

GRE Mathematics Practice Test 18

Upper-Intermediate Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

Intermediate reinforcement, combining standard applications with selected more demanding arguments. This is cycle 3 of 19, position 3 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Let C be the unit circle $x^2 + y^2 = 1$, $z = 4$, counterclockwise viewed from above. Evaluate $\int_C -4y(x^2 + y^2)^3 dx + (4x(x^2 + y^2)^3) dy + 4z dz$.

- (A) 8π
- (B) -8π
- (C) 16π
- (D) 4π
- (E) $1 + 8\pi$

2. An invertible 20×20 matrix A has determinant 2. Find $\det(\text{adj } A)$.

- (A) 524286
- (B) 524290
- (C) 524289
- (D) 524287
- (E) 524288

3. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^4 y^3 dV$.

- (A) $10 \left(\frac{4! 3!}{10!} \right)$
- (B) $6 \left(\frac{4! 3!}{10!} \right)$
- (C) $\frac{4! 3!}{10!}$
- (D) $\frac{1}{20}$
- (E) $\frac{1}{6} \left(\frac{4! 3!}{10!} \right)$

Questions 4–6

4. A coin success probability p has uniform prior on $[0, 1]$. After observing 5 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?

- (A) $\frac{5}{6}$
- (B) $\frac{8}{9}$
- (C) $\frac{2}{9}$
- (D) $\frac{2}{3}$
- (E) $\frac{1}{3}$

5. Find the total outward flux of $\mathbf{F} = (4x^4y, y^4z, xz^4)$ through the boundary of $[0, 1]^3$.

- (A) 4
- (B) $\frac{3}{2}$
- (C) -3
- (D) 3
- (E) 6

6. On P_{21} , the real polynomials of degree at most 21, define $T(p) = x^2p'' - 0p$. Find its rank.

- (A) 19
- (B) 21
- (C) 22
- (D) 20
- (E) 18

Questions 7–9

7. Let $F(x) = \int_0^x (x-t)^3 e^{4t} dt$. Find $F^{(4)}(0)$.

- (A) $\frac{1}{6}$
- (B) 6
- (C) 0
- (D) 24
- (E) 12

8. A random variable has mean 0 and variance 5. What upper bound does the Chebyshev inequality give for $P(|X| \geq 10)$?

- (A) $\frac{1}{60}$
- (B) $\frac{1}{20}$
- (C) $\frac{19}{20}$
- (D) $\frac{21}{40}$
- (E) $\frac{1}{40}$

9. If $y'' + 16y = 4$ and $y(0) = y'(0) = 0$, find $y(\pi/4)$.

- (A) $\frac{1}{2}$
- (B) $\frac{3}{2}$
- (C) $-\frac{1}{2}$
- (D) $\frac{1}{4}$
- (E) 1

Questions 10–12

10. Subspaces U, V of $\mathbb{R}^{22}$ have dimensions 5 and 6. What is the smallest possible value of $\dim(U \cap V)$?

- (A) 1
- (B) 4
- (C) 2
- (D) 0
- (E) 3

11. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{4t} \cos(4t)$, and $y = e^{4t} \sin(4t)$, find d^2f/dt^2 at $t = 1$.

- (A) 1
- (B) 0
- (C) -1
- (D) 64
- (E) 8

12. A box is chosen uniformly from two boxes. Box A contains 10 red and 12 blue balls; box B contains 12 red and 10 blue balls. Two balls are drawn without replacement and both are red. What is the conditional probability that A was chosen?

- (A) $\frac{15}{74}$
- (B) $\frac{15}{37}$
- (C) $\frac{22}{37}$
- (D) $\frac{26}{37}$
- (E) $\frac{5}{37}$

Questions 13–15

13. Evaluate $\int_0^{\log 2} e^x (e^x + 4)^4 dx$.

- (A) $\frac{6^5}{5}$
- (B) $5 \left(\frac{6^5 - 5^5}{5} \right)$
- (C) $\frac{6^5 - 5^5}{5}$
- (D) $6^4 - 5^4$
- (E) $-\frac{6^5 - 5^5}{5}$

14. Let V consist of real polynomials of degree at most 9 that vanish at 14, 15, and 17. Find $\dim V$.

- (A) 5
- (B) 8
- (C) 7
- (D) 6
- (E) 9

15. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-4)^j}{j^1 (\log j)^m}$ converge absolutely at $x = 3$? Give the least such m , or 0 if none exists.

