

GRE Mathematics Practice Test 19

Bridge to Standard

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A bridge toward standard practice, with routine multi-step questions and a smaller number of challenging concepts. This is cycle 3 of 19, position 4 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

zarimion.com/gre

100 practice tests, complete answer keys, and future updates.

Please retain the source attribution when sharing this material.



This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Find the coefficient of x^3 in $e^{2x} \log(1 + 4x)$.

- (A) $\frac{40}{3}$
- (B) $\frac{64}{3}$
- (C) $\frac{52}{3}$
- (D) $-\frac{40}{3}$
- (E) 80

2. For real t , let $M(t) = \begin{pmatrix} t-15 & 5 & 0 \\ 0 & t-15 & 1 \\ 0 & 0 & t-15 \end{pmatrix}$. At $t = 15$, what is $\text{rank } M(t)$?

- (A) 4
- (B) 1
- (C) 3
- (D) 2
- (E) 0

3. Evaluate $\int_0^1 x^3(\log x)^2 dx$.

- (A) $-\frac{(-1)^2 2!}{4^3}$
- (B) $8 \left(\frac{(-1)^2 2!}{4^3} \right)$
- (C) $\frac{1}{3} \left(\frac{(-1)^2 2!}{4^3} \right)$
- (D) $4 \left(\frac{(-1)^2 2!}{4^3} \right)$
- (E) $\frac{(-1)^2 2!}{4^3}$

Questions 4–6

4. A coin success probability p has uniform prior on $[0, 1]$. After observing 6 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?

- (A) $\frac{3}{10}$
- (B) $\frac{17}{20}$
- (C) $\frac{7}{30}$
- (D) $\frac{7}{10}$
- (E) $\frac{7}{20}$

5. Let $\mathbf{F} = (4y^3, z^3, x^3)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (0, 0, 1)$.

- (A) -11
- (B) -6
- (C) -24
- (D) -12
- (E) 12

6. Let $u = (3, 3)^T$ and $v = (1, 0)^T$. Find the squared norm of the orthogonal projection of v onto $\text{span}(u)$.

- (A) 1
- (B) $\frac{3}{2}$
- (C) 2
- (D) $\frac{1}{2}$
- (E) 0

Questions 7–9

7. Evaluate $\lim_{x \rightarrow 0} (1 + 4x)^{1/(2x)}$.

- (A) e^2
- (B) 4
- (C) e^5
- (D) $e^{\frac{1}{2}}$
- (E) e^{16}

8. A random variable has mean 0 and variance 6. What upper bound does the Chebyshev inequality give for $P(|X| \geq 12)$?

- (A) $\frac{1}{24}$
- (B) $\frac{1}{72}$
- (C) $\frac{23}{24}$
- (D) $\frac{25}{48}$
- (E) $\frac{1}{48}$

9. Evaluate $\int_0^{\log 2} e^x (e^x + 4)^3 dx$.

- (A) $-\frac{6^4 - 5^4}{4}$
- (B) $6^3 - 5^3$
- (C) $\frac{6^4}{4}$
- (D) $4 \left(\frac{6^4 - 5^4}{4} \right)$
- (E) $\frac{6^4 - 5^4}{4}$

Questions 10–12

10. An idempotent 21×21 real matrix P has rank 2. What is $\text{tr } P$?

- (A) 1
- (B) 3
- (C) 0
- (D) 4
- (E) 2

11. Evaluate $\int_0^1 x^4(1-x)^2 dx$.

- (A) $\frac{1}{3} \left(\frac{4! 2!}{7!} \right)$
- (B) $\frac{4! 2!}{7!}$
- (C) $7 \left(\frac{4! 2!}{7!} \right)$
- (D) $5 \left(\frac{4! 2!}{7!} \right)$
- (E) $2 \left(\frac{4! 2!}{7!} \right)$

12. A box is chosen uniformly from two boxes. Box A contains 11 red and 13 blue balls; box B contains 13 red and 11 blue balls. Two balls are drawn without replacement and both are red. What is the conditional probability that A was chosen?

