

GRE Mathematics Practice Test 17

Advanced Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A demanding practice form with frequent multi-step arguments and a mix of computational and conceptual questions. This is cycle 3 of 19, position 2 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Let C be the unit circle $x^2 + y^2 = 1$, $z = 5$, counterclockwise viewed from above. Evaluate $\int_C -4y(x^2 + y^2)^4 dx + (4x(x^2 + y^2)^4) dy + 3z dz$.

- (A) 8π
- (B) 4π
- (C) $1 + 8\pi$
- (D) 16π
- (E) -8π

2. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 52$ and $x + y = 17$. Find $x^2 + y^2$.

- (A) 184
- (B) 187
- (C) 183
- (D) 185
- (E) 186

3. Find the curvature of the helix $r(t) = (4 \cos t, 4 \sin t, 5t)$.

- (A) $\frac{8}{41}$
- (B) $-\frac{4}{41}$
- (C) $\frac{2}{41}$
- (D) $\frac{4}{41}$
- (E) $\frac{45}{41}$

Questions 4–6

4. A coin success probability p has uniform prior on $[0, 1]$. After observing 4 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?

- (A) $\frac{13}{16}$
- (B) $\frac{3}{8}$
- (C) $\frac{5}{8}$
- (D) $\frac{5}{24}$
- (E) $\frac{5}{16}$

5. If $y'' - 8y' + 16y = 5e^{4x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $-\frac{5e^4}{2}$
- (B) $\frac{5e^4}{4}$
- (C) $5e^4$
- (D) $\frac{5e^4}{2}$
- (E) $1 + \frac{5e^4}{2}$

6. A sequence satisfies $u_0 = 13$ and $u_{k+1} = 2u_k + 4$ for all $k \geq 0$. Find u_8 .

- (A) 4348
- (B) 4350
- (C) 4346
- (D) 4347
- (E) 4349

Questions 7–9

7. The region $0 \leq x \leq 1$, $0 \leq y \leq 4x^5$ is revolved about the line $x = -3$. Find its volume.
- (A) $\frac{18\pi}{7}$
(B) $-\frac{36\pi}{7}$
(C) $1 + \frac{36\pi}{7}$
(D) $\frac{72\pi}{7}$
(E) $\frac{36\pi}{7}$
8. A random variable has mean 0 and variance 4. What upper bound does the Chebyshev inequality give for $P(|X| \geq 8)$?
- (A) $\frac{17}{32}$
(B) $\frac{1}{48}$
(C) $\frac{1}{32}$
(D) $\frac{15}{16}$
(E) $\frac{1}{16}$
9. Find the coefficient of x^5 in the Taylor series of $(1 - 4x)^{-2}$ at zero.
- (A) $\frac{1}{4} (6 \cdot 4^5)$
(B) $6 \cdot 4^5$
(C) $\frac{5}{6} (6 \cdot 4^5)$
(D) $\frac{1}{6} (6 \cdot 4^5)$
(E) $-6 \cdot 4^5$

Questions 10–12

10. Linear maps $T : \mathbb{R}^{20} \rightarrow \mathbb{R}^{20}$ and $U : \mathbb{R}^{20} \rightarrow \mathbb{R}^{20}$ have ranks 5 and 3. What is the largest possible rank of UT ?

- (A) 4
- (B) 2
- (C) 1
- (D) 5
- (E) 3

11. Let $F(x) = \int_0^{x^5} e^{4t} dt$. Find $F'(1)$.

- (A) $\frac{e^4}{5}$
- (B) e^{20}
- (C) $5e^4$
- (D) e^4
- (E) $20e^4$

12. A box is chosen uniformly from two boxes. Box A contains 9 red and 11 blue balls; box B contains 11 red and 9 blue balls. Two balls are drawn without replacement and both are red. What is the conditional probability that A was chosen?