- (A) 2
- (B) 4
- (C) 0
- (D) 3
- (E) 1

Questions 16–18

16. Let $X_1, \dots, X_{10}$ be independent uniform random variables on $[0, 1]$. What is $E[(\max_i X_i)^2]$?

- (A) $\frac{1}{6}$
- (B) $\frac{5}{6}$
- (C) $\frac{11}{12}$
- (D) $\frac{5}{18}$
- (E) $\frac{5}{12}$

17. The region $0 \leq x \leq 1$, $0 \leq y \leq 4x^4$ is revolved about the line $x = -4$. Find its volume.

- (A) $-\frac{116\pi}{15}$
- (B) $1 + \frac{116\pi}{15}$
- (C) $\frac{116\pi}{15}$
- (D) $\frac{232\pi}{15}$
- (E) $\frac{58\pi}{15}$

18. The roots r, s of $x^2 - 21x + 98 = 0$ are counted with multiplicity. Find $r^3 + s^3$.

- (A) 3086
- (B) 3085
- (C) 3088
- (D) 3089
- (E) 3087

Questions 19–21

19. Find the coefficient of x^4 in the Taylor series of $(1 - 4x)^{-2}$ at zero.

- (A) $\frac{4}{5} (5 \cdot 4^4)$
- (B) $5 \cdot 4^4$
- (C) $\frac{1}{4} (5 \cdot 4^4)$
- (D) $2 (5 \cdot 4^4)$
- (E) $\frac{1}{5} (5 \cdot 4^4)$

20. Independent trials have success probability $1/10$. Let T count trials up to and including the first success. Find $E[T]$.

- (A) 16
- (B) 10
- (C) 7
- (D) 4
- (E) 13

21. Let C be $x^2/16 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C \left(-\frac{4y^3}{3}\right) dx + \left(\frac{x^3}{3}\right) dy$.

- (A) 10π
- (B) -20π
- (C) 20π
- (D) 40π
- (E) $1 + 20\pi$

Questions 22–24

22. For real t , let $M(t) = \begin{pmatrix} t-14 & 7 & 0 \\ 0 & t-14 & 1 \\ 0 & 0 & t-14 \end{pmatrix}$. At $t = 14$, what is $\dim \ker(M(t)^2) - \dim \ker M(t)$?

- (A) 2
- (B) 3
- (C) 4
- (D) 1
- (E) 0

23. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 4z^2 = x + 4y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) -16
- (B) 32
- (C) -8
- (D) -4
- (E) -32

24. A continuous $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(q) = q^5$ for every rational q . Find $f(2)$.

- (A) 2^4
- (B) 2^7
- (C) 2^8
- (D) 2^6
- (E) 2^5

Questions 25–27

25. Evaluate $\int_0^1 x^4(1-x)^3 dx$.

- (A) $5 \left(\frac{4! 3!}{8!} \right)$
- (B) $\frac{4! 3!}{8!}$
- (C) $14 \left(\frac{4! 3!}{8!} \right)$
- (D) $\frac{1}{4} \left(\frac{4! 3!}{8!} \right)$
- (E) $8 \left(\frac{4! 3!}{8!} \right)$

26. Let $u = (3, 4)^T$ and $v = (1, 0)^T$. Find the squared distance from $(3, 0)^T$ to $\text{span}(u)$.

- (A) $\frac{144}{25}$
- (B) $\frac{169}{25}$
- (C) $\frac{263}{50}$
- (D) $\frac{313}{50}$
- (E) $\frac{119}{25}$

27. For $f(x, y) = x^2 + 4xy + 4y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) $2\sqrt{5}$
- (B) 20
- (C) 6
- (D) -4
- (E) -2

Questions 28–30

28. Find the radius of convergence of $\sum_{k=1}^{\infty} k^5 x^k / 5^k$.

- (A) 3
- (B) 7
- (C) 4
- (D) 6
- (E) 5

29. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 4x)^{1/(3x)} - e^{4/3}}{x}$.