- (A) $\frac{55}{266}$
- (B) $\frac{94}{133}$
- (C) $\frac{78}{133}$
- (D) $\frac{55}{133}$
- (E) $\frac{55}{399}$

Questions 13–15

13. Evaluate $\iiint_B 4(x^2 + y^2 + z^2)^{3/2} dV$, where B is the unit ball.

- (A) $\frac{8\pi}{3}$
- (B) $\frac{16\pi}{3}$
- (C) $-\frac{8\pi}{3}$
- (D) $1 + \frac{8\pi}{3}$
- (E) $\frac{4\pi}{3}$

14. For $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $b = \begin{bmatrix} 15 \\ 5 \\ 20 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find x_2 .

- (A) 4
- (B) 3
- (C) 6
- (D) 5
- (E) 7

15. Solve $y'' - 11y' + 28y = 0$ with $y(0) = 1$, $y'(0) = 0$. Find $y(1)$.

- (A) $\frac{(-7 + 4e^3)e^4}{3}$
- (B) $\frac{(7 - 4e^3)e^4}{3} + 1$
- (C) $\frac{(7 - 4e^3)e^4}{3}$
- (D) $\frac{(7 - 4e^3)e^4}{6}$
- (E) $\frac{2(7 - 4e^3)e^4}{3}$

Questions 16–18

16. Let $X_1, \dots, X_{11}$ be independent uniform random variables on $[0, 1]$. What is $E[\max_i X_i]$?

- (A) $\frac{11}{12}$
- (B) $\frac{1}{12}$
- (C) $\frac{11}{24}$
- (D) $\frac{23}{24}$
- (E) $\frac{11}{36}$

17. Under $x = u + 4v$, $y = u - 3v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) $\frac{1}{7}$
- (B) 7
- (C) 49
- (D) 12
- (E) 1

18. What is the coefficient of x^3 in $(x + 3)^5$?

- (A) 92
- (B) 89
- (C) 88
- (D) 90
- (E) 91

Questions 19–21

19. Two solutions of $y'' + 4x^1y' + 3y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) e^{-2}
- (B) $\frac{2}{e^2}$
- (C) $-\frac{1}{e^2}$
- (D) $e^{-2} + 1$
- (E) $\frac{1}{2e^2}$

20. Independent trials have success probability $1/11$. Let T count trials up to and including the first success. Find $E[T]$.

- (A) 11
- (B) 5
- (C) 14
- (D) 8
- (E) 17

21. If $y'' + 9y = 0$, $y(0) = 4$, and $y'(0) = 5$, find $y(\pi/6)$.

- (A) $\frac{10}{3}$
- (B) $\frac{5}{3}$
- (C) $\frac{5}{6}$
- (D) $-\frac{5}{3}$
- (E) $\frac{8}{3}$

Questions 22–24

22. A matrix A satisfies $(A - 16I)(A - 18I) = 0$, and its minimal polynomial has degree two. If $A^2 = uA + vI$, find u .

- (A) 35
- (B) 34
- (C) 36
- (D) 32
- (E) 33

23. Evaluate $\lim_{x \rightarrow 0} \frac{e^{4x} - \sum_{j=0}^2 (4x)^j / j!}{x^3}$.

- (A) $\frac{1}{4} \left(\frac{4^3}{3!} \right)$
- (B) $3 \left(\frac{4^3}{3!} \right)$
- (C) $\frac{4^3}{3!}$
- (D) $-\frac{4^3}{3!}$
- (E) $6 \left(\frac{4^3}{3!} \right)$

24. A continuous $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(q) = q^6$ for every rational q . Find $f(2)$.

- (A) 2^7
- (B) 2^9
- (C) 2^6
- (D) 2^5
- (E) 2^8

Questions 25–27

25. Evaluate $\int_C (3x^2 + 4y) dx + (4x + 3y^2) dy$, where C is the line segment from $(0, 0)$ to $(5, 1)$.

- (A) -146
- (B) 73
- (C) 147
- (D) 146
- (E) 292

26. Let $V = \begin{bmatrix} 1 & 15 & 225 \\ 1 & 17 & 289 \\ 1 & 20 & 400 \end{bmatrix}$. Find $\det V$.

- (A) 29
- (B) 30
- (C) 32
- (D) 31
- (E) 28

27. Find the upward flux of $\mathbf{F} = (0, 0, z)$ through $z = 4x^2 + 3y^2$, $0 \leq x, y \leq 1$.

- (A) $\frac{10}{3}$
- (B) $-\frac{7}{3}$
- (C) $\frac{14}{3}$
- (D) $\frac{7}{6}$
- (E) $\frac{7}{3}$