- (A) $\frac{12}{91}$
- (B) $\frac{36}{91}$
- (C) $\frac{18}{91}$
- (D) $\frac{127}{182}$
- (E) $\frac{55}{91}$

Questions 13–15

13. Find the total outward flux of $\mathbf{F} = (4x^5y, y^5z, xz^5)$ through the boundary of $[0, 1]^3$.
- (A) 6
(B) -3
(C) $\frac{3}{2}$
(D) 3
(E) 4
14. Fix a nonzero vector $u \in \mathbb{R}^{19}$. What is the dimension of the space of skew-symmetric 19×19 matrices A satisfying $Au = 0$?
- (A) 152
(B) 153
(C) 151
(D) 155
(E) 154
15. Find the coefficient of x^6 in $\cos(4x)/(1 - 4x^2)$.
- (A) $-\frac{1261}{45}$
(B) $-\frac{256}{45}$
(C) $\frac{1216}{45}$
(D) -19456
(E) $-\frac{1216}{45}$

Questions 16–18

16. Let $X_1, \dots, X_9$ be independent uniform random variables on $[0, 1]$. What is $E[(\max_i X_i)^2]$?

- (A) $\frac{3}{11}$
- (B) $\frac{9}{22}$
- (C) $\frac{10}{11}$
- (D) $\frac{2}{11}$
- (E) $\frac{9}{11}$

17. The solution of $y' = 4x^5y^3$ has $y(0) = 1/5$. Find $y(1)$.

- (A) $\frac{2\sqrt{213}}{71}$
- (B) $\frac{\sqrt{213}}{142}$
- (C) $-\frac{\sqrt{213}}{71}$
- (D) $\frac{\sqrt{213}}{71}$
- (E) $\frac{\sqrt{213}}{71} + 1$

18. Let $A = \begin{bmatrix} 1 & 13 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 3)$ entry of A^6 ?

- (A) 779
- (B) 781
- (C) 782
- (D) 778
- (E) 780

Questions 19–21

19. For $D = \{(x, y) : x^2 + y^2 \leq 4\}$, evaluate $\iint_D (x^2 + y^2)^4 dA$.
- (A) $\frac{1024\pi}{9}$
(B) $\frac{256\pi}{5}$
(C) $\frac{1024\pi}{5}$
(D) $\frac{2048\pi}{5}$
(E) 1024π
20. Independent trials have success probability $1/9$. Let T count trials up to and including the first success. Find $E[T^2]$.
- (A) 102
(B) 204
(C) 153
(D) 255
(E) 51
21. Evaluate $\iiint_{x^2+y^2+z^2 \leq 4} (x^2 + y^2 + z^2)^4 dV$.
- (A) $\frac{1}{3} \left(\frac{4\pi 4^{\frac{11}{2}}}{11} \right)$
(B) $\frac{4\pi 4^{\frac{11}{2}}}{11}$
(C) $\frac{11}{36} \left(\frac{4\pi 4^{\frac{11}{2}}}{11} \right)$
(D) $11 \left(\frac{4\pi 4^{\frac{11}{2}}}{11} \right)$
(E) $\frac{1}{2} \left(\frac{4\pi 4^{\frac{11}{2}}}{11} \right)$

Questions 22–24

22. An invertible 19×19 matrix A has determinant 4. Find $\det(\text{adj } A)$.

- (A) 2^{35}
- (B) 2^{36}
- (C) 2^{37}
- (D) $2^{36} \cdot 3$
- (E) $2^{36} \cdot 5$

23. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(4x) - \sum_{j=0}^3 (-1)^j (4x)^{2j+1} / (2j+1)!}{x^9}$.

- (A) $4 \left(\frac{(-1)^4 4^9}{9!} \right)$
- (B) $\frac{(-1)^4 4^9}{9!}$
- (C) $-\frac{(-1)^4 4^9}{9!}$
- (D) $\frac{1}{4} \left(\frac{(-1)^4 4^9}{9!} \right)$
- (E) $9 \left(\frac{(-1)^4 4^9}{9!} \right)$

24. A continuous $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(q) = q^4$ for every rational q . Find $f(2)$.

- (A) 2^7
- (B) 2^6
- (C) 2^5
- (D) 2^4
- (E) 2^3

Questions 25–27

25. The solution of $y' = 4x^4y^2$, $y(0) = 1$, has its first positive singularity at which x ?
- (A) $\frac{2^{\frac{3}{5}}\sqrt[5]{5}}{2}$
(B) $\frac{5}{4}$
(C) $\frac{2^{\frac{2}{5}}\sqrt[5]{5}}{2}$
(D) $\frac{2^{\frac{2}{5}} \cdot 5^{\frac{4}{5}}}{5}$
(E) $\frac{2^{\frac{3}{5}}}{2}$
26. On P_{20} , the real polynomials of degree at most 20, define $T(p) = x^2p'' - 2xp' + 2p$. Find its rank.
- (A) 20
(B) 21
(C) 18
(D) 19
(E) 17
27. Find the upward flux of $\mathbf{F} = (x, y, z)$ through $z = 4x^2 + 5y^2$, $0 \leq x, y \leq 1$.
- (A) 3
(B) -6
(C) -2
(D) $-\frac{3}{2}$
(E) -3