- (A) $-\frac{8e^{\frac{4}{3}}}{3}$
- (B) $-\frac{2e^{\frac{4}{3}}}{3}$
- (C) $\frac{8e^{\frac{4}{3}}}{3}$
- (D) $e^{\frac{4}{3}}$
- (E) $-\frac{16e^{\frac{4}{3}}}{3}$

30. An idempotent 20×20 real matrix P has rank 1. What is $\det(I + P)$?

- (A) 3
- (B) 0
- (C) 4
- (D) 1
- (E) 2

Questions 31–33

31. If $y' = y(1 - y/4)$ and $y(0) = 2$, find $y(-\log 4)$.

- (A) $\frac{4}{5}$
- (B) $-\frac{4}{5}$
- (C) $\frac{2}{5}$
- (D) $\frac{9}{5}$
- (E) $\frac{8}{5}$

32. On $[0, 1]$, f is 5 on rationals and $1/5$ on irrationals. Find its upper Darboux integral minus its lower Darboux integral.

- (A) $\frac{29}{5}$
- (B) $\frac{34}{5}$
- (C) $\frac{48}{5}$
- (D) $\frac{12}{5}$
- (E) $\frac{24}{5}$

33. For $D = \{(x, y) : x^2 + y^2 \leq 4\}$, evaluate $\iint_D (x^2 + y^2)^3 dA$.

- (A) 64π
- (B) 16π
- (C) 128π
- (D) 256π
- (E) $\frac{256\pi}{7}$

Questions 34–36

34. For $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 14 \\ 7 \\ 21 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find x_2 .

- (A) 3
- (B) 4
- (C) $\frac{9}{2}$
- (D) $\frac{7}{2}$
- (E) $\frac{5}{2}$

35. The local inverse $y(x)$ of $x = 4y + y^3$ satisfies $y(0) = 0$. Find $y^{(3)}(0)$.

- (A) $\frac{1}{4} \left(-\frac{6}{4^4} \right)$
- (B) $-\frac{6}{4^4}$
- (C) $\frac{1}{6} \left(-\frac{6}{4^4} \right)$
- (D) $4 \left(-\frac{6}{4^4} \right)$
- (E) $\frac{6}{4^4}$

36. A set with 5 elements has the discrete topology. How many of its subsets are simultaneously open and closed?

- (A) 2^5
- (B) 2^4
- (C) 2^7
- (D) 2^8
- (E) 2^6

Questions 37–39

37. If $y'' - 8y' + 16y = 4e^{4x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $-2e^4$
- (B) $4e^4$
- (C) $2e^4$
- (D) $1 + 2e^4$
- (E) e^4

38. What is the coefficient of x^7 in $(x + 3)^6(1 - x)^3$?

- (A) -82
- (B) -84
- (C) -85
- (D) -83
- (E) -86

39. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 4x$ and has density $\rho(x, y) = x^2y$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{5}{12}$
- (B) $\frac{5}{6}$
- (C) $-\frac{5}{6}$
- (D) $\frac{5}{3}$
- (E) $\frac{11}{6}$

Questions 40–42

40. Let $\zeta = e^{2\pi i/7}$. Find $\sum_{k=0}^6 \zeta^k$.

- (A) 4
- (B) 1
- (C) 3
- (D) 0
- (E) 2

41. Evaluate $\int_0^1 \int_0^x x^4 y^4 dy dx$.

- (A) $\frac{1}{50}$
- (B) $\frac{1}{60}$
- (C) $\frac{1}{25}$
- (D) $\frac{1}{10}$
- (E) $\frac{1}{45}$

42. In the additive group $\mathbb{Z}/36\mathbb{Z}$, find the index of the subgroup generated by $[6]$.

- (A) 4
- (B) 7
- (C) 6
- (D) 5
- (E) 8

Questions 43–45

43. Evaluate $\lim_{x \rightarrow 0} \frac{e^{4x} - \sum_{j=0}^3 (4x)^j / j!}{x^4}$.

- (A) $24 \left(\frac{4^4}{4!} \right)$
- (B) $-\frac{4^4}{4!}$
- (C) $4 \left(\frac{4^4}{4!} \right)$
- (D) $\frac{4^4}{4!}$
- (E) $\frac{1}{4} \left(\frac{4^4}{4!} \right)$

44. An analytic function has real part $u(x, y) = x^2 - y^2 + 5x$ and satisfies $f(0) = 0$. Find $\text{Im } f(1 + i)$.

- (A) 5
- (B) 3
- (C) 7
- (D) 9
- (E) 11

45. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(4 + 4j)(8 + 4j)}$.