Questions 28–30

28. Find the radius of convergence of $\sum_{k=1}^{\infty} k^6 x^k / 6^k$.

- (A) 6
- (B) 10
- (C) 2
- (D) 8
- (E) 4

29. Find the coefficient of x^6 in $\cos(4x)/(1 - 2x^2)$.

- (A) -6016
- (B) $-\frac{256}{45}$
- (C) $-\frac{421}{45}$
- (D) $-\frac{376}{45}$
- (E) $\frac{376}{45}$

30. Let $u = (1, 2, \dots, 21)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $\det B$.

- (A) 230
- (B) 234
- (C) 232
- (D) 233
- (E) 231

Questions 31–33

31. The region $0 \leq x \leq 1$, $0 \leq y \leq 4x^3$ is revolved about the x -axis. Find its volume.

- (A) $1 + \frac{16\pi}{7}$
- (B) $\frac{16\pi}{7}$
- (C) $\frac{8\pi}{7}$
- (D) $-\frac{16\pi}{7}$
- (E) $\frac{32\pi}{7}$

32. On $[0, 1]$, f is 6 on rationals and $1/6$ on irrationals. Find its upper Darboux integral minus its lower Darboux integral.

- (A) $\frac{47}{6}$
- (B) $\frac{41}{6}$
- (C) $\frac{35}{6}$
- (D) $\frac{12}{35}$
- (E) $\frac{3}{35}$

33. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 4z^2 = x + 3y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) -12
- (B) -8
- (C) 24
- (D) -3
- (E) -24

Questions 34–36

34. Let D be a diagonal 21×21 real matrix with distinct diagonal entries. What is the dimension of the space of matrices X satisfying $XD = DX$?

- (A) 21
- (B) 20
- (C) 23
- (D) 19
- (E) 22

35. Evaluate $\int_0^{\pi/2} \sin^7 t \cos^9 t \, dt$.

- (A) $\frac{1}{2} \left(\frac{3! 4!}{2 \cdot 8!} \right)$
- (B) $2 \left(\frac{3! 4!}{2 \cdot 8!} \right)$
- (C) $35 \left(\frac{3! 4!}{2 \cdot 8!} \right)$
- (D) $8 \left(\frac{3! 4!}{2 \cdot 8!} \right)$
- (E) $\frac{3! 4!}{2 \cdot 8!}$

36. A set with 6 elements has the discrete topology. How many of its subsets are simultaneously open and closed?

- (A) 2^6
- (B) 2^5
- (C) 2^8
- (D) 2^9
- (E) 2^7

Questions 37–39

37. The solution of $y' = 4x^3y^2$ has $y(0) = 1/5$. Find $y(1)$.

- (A) $\frac{5}{4}$
- (B) $-\frac{1}{4}$
- (C) $\frac{1}{8}$
- (D) $\frac{1}{2}$
- (E) $\frac{1}{4}$

38. What is the remainder when $x^2 + 1$ is divided by $x - 16$?

- (A) 258
- (B) 256
- (C) 257
- (D) 259
- (E) 255

39. For $D = \{(x, y) : x^2 + y^2 \leq 4\}$, evaluate $\iint_D (x^2 + y^2)^2 dA$.

- (A) 64π
- (B) $\frac{64\pi}{5}$
- (C) $\frac{16\pi}{3}$
- (D) $\frac{64\pi}{3}$
- (E) $\frac{128\pi}{3}$

Questions 40–42

40. Let $\zeta = e^{2\pi i/8}$. Find $\sum_{k=0}^7 \zeta^k$.

- (A) 3
- (B) 1
- (C) 2
- (D) 0
- (E) 4

41. Evaluate $\int_0^\infty x^3 e^{-4x} dx$.

- (A) $4 \left(\frac{3!}{4^4} \right)$
- (B) $\frac{1}{4} \left(\frac{3!}{4^4} \right)$
- (C) $256 \left(\frac{3!}{4^4} \right)$
- (D) $\frac{1}{3} \left(\frac{3!}{4^4} \right)$
- (E) $\frac{3!}{4^4}$

42. Let $P = \text{diag}(\underbrace{1, \dots, 1}_2, \underbrace{0, \dots, 0}_{20})$. What is the dimension of the space of 22×22 real matrices X satisfying $PX = X$?