Questions 28–30

28. Find the radius of convergence of $\sum_{k=1}^{\infty} k^4 x^k / 4^k$.

- (A) 5
- (B) 6
- (C) 2
- (D) 3
- (E) 4

29. Two solutions of $y'' + 4x^5 y' + 5y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $e^{-\frac{2}{3}} + 1$
- (B) $e^{-\frac{2}{3}}$
- (C) $\frac{2}{e^{\frac{2}{3}}}$
- (D) $-\frac{1}{e^{\frac{2}{3}}}$
- (E) $\frac{1}{2e^{\frac{2}{3}}}$

30. Subspaces U, V of $\mathbb{R}^{21}$ have dimensions 6 and 7. What is the smallest possible value of $\dim(U \cap V)$?

- (A) 4
- (B) 1
- (C) 0
- (D) 3
- (E) 2

Questions 31–33

31. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-4)^j}{j^{\frac{4}{3}} (\log j)^m}$ converge absolutely at $x = 3$? Give the least such m , or 0 if none exists.

- (A) 1
- (B) 4
- (C) 2
- (D) 3
- (E) 0

32. On $[0, 1]$, f is 4 on rationals and $1/4$ on irrationals. Find its upper Darboux integral minus its lower Darboux integral.

- (A) $\frac{23}{4}$
- (B) $\frac{15}{2}$
- (C) $\frac{15}{4}$
- (D) $\frac{19}{4}$
- (E) $\frac{15}{8}$

33. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 4x$ and has density $\rho(x, y) = x^3 y$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{3}{7}$
- (B) $\frac{13}{7}$
- (C) $\frac{6}{7}$
- (D) $-\frac{6}{7}$
- (E) $\frac{12}{7}$

Questions 34–36

34. Let V consist of real polynomials of degree at most 10 that have both value and first derivative zero at 13, 14, and 16. Find $\dim V$.

- (A) 3
- (B) 5
- (C) 6
- (D) 4
- (E) 7

35. Let $F(x) = \int_0^x (x-t)^4 e^{4t} dt$. Find $F^{(5)}(0)$.

- (A) 120
- (B) $\frac{1}{24}$
- (C) 96
- (D) 24
- (E) 0

36. A set with 4 elements has the discrete topology. How many of its subsets are simultaneously open and closed?

- (A) 2^6
- (B) 2^5
- (C) 2^7
- (D) 2^3
- (E) 2^4

Questions 37–39

37. Let $f(x, y) = e^{4x+5y}$. Find $\partial^5 f / \partial x^4 \partial y$ at $(0, 0)$.

- (A) $\frac{4}{5} (5 \cdot 4^4)$
- (B) $\frac{1}{5} (5 \cdot 4^4)$
- (C) $5 \cdot 4^4$
- (D) $\frac{625}{256} (5 \cdot 4^4)$
- (E) $\frac{125}{64} (5 \cdot 4^4)$

38. The roots r, s of $x^2 - 17x + 52 = 0$ are counted with multiplicity. Find $r^3 + s^3$.

- (A) 2263
- (B) 2259
- (C) 2261
- (D) 2260
- (E) 2262

39. If $y' + 4y = 5x^5 e^{-4x}$ and $y(0) = 3$, find $y(1)$.

- (A) $\frac{23}{6e^4}$
- (B) $\frac{23}{3e^4}$
- (C) $\frac{23}{12e^4}$
- (D) $-\frac{23}{6e^4}$
- (E) $\frac{23}{6e^4} + 1$

