

GRE Mathematics Practice Test 20

Standard-Level Target

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A mixed standard-level target, intended to approximate public GRE Mathematics practice in scope and overall demand. This is cycle 3 of 19, position 5 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

zarimion.com/gre

100 practice tests, complete answer keys, and future updates.

Please retain the source attribution when sharing this material.



This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Let C be the unit circle $x^2 + y^2 = 1$, $z = 2$, counterclockwise viewed from above. Evaluate $\int_C -4y(x^2 + y^2) dx + (4x(x^2 + y^2)) dy$.

- (A) 16π
- (B) 8π
- (C) 4π
- (D) $1 + 8\pi$
- (E) -8π

2. A matrix A satisfies $(A - 17I)(A - 19I) = 0$, and its minimal polynomial has degree two. If $A^2 = uA + vI$, find u .

- (A) 37
- (B) 35
- (C) 38
- (D) 34
- (E) 36

3. Evaluate $\int_0^\infty x^2 e^{-4x} dx$.

- (A) $\frac{2!}{4^3}$
- (B) $4 \left(\frac{2!}{4^3} \right)$
- (C) $64 \left(\frac{2!}{4^3} \right)$
- (D) $\frac{1}{2} \left(\frac{2!}{4^3} \right)$
- (E) $\frac{1}{4} \left(\frac{2!}{4^3} \right)$

Questions 4–6

4. A coin success probability p has uniform prior on $[0, 1]$. After observing 7 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?

- (A) $\frac{3}{11}$
- (B) $\frac{19}{22}$
- (C) $\frac{8}{33}$
- (D) $\frac{8}{11}$
- (E) $\frac{4}{11}$

5. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(4j)!}{(j!)^4} x^{2j}$?

- (A) $4(4^{-2})$
- (B) 4^{-4}
- (C) $4^{-\frac{1}{2}}$
- (D) 4^2
- (E) 4^{-2}

6. Let $V = \begin{bmatrix} 1 & 16 & 256 \\ 1 & 18 & 324 \\ 1 & 21 & 441 \end{bmatrix}$. Find $\det V$.

- (A) 29
- (B) 31
- (C) 28
- (D) 32
- (E) 30

Questions 7–9

7. For $x > 0$, $xy' = 4y$ and $y(1) = 1$. Find $y(e^2)$.

- (A) $-e^8$
- (B) e^8
- (C) $\frac{e^8}{2}$
- (D) $2e^8$
- (E) $1 + e^8$

8. A random variable has mean 0 and variance 7. What upper bound does the Chebyshev inequality give for $P(|X| \geq 14)$?

- (A) $\frac{29}{56}$
- (B) $\frac{1}{28}$
- (C) $\frac{1}{56}$
- (D) $\frac{27}{28}$
- (E) $\frac{1}{84}$

9. Evaluate $\int_0^{\pi/2} \sin^5 t \cos^9 t \, dt$.

- (A) $15 \left(\frac{2! 4!}{2 \cdot 7!} \right)$
- (B) $\frac{2! 4!}{2 \cdot 7!}$
- (C) $\frac{1}{2} \left(\frac{2! 4!}{2 \cdot 7!} \right)$
- (D) $7 \left(\frac{2! 4!}{2 \cdot 7!} \right)$
- (E) $2 \left(\frac{2! 4!}{2 \cdot 7!} \right)$

Questions 10–12

10. Let $u = (1, 2, \dots, 22)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $\det B$.

- (A) 255
- (B) 254
- (C) 253
- (D) 252
- (E) 256

11. For $x > 0$, $x^2y'' + (1 - 6)xy' + 8y = 0$, $y(1) = 0$, $y'(1) = 4$. Find $y(e)$.

- (A) $-2(1 - e^2)e^2$
- (B) $2(1 - e^2)e^2$
- (C) $\frac{e^4}{6}$
- (D) $-e^2 + e^4$
- (E) $e^2 + e^4$

12. A box is chosen uniformly from two boxes. Box A contains 12 red and 14 blue balls; box B contains 14 red and 12 blue balls. One ball is drawn and is red. What is the conditional probability that A was chosen?

- (A) $\frac{7}{13}$
- (B) $\frac{2}{13}$
- (C) $\frac{3}{13}$
- (D) $\frac{19}{26}$
- (E) $\frac{6}{13}$

Questions 13–15

13. What is the maximum of x^2y^4 over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\left(\frac{2}{6}\right)^2 \left(\frac{4}{6}\right)^4$
- (B) $6 \left(\left(\frac{2}{6}\right)^2 \left(\frac{4}{6}\right)^4 \right)$
- (C) $\left(\frac{2}{6}\right)^4 \left(\frac{4}{6}\right)^2$
- (D) $\frac{1}{26}$
- (E) $\frac{81}{8} \left(\left(\frac{2}{6}\right)^2 \left(\frac{4}{6}\right)^4 \right)$

14. Let D be a diagonal 22×22 real matrix with distinct diagonal entries. What is the dimension of the space of matrices X satisfying $XD = DX$?