- (A) $\frac{1}{8}$
- (B) 1
- (C) $\frac{1}{32}$
- (D) $\frac{1}{16}$
- (E) $\frac{1}{4}$

Questions 46–48

46. How many invertible 3×3 matrices over $\mathbb{F}_5$ are there whose first column is $(1, 0, 0)^T$?

- (A) $\frac{5^3 5^3 5^3}{5^3 - 1}$
(B) $\frac{(5^3 - 5^1)(5^3 - 5^2)(5^3 - 5^3)}{5^3 - 1}$
(C) $\frac{(5^3 - 5^0)(5^3 - 5^1)(5^3 - 5^2)}{5^3 - 1}$
(D) $\frac{(5^3 - 5^0)(5^3 - 5^1)}{5^3 - 1}$
(E) $\frac{(5^3 - 5^0)(5^3 - 5^0)(5^3 - 5^0)}{5^3 - 1}$

47. For $\mathbf{F} = (4x^4y, y^4z, xz^4)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) 34
(B) 136
(C) 68
(D) -68
(E) 69

48. What is the coefficient of x^2 in $(1 + 2x)^5$?

- (A) 66
(B) 53
(C) 40
(D) 27
(E) 14

Questions 49–51

49. The solution of $y' = 4x^3y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{2^{\frac{3}{4}}}{2}$
- (B) 2
- (C) 1
- (D) $\frac{1}{2}$
- (E) $\frac{\sqrt{2}}{2}$

50. How many permutations in S_{20} consist of one 6-cycle and 14 fixed points?

- (A) $2^7 \cdot 3^2 \cdot 5^2 \cdot 17 \cdot 19$
- (B) $2^6 \cdot 3^2 \cdot 5^3 \cdot 17 \cdot 19$
- (C) $2^5 \cdot 3^2 \cdot 5^2 \cdot 17 \cdot 19$
- (D) $2^6 \cdot 3^2 \cdot 5^2 \cdot 17 \cdot 19$
- (E) $2^6 \cdot 3^3 \cdot 5^2 \cdot 17 \cdot 19$

51. For the curve $x = t^4$, $y = t^5 + 4t$, find d^2y/dx^2 at $t = 1$.

- (A) $\frac{9}{4}$
- (B) -1
- (C) $\frac{17}{16}$
- (D) $-\frac{7}{16}$
- (E) $-\frac{7}{4}$

Questions 52–54

52. How many edges does the complete bipartite graph $K_{5,7}$ have?

- (A) 46
- (B) 13
- (C) 24
- (D) 57
- (E) 35

53. Find the curvature of the helix $r(t) = (4 \cos t, 4 \sin t, 4t)$.

- (A) $\frac{1}{8}$
- (B) $\frac{1}{16}$
- (C) $\frac{9}{8}$
- (D) $\frac{1}{4}$
- (E) $-\frac{1}{8}$

54. How many maximal proper subgroups does the cyclic group of order 216 have?

- (A) 2
- (B) 3
- (C) 4
- (D) 0
- (E) 1

Questions 55–57

55. Evaluate $\int_0^1 \int_x^1 4y^4 \, dy \, dx$.

- (A) $\frac{4}{5}$
- (B) $\frac{2}{3}$
- (C) 2
- (D) 1
- (E) $\frac{2}{15}$

56. How many functions from a set of 10 elements onto a set of 3 elements exist?

- (A) $3^{10} - 2^{10}$
- (B) $3^{10} - 3(2^{10})$
- (C) 3^{10}
- (D) $3^{10} - 3(2^{10}) + 3$
- (E) $3^{10} - 3(2^{10}) - 3$

