatrix} 1 & 12 & 0 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 3)$ entry of A^5 ?

- (A) 718
- (B) 719
- (C) 720
- (D) 721
- (E) 722

39. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(4+6j)(10+6j)}$.

- (A) $\frac{3}{2}$
- (B) $\frac{1}{6}$
- (C) $\frac{1}{4}$
- (D) $\frac{1}{10}$
- (E) $\frac{1}{24}$

Questions 40–42

40. Let $\zeta = e^{2\pi i/5}$. Find $\sum_{k=0}^4 \zeta^k$.

- (A) 0
- (B) 2
- (C) 4
- (D) 1
- (E) 3

41. Evaluate $\lim_{x \rightarrow 0} \frac{e^{4x} - \sum_{j=0}^5 (4x)^j / j!}{x^6}$.

- (A) $720 \left(\frac{4^6}{6!} \right)$
- (B) $6 \left(\frac{4^6}{6!} \right)$
- (C) $\frac{4^6}{6!}$
- (D) $\frac{1}{4} \left(\frac{4^6}{6!} \right)$
- (E) $-\frac{4^6}{6!}$

42. Let $G = \mathbb{Z}/102\mathbb{Z} \times \mathbb{Z}/72\mathbb{Z}$. Find the order of G/H , where $H = \{g \in G : 6g = 0\}$.

- (A) 205
- (B) 202
- (C) 204
- (D) 206
- (E) 203

Questions 43–45

43. Find the coefficient of x^6 in the Taylor series of $(1 - 4x)^{-2}$ at zero.
- (A) $7 \cdot 4^6$
(B) $\frac{1}{4} (7 \cdot 4^6)$
(C) $\frac{6}{7} (7 \cdot 4^6)$
(D) $2 (7 \cdot 4^6)$
(E) $\frac{1}{7} (7 \cdot 4^6)$
44. An analytic function has real part $u(x, y) = x^2 - y^2 + 3x$ and satisfies $f(0) = 0$. Find $\text{Im } f(1 + i)$.
- (A) 6
(B) 5
(C) 7
(D) 4
(E) 3
45. Evaluate $\int_C (6x^5 + 4y) dx + (4x + 6y^5) dy$, where C is $r(t) = (t, 2t^2)$, $0 \leq t \leq 1$.
- (A) 73
(B) 74
(C) $\frac{73}{2}$
(D) -73
(E) 146

Questions 46–48

46. How many group homomorphisms $\mathbb{Z}/102\mathbb{Z} \rightarrow \mathbb{Z}/72\mathbb{Z}$ have image of order 6?

- (A) 4
- (B) 0
- (C) 1
- (D) 3
- (E) 2

47. Let $f(x, y) = x^2 + 8xy + (22)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 22
- (B) 6
- (C) 24
- (D) 88
- (E) 12

48. Find the coefficient of x^3 in $(1 + x)^3(1 - x)^3(1 + 2x)^2$.

- (A) -16
- (B) -4
- (C) -12
- (D) -20
- (E) -8

Questions 49–51

49. If $y'' + 36y = 4$ and $y(0) = y'(0) = 0$, find $y(\pi/6)$.

- (A) $\frac{4}{9}$
- (B) $-\frac{2}{9}$
- (C) $\frac{2}{9}$
- (D) $\frac{1}{9}$
- (E) $\frac{11}{9}$

50. How many 2-dimensional subspaces does $\mathbb{F}_5^7$ have?

- (A) $\frac{(5^6 - 1)(5^6 - 5)}{(5^2 - 1)(5^2 - 5)}$
- (B) $\frac{(5^8 - 1)(5^8 - 5)}{(5^2 - 1)(5^2 - 5)}$
- (C) $\frac{(5^9 - 1)(5^9 - 5)}{(5^2 - 1)(5^2 - 5)}$
- (D) $\frac{(5^7 - 1)(5^7 - 5)}{(5^2 - 1)(5^2 - 5)}$
- (E) $\frac{(5^{10} - 1)(5^{10} - 5)}{(5^2 - 1)(5^2 - 5)}$

51. For $\mathbf{F} = (4x^6y, y^6z, xz^6)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) 492
- (B) 246
- (C) -246
- (D) 247
- (E) 123

Questions 52–54

52. How many distinct unoriented cycles of length 4 are in $K_{3,5}$?

- (A) 20
- (B) 10
- (C) 50
- (D) 40
- (E) 30

53. Find the upward flux of $\mathbf{F} = (x, y, z)$ through $z = 4x^2 + 6y^2$, $0 \leq x, y \leq 1$.

- (A) $-\frac{10}{3}$
- (B) $-\frac{20}{3}$
- (C) $\frac{10}{3}$
- (D) $-\frac{5}{3}$
- (E) $-\frac{7}{3}$