- (A) 45
- (B) 42
- (C) 44
- (D) 46
- (E) 43

Questions 43–45

43. Let $f(x) = \log(1 + 4x)$. Find $f^{(3)}(0)$.

- (A) $-(-1)^2 2! 4^3$
- (B) $\frac{1}{3} \left((-1)^2 2! 4^3 \right)$
- (C) $(-1)^2 2! 4^3$
- (D) $\frac{1}{4} \left((-1)^2 2! 4^3 \right)$
- (E) $3 \left((-1)^2 2! 4^3 \right)$

44. An analytic function has real part $u(x, y) = x^2 - y^2 + 6x$ and satisfies $f(0) = 0$. Find $\text{Im } f(1 + i)$.

- (A) 12
- (B) 8
- (C) 10
- (D) 4
- (E) 6

45. Let $f(x, y) = x^2 + 8xy + (19)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 76
- (B) 12
- (C) 3
- (D) 19
- (E) 6

Questions 46–48

46. How many group homomorphisms $\mathbb{Z}/120\mathbb{Z} \rightarrow \mathbb{Z}/84\mathbb{Z}$ are there?

- (A) 10
- (B) 12
- (C) 14
- (D) 11
- (E) 13

47. Let $f(x, y) = e^{4x+3y}$. Find $\partial^3 f / \partial x^2 \partial y$ at $(0, 0)$.

- (A) $\frac{4}{3} (3 \cdot 4^2)$
- (B) $\frac{9}{16} (3 \cdot 4^2)$
- (C) $3 \cdot 4^2$
- (D) $\frac{3}{4} (3 \cdot 4^2)$
- (E) $\frac{1}{3} (3 \cdot 4^2)$

48. What is the coefficient of x^2 in $(1 + 2x)^6$?

- (A) 40
- (B) 20
- (C) 80
- (D) 60
- (E) 100

Questions 49–51

49. What is the maximum of x^3y^4 over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\left(\frac{3}{7}\right)^4 \left(\frac{4}{7}\right)^3$
- (B) $\frac{1}{2^7}$
- (C) $\left(\frac{3}{7}\right)^3 \left(\frac{4}{7}\right)^4$
- (D) $\frac{16807}{576} \left(\left(\frac{3}{7}\right)^3 \left(\frac{4}{7}\right)^4\right)$
- (E) $7 \left(\left(\frac{3}{7}\right)^3 \left(\frac{4}{7}\right)^4\right)$

50. How many 1-dimensional subspaces does $\mathbb{F}_5^4$ have?

- (A) $\frac{5^4 - 1}{5 - 1}$
- (B) $\frac{5^3 - 1}{5 - 1}$
- (C) $\frac{5^7 - 1}{5 - 1}$
- (D) $\frac{5^6 - 1}{5 - 1}$
- (E) $\frac{5^5 - 1}{5 - 1}$

51. For the curve $x = t^3$, $y = t^4 + 4t$, find d^2y/dx^2 at $t = 1$.

- (A) $\frac{8}{3}$
- (B) $-\frac{8}{9}$
- (C) $-\frac{4}{9}$
- (D) $\frac{4}{3}$
- (E) $-\frac{4}{3}$

Questions 52–54

52. How many edges does the complete bipartite graph $K_{6,8}$ have?

- (A) 16
- (B) 80
- (C) 48
- (D) 64
- (E) 32

53. Evaluate $\iiint_{x^2+y^2+z^2 \leq 4} (x^2 + y^2 + z^2)^2 dV$.

- (A) $\frac{7}{20} \left(\frac{4\pi 4^{\frac{7}{2}}}{7} \right)$
- (B) $\frac{1}{3} \left(\frac{4\pi 4^{\frac{7}{2}}}{7} \right)$
- (C) $\frac{1}{2} \left(\frac{4\pi 4^{\frac{7}{2}}}{7} \right)$
- (D) $7 \left(\frac{4\pi 4^{\frac{7}{2}}}{7} \right)$
- (E) $\frac{4\pi 4^{\frac{7}{2}}}{7}$

54. How many monic irreducible polynomials of degree 2 are there over $\mathbb{F}_5$?

- (A) 12
- (B) 9
- (C) 11
- (D) 8
- (E) 10

Questions 55–57

55. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-4)^j}{j^{\frac{2}{3}} (\log j)^m}$ converge absolutely at $x = 3$? Give the least such m , or 0 if none exists.