- (A) 20
- (B) 24
- (C) 21
- (D) 22
- (E) 23

15. Find the coefficient of x^2 in the Taylor series of $(1 - 4x)^{-2}$ at zero.

- (A) $2(3 \cdot 4^2)$
- (B) $3 \cdot 4^2$
- (C) $\frac{2}{3}(3 \cdot 4^2)$
- (D) $\frac{1}{4}(3 \cdot 4^2)$
- (E) $\frac{1}{3}(3 \cdot 4^2)$

Questions 16–18

16. Let $X_1, \dots, X_{12}$ be independent uniform random variables on $[0, 1]$. What is $E[\max_i X_i]$?
- (A) $\frac{4}{13}$
 - (B) $\frac{25}{26}$
 - (C) $\frac{1}{13}$
 - (D) $\frac{6}{13}$
 - (E) $\frac{12}{13}$
17. For $y' = 4y(1 - y/2)$, what is the derivative of the right-hand side with respect to y at the positive equilibrium?
- (A) 4
 - (B) -8
 - (C) -2
 - (D) -4
 - (E) -3
18. What is the remainder when $x^2 + 1$ is divided by $x - 17$?
- (A) 290
 - (B) 291
 - (C) 289
 - (D) 288
 - (E) 292

Questions 19–21

19. Solve $y'' - 10y' + 24y = 0$ with $y(0) = 1$, $y'(0) = 0$. Find $y(1)$.

- (A) $(3 - 2e^2)e^4$
- (B) $(3 - 2e^2)e^4 + 1$
- (C) $\left(\frac{3}{2} - e^2\right)e^4$
- (D) $(6 - 4e^2)e^4$
- (E) $(-3 + 2e^2)e^4$

20. Independent trials have success probability $1/12$. Let T count trials up to and including the first success. Find $E[T]$.

- (A) 12
- (B) 8
- (C) 16
- (D) 4
- (E) 20

21. Evaluate $\iiint_{x^2+y^2+z^2 \leq 4} (x^2 + y^2 + z^2)^1 dV$.

- (A) $\frac{1}{3} \left(\frac{4\pi 4^{\frac{5}{2}}}{5} \right)$
- (B) $5 \left(\frac{4\pi 4^{\frac{5}{2}}}{5} \right)$
- (C) $\frac{5}{12} \left(\frac{4\pi 4^{\frac{5}{2}}}{5} \right)$
- (D) $\frac{1}{2} \left(\frac{4\pi 4^{\frac{5}{2}}}{5} \right)$
- (E) $\frac{4\pi 4^{\frac{5}{2}}}{5}$

Questions 22–24

22. Let $P = \text{diag}(\underbrace{1, \dots, 1}_3, \underbrace{0, \dots, 0}_{20})$. What is the dimension of the space of 23×23 real matrices X satisfying $PX = X$?

- (A) 71
- (B) 70
- (C) 69
- (D) 67
- (E) 68

23. Evaluate $\lim_{x \rightarrow 0} \frac{e^{4x} - \sum_{j=0}^1 (4x)^j / j!}{x^2}$.

- (A) $2 \left(\frac{4^2}{2!} \right)$
- (B) $\frac{1}{4} \left(\frac{4^2}{2!} \right)$
- (C) $\frac{1}{2} \left(\frac{4^2}{2!} \right)$
- (D) $\frac{4^2}{2!}$
- (E) $-\frac{4^2}{2!}$

24. A continuous $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(q) = q^7$ for every rational q . Find $f(2)$.

- (A) 2^9
- (B) 2^{10}
- (C) 2^7
- (D) 2^8
- (E) 2^6

Questions 25–27

25. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 4z^2 = x + 2y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.
- (A) 16
(B) -8
(C) -16
(D) -32
(E) -2
26. A 3×3 matrix A has characteristic polynomial $(t - 16)(t + 1)(t - 2)$. What is $\text{tr}(A^2)$?
- (A) 263
(B) 259
(C) 261
(D) 262
(E) 260
27. Let $f(x, y) = e^{4x+2y}$. Find $\partial^2 f / \partial x^1 \partial y$ at $(0, 0)$.
- (A) $2 \cdot 4^1$
(B) $-1(2 \cdot 4^1)$
(C) $\frac{1}{2}(2 \cdot 4^1)$
(D) $2(2 \cdot 4^1)$
(E) $\frac{9}{8}(2 \cdot 4^1)$

Questions 28–30

28. Find the radius of convergence of $\sum_{k=1}^{\infty} k^7 x^k / 7^k$.

- (A) 3
- (B) 5
- (C) 7
- (D) 11
- (E) 9

29. Let C be $x^2/4 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-y) dx + (4x) dy$.

- (A) $1 + 10\pi$
- (B) -10π
- (C) 10π
- (D) 5π
- (E) 20π

30. Let N be the 22×22 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\text{rank } N^1$.

- (A) 23
- (B) 21
- (C) 19
- (D) 20
- (E) 22

Questions 31–33

31. Evaluate $\int_C (2x + 4y