Questions 40–42

40. Let $\zeta = e^{2\pi i/6}$. Find $\sum_{k=0}^5 \zeta^k$.

- (A) 0
- (B) 4
- (C) 3
- (D) 2
- (E) 1

41. Evaluate $\int_0^1 x^3(\log x)^4 dx$.

- (A) $4 \left(\frac{(-1)^4 4!}{4^5} \right)$
- (B) $\frac{1}{5} \left(\frac{(-1)^4 4!}{4^5} \right)$
- (C) $\frac{32}{3} \left(\frac{(-1)^4 4!}{4^5} \right)$
- (D) $\frac{(-1)^4 4!}{4^5}$
- (E) $-\frac{(-1)^4 4!}{4^5}$

42. What is the largest integer k such that 3^k divides $22!$?

- (A) 7
- (B) 10
- (C) 8
- (D) 11
- (E) 9

Questions 43–45

43. What is the maximum of x^5y^4 over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $9 \left(\left(\frac{5}{9} \right)^5 \left(\frac{4}{9} \right)^4 \right)$
- (B) $\left(\frac{5}{9} \right)^4 \left(\frac{4}{9} \right)^5$
- (C) $\frac{1}{29}$
- (D) $\left(\frac{5}{9} \right)^5 \left(\frac{4}{9} \right)^4$
- (E) $\frac{20}{81}$

44. An analytic function has real part $u(x, y) = x^2 - y^2 + 4x$ and satisfies $f(0) = 0$. Find $\text{Im } f(1 + i)$.

- (A) 6
- (B) 8
- (C) 4
- (D) 10
- (E) 2

45. Find the coefficient of x^4 in $e^{4x} \log(1 + 4x)$.

- (A) 0
- (B) -1
- (C) 4
- (D) 1
- (E) -64

Questions 46–48

46. In the ring $\mathbb{Z}/102\mathbb{Z}$, how many elements satisfy $x^2 = 1$?

- (A) 4
- (B) 5
- (C) 6
- (D) 2
- (E) 3

47. Evaluate $\int_C (5x^4 + 4y) dx + (4x + 5y^4) dy$, where C is $r(t) = (t, 3t^2)$, $0 \leq t \leq 1$.

- (A) -256
- (B) 512
- (C) 128
- (D) 257
- (E) 256

48. Find the coefficient of x^3 in $(1+x)^4(1-x)^4(1+2x)^2$.

- (A) -11
- (B) -26
- (C) -16
- (D) -6
- (E) -21

Questions 49–51

49. Evaluate $\int_0^{\log 2} e^x (e^x + 4)^5 dx$.

- (A) $\frac{6^6 - 5^6}{6}$
- (B) $6 \left(\frac{6^6 - 5^6}{6} \right)$
- (C) $-\frac{6^6 - 5^6}{6}$
- (D) $6^5 - 5^5$
- (E) $\frac{6^6}{6}$

50. A permutation consists of disjoint cycles of lengths 7 and 6, together with a disjoint transposition when the sum of these two lengths is odd. It fixes all other points. What is its order?

- (A) 43
- (B) 42
- (C) 40
- (D) 41
- (E) 44

51. Evaluate $\iiint_B 4z^2(x^2 + y^2 + z^2)^5 dV$, where B is the unit ball.

- (A) $\frac{16\pi}{45}$
- (B) $\frac{32\pi}{45}$
- (C) $-\frac{16\pi}{45}$
- (D) $\frac{8\pi}{45}$
- (E) $1 + \frac{16\pi}{45}$

Questions 52–54

52. How many edges does the complete bipartite graph $K_{4,6}$ have?

- (A) 16
- (B) 40
- (C) 24
- (D) 32
- (E) 8

53. Evaluate $\lim_{x \rightarrow 0} \frac{e^{4x} - \sum_{j=0}^4 (4x)^j / j!}{x^5}$.

- (A) $-\frac{4^5}{5!}$
- (B) $\frac{4^5}{5!}$
- (C) $\frac{1}{4} \left(\frac{4^5}{5!} \right)$
- (D) $5 \left(\frac{4^5}{5!} \right)$
- (E) $120 \left(\frac{4^5}{5!} \right)$