54. How many monic irreducible polynomials of degree 5 are there over $\mathbb{F}_5$?

- (A) $2(5^5 - 5)$
- (B) $\frac{3(5^5 - 5)}{5}$
- (C) $\frac{2(5^5 - 5)}{5}$
- (D) $\frac{5^5 - 5}{5}$
- (E) $5^5 - 5$

Questions 55–57

55. If $y' = 8xy$ and $y(0) = 1$, find the coefficient of x^{12} in the Maclaurin series of y .

- (A) $\frac{128}{45}$
- (B) $\frac{512}{45}$
- (C) $-\frac{256}{45}$
- (D) $\frac{256}{45}$
- (E) $\frac{301}{45}$

56. How many functions from a set of 8 elements onto a set of 3 elements exist?

- (A) $3^8 - 2^8$
- (B) $3^8 - 3(2^8) + 3$
- (C) 3^8
- (D) $3^8 - 3(2^8)$
- (E) $3^8 - 3(2^8) - 3$

57. Evaluate $\int_0^1 \int_x^1 4y^6 \, dy \, dx$.

- (A) $\frac{1}{2}$
- (B) $\frac{4}{7}$
- (C) $\frac{2}{3}$
- (D) 2
- (E) $\frac{1}{14}$

Questions 58–60

58. How many elements z of $\mathbb{F}_{11^4}$ satisfy $z^{121} = z$?

- (A) 121
- (B) 119
- (C) 123
- (D) 120
- (E) 122

59. If $y' = y(1 - y/6)$ and $y(0) = 3$, find $y(-\log 4)$.

- (A) $\frac{11}{5}$
- (B) $\frac{3}{5}$
- (C) $\frac{12}{5}$
- (D) $-\frac{6}{5}$
- (E) $\frac{6}{5}$

60. How many nonnegative integer triples sum to 16, if each coordinate is at most 8?

- (A) 15
- (B) 75
- (C) 60
- (D) 45
- (E) 30

Questions 61–63

61. Let $f(x, y) = e^{4x+6y}$. Find $\partial^6 f / \partial x^5 \partial y$ at $(0, 0)$.

- (A) $\frac{2}{3} (6 \cdot 4^5)$
- (B) $6 \cdot 4^5$
- (C) $\frac{1}{6} (6 \cdot 4^5)$
- (D) $\frac{243}{32} (6 \cdot 4^5)$
- (E) $\frac{81}{16} (6 \cdot 4^5)$

62. How many generators does the cyclic group of order 15625 have?

- (A) $5^8(5 - 1)$
- (B) $5^6(5 - 1)$
- (C) $5^7(5 - 1)$
- (D) $5^4(5 - 1)$
- (E) $5^5(5 - 1)$

63. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 4x$ and has density $\rho(x, y) = x^4 y$. Find its centroid coordinate $\bar{x}$.

- (A) $-\frac{7}{8}$
- (B) $\frac{15}{8}$
- (C) $\frac{7}{8}$
- (D) $\frac{7}{16}$
- (E) $\frac{7}{4}$

Questions 64–66

64. A tree has 8 vertices of degree 3; every other vertex has degree 1. How many leaves does it have?

- (A) 4
- (B) 10
- (C) 16
- (D) 13
- (E) 7

65. Let $F(x) = \int_0^{x^6} e^{4t} dt$. Find $F'(1)$.

- (A) e^{24}
- (B) $\frac{e^4}{6}$
- (C) $24e^4$
- (D) e^4
- (E) $6e^4$

66. Let $R = (\mathbb{Z}/30\mathbb{Z})/([5])$, where $([5])$ is the ideal generated by $[5]$. How many units does R have?

- (A) 4
- (B) 5
- (C) 6
- (D) 3
- (E) 2

Answer Key

Practice Test 16 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	E	23	E	45	A
2	D	24	D	46	E
3	A	25	C	47	C
4	B	26	B	48	C
5	C	27	B	49	C
6	E	28	D	50	D
7	D	29	D	51	B
8	A	30	A	52	E
9	E	31	D	53	A
10	D	32	D	54	D
11	A	33	B	55	D
12	A	34	D	56	B
13	E	35	C	57	A
14	B	36	E	58	A
15	C	37	B	59	E
16	B	38	C	60	D
17	C	39	E	61	B
18	E	40	A	62	E
19	B	41	C	63	C
20	D	42	C	64	B
21	C	43	A	65	E
22	A	44	B	66	A

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