57. If $y' = 8xy$ and $y(0) = 1$, find the coefficient of x^8 in the Maclaurin series of y .

- (A) $\frac{32}{3}$
- (B) $\frac{35}{3}$
- (C) $-\frac{32}{3}$
- (D) $\frac{64}{3}$
- (E) $\frac{16}{3}$

Questions 58–60

58. How many residues x modulo 36 satisfy $6x \equiv 42 \pmod{36}$ and $2x \equiv 0 \pmod{36}$?

- (A) 4
- (B) 0
- (C) 2
- (D) 3
- (E) 1

59. Two solutions of $y'' + 4x^4y' + 4y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $-\frac{1}{e^{\frac{4}{5}}}$
- (B) $\frac{2}{e^{\frac{4}{5}}}$
- (C) $\frac{1}{2e^{\frac{4}{5}}}$
- (D) $e^{-\frac{4}{5}} + 1$
- (E) $e^{-\frac{4}{5}}$

60. How many nonnegative integer triples sum to 20, if each coordinate is at most 10?

- (A) 44
- (B) 22
- (C) 110
- (D) 88
- (E) 66

Questions 61–63

61. Evaluate $\int_0^{\pi/2} \sin^9 t \cos^9 t \, dt$.

- (A) $\frac{4! 4!}{2 \cdot 9!}$
- (B) $9 \left(\frac{4! 4!}{2 \cdot 9!} \right)$
- (C) $2 \left(\frac{4! 4!}{2 \cdot 9!} \right)$
- (D) $\frac{1}{2} \left(\frac{4! 4!}{2 \cdot 9!} \right)$
- (E) $70 \left(\frac{4! 4!}{2 \cdot 9!} \right)$

62. The field K has 5^4 elements. Find the largest possible multiplicative order of a nonzero element of K .

- (A) 5^4
- (B) $5^5 - 1$
- (C) $5^4 + 1$
- (D) $5^3 - 1$
- (E) $5^4 - 1$

63. Evaluate $\iiint_{x^2+y^2+z^2 \leq 4} (x^2 + y^2 + z^2)^3 \, dV$.

- (A) $\frac{9}{28} \left(\frac{4\pi 4^{\frac{9}{2}}}{9} \right)$
- (B) $9 \left(\frac{4\pi 4^{\frac{9}{2}}}{9} \right)$
- (C) $\frac{1}{2} \left(\frac{4\pi 4^{\frac{9}{2}}}{9} \right)$
- (D) $\frac{4\pi 4^{\frac{9}{2}}}{9}$
- (E) $\frac{1}{3} \left(\frac{4\pi 4^{\frac{9}{2}}}{9} \right)$

Questions 64–66

64. A tree has 14 vertices. What is the sum of its vertex degrees?

- (A) 18
- (B) 42
- (C) 10
- (D) 34
- (E) 26

65. Let $f(x, y) = e^{4x+4y}$. Find $\partial^4 f / \partial x^3 \partial y$ at $(0, 0)$.

- (A) $\frac{1}{2} (4 \cdot 4^3)$
- (B) $-1 (4 \cdot 4^3)$
- (C) $4 \cdot 4^3$
- (D) $\frac{1}{4} (4 \cdot 4^3)$
- (E) $2 (4 \cdot 4^3)$

66. Let $G = \mathbb{Z}/114\mathbb{Z} \times \mathbb{Z}/80\mathbb{Z}$. Find the order of G/H , where $H = \{g \in G : 2g = 0\}$.

- (A) 2278
- (B) 2282
- (C) 2279
- (D) 2281
- (E) 2280

Answer Key

Practice Test 18 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	A	23	E	45	D
2	E	24	E	46	C
3	C	25	B	47	C
4	D	26	A	48	C
5	D	27	A	49	C
6	D	28	E	50	D
7	B	29	A	51	D
8	B	30	E	52	E
9	A	31	A	53	A
10	D	32	E	54	A
11	B	33	A	55	B
12	B	34	D	56	D
13	C	35	B	57	A
14	C	36	A	58	B
15	A	37	C	59	E
16	B	38	B	60	E
17	C	39	B	61	A
18	E	40	D	62	E
19	B	41	A	63	D
20	B	42	C	64	E
21	C	43	D	65	C
22	D	44	C	66	E

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