- (A) 1
- (B) 3
- (C) 0
- (D) 2
- (E) 4

56. How many functions from a set of 11 elements onto a set of 3 elements exist?

- (A) 3^{11}
- (B) $3^{11} - 2^{11}$
- (C) $3^{11} - 3(2^{11})$
- (D) $3^{11} - 3(2^{11}) - 3$
- (E) $3^{11} - 3(2^{11}) + 3$

57. The function $y(x)$ satisfies $y^3 + 4y = 2x$ and $y(0) = 0$. Find $y'(0)$.

- (A) 0
- (B) $-\frac{1}{2}$
- (C) 4
- (D) $\frac{1}{2}$
- (E) $\frac{2}{7}$

Questions 58–60

58. How many elements z of $\mathbb{F}_{3^4}$ satisfy $z^3 = z$?

- (A) 4
- (B) 2
- (C) 3
- (D) 1
- (E) 5

59. For $x > 0$, $x^2 y'' + (1 - 7)xy' + 12y = 0$, $y(1) = 0$, $y'(1) = 4$. Find $y(e)$.

- (A) $\frac{e^4}{7}$
- (B) $e^3 + e^4$
- (C) $4(1 - e)e^3$
- (D) $-e^3 + e^4$
- (E) $-4(1 - e)e^3$

60. How many nonnegative integer triples (a, b, c) satisfy $a + b + c = 11$?

- (A) 130
- (B) 52
- (C) 104
- (D) 26
- (E) 78

Questions 61–63

61. For $y' = 4y(1 - y/3)$, what is the derivative of the right-hand side with respect to y at the positive equilibrium?

- (A) -4
- (B) -2
- (C) 4
- (D) -3
- (E) -8

62. How many generators does the cyclic group of order 5 have?

- (A) 2
- (B) 4
- (C) 6
- (D) 5
- (E) 3

63. For $x > 0$, $xy' = 4y$ and $y(1) = 1$. Find $y(e^3)$.

- (A) $2e^{12}$
- (B) e^{12}
- (C) $1 + e^{12}$
- (D) $-e^{12}$
- (E) $\frac{e^{12}}{2}$

Questions 64–66

64. A tree has 15 vertices. What is the sum of its vertex degrees?

- (A) 37
- (B) 28
- (C) 46
- (D) 10
- (E) 19

65. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(4j)!}{(j!)^4} x^{3j}$?

- (A) 4^{-4}
- (B) $4^{-\frac{4}{3}}$
- (C) $4^{\frac{4}{3}}$
- (D) $4^{-\frac{3}{4}}$
- (E) $\frac{1}{4}$

66. Let $R = (\mathbb{Z}/16\mathbb{Z})/([8])$, where $([8])$ is the ideal generated by $[8]$. How many elements does R have?

- (A) 8
- (B) 7
- (C) 9
- (D) 10
- (E) 6

Answer Key

Practice Test 19 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	A	23	C	45	B
2	D	24	C	46	B
3	E	25	D	47	C
4	D	26	B	48	D
5	D	27	E	49	C
6	D	28	A	50	A
7	A	29	D	51	C
8	A	30	C	52	C
9	E	31	B	53	E
10	E	32	E	54	E
11	B	33	E	55	C
12	D	34	A	56	E
13	A	35	E	57	D
14	D	36	A	58	C
15	C	37	E	59	E
16	A	38	C	60	E
17	B	39	D	61	A
18	D	40	D	62	B
19	A	41	E	63	B
20	A	42	C	64	B
21	B	43	C	65	B
22	B	44	B	66	A

zarimion.com/gre

100 practice tests, complete answer keys, and future updates.
Please retain the source attribution when sharing this material.