54. In $R = \mathbb{F}_7[x]/(x^4)$, how many elements are units?

- (A) $7^4 - 7^0$
- (B) $7^4 - 7^3$
- (C) $7^6 - 7^5$
- (D) $7^5 - 7^3$
- (E) $7^5 - 7^4$

Questions 55–57

55. The local inverse $y(x)$ of $x = 4y + y^3$ satisfies $y(0) = 0$. Find $y^{(5)}(0)$.

- (A) $\frac{360}{4^7}$
- (B) $\frac{1}{120} \left(\frac{360}{4^7} \right)$
- (C) $-\frac{360}{4^7}$
- (D) $\frac{1}{4} \left(\frac{360}{4^7} \right)$
- (E) $4 \left(\frac{360}{4^7} \right)$

56. How many functions from a set of 9 elements onto a set of 3 elements exist?

- (A) $3^9 - 2^9$
- (B) $3^9 - 3(2^9) - 3$
- (C) $3^9 - 3(2^9)$
- (D) 3^9
- (E) $3^9 - 3(2^9) + 3$

57. If $y'' + 25y = 4$ and $y(0) = y'(0) = 0$, find $y(\pi/5)$.

- (A) $-\frac{8}{25}$
- (B) $\frac{33}{25}$
- (C) $\frac{4}{25}$
- (D) $\frac{8}{25}$
- (E) $\frac{16}{25}$

Questions 58–60

58. Integers x satisfy $x \equiv 8 \pmod{11}$ and $x \equiv 7 \pmod{16}$. Find the sum of all such x with $0 \leq x < 352$.

- (A) 477
- (B) 476
- (C) 479
- (D) 480
- (E) 478

59. Let C be $x^2/25 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C \left(-\frac{4y^3}{3}\right) dx + \left(\frac{x^3}{3}\right) dy$.

- (A) $\frac{145\pi}{4}$
- (B) $1 + \frac{145\pi}{4}$
- (C) $-\frac{145\pi}{4}$
- (D) $\frac{145\pi}{2}$
- (E) $\frac{145\pi}{8}$

60. How many nonnegative integer triples sum to 18, if each coordinate is at most 9?

- (A) 91
- (B) 55
- (C) 19
- (D) 73
- (E) 37

Questions 61–63

61. Find the directional derivative of $f(x, y, z) = x^2y + 5xz^2 + 4y^2z$ at $(1, 1, 1)$ in the direction $(1, 3, 0)$.

- (A) $\frac{34\sqrt{10}}{5}$
- (B) $\frac{17\sqrt{10}}{10}$
- (C) $\frac{17\sqrt{10}}{5}$
- (D) $-\frac{17\sqrt{10}}{5}$
- (E) $1 + \frac{17\sqrt{10}}{5}$

62. The symmetric group S_{19} acts on the two-element subsets of $\{1, \dots, 19\}$. What is the index of the setwise stabilizer of $\{1, 2\}$?

- (A) 173
- (B) 170
- (C) 172
- (D) 169
- (E) 171

63. Under $x = u + 4v$, $y = u - 5v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) 81
- (B) 9
- (C) $\frac{1}{9}$
- (D) 20
- (E) 1

Questions 64–66

- 64.** A tree has 9 vertices of degree 3; every other vertex has degree 1. How many leaves does it have?
- (A) 5
 - (B) 14
 - (C) 8
 - (D) 11
 - (E) 17
- 65.** Let $f(x, y) = x^2 + 8xy + (21)y^2$. What is the product of the eigenvalues of the Hessian of f ?
- (A) 5
 - (B) 21
 - (C) 20
 - (D) 10
 - (E) 84
- 66.** In $\mathbb{Z}/366\mathbb{Z}$, how many residues are not idempotent ($x^2 \neq x$)?
- (A) 359
 - (B) 356
 - (C) 360
 - (D) 358
 - (E) 357

Answer Key

Practice Test 17 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	A	23	B	45	A
2	D	24	D	46	A
3	D	25	A	47	E
4	C	26	D	48	C
5	D	27	E	49	A
6	A	28	E	50	B
7	E	29	B	51	A
8	E	30	C	52	C
9	B	31	A	53	B
10	E	32	C	54	B
11	C	33	C	55	A
12	B	34	B	56	E
13	D	35	D	57	D
14	B	36	E	58	E
15	E	37	C	59	A
16	E	38	C	60	B
17	D	39	A	61	C
18	E	40	A	62	E
19	C	41	D	63	B
20	C	42	E	64	D
21	B	43	D	65	C
22	B	44	A	66	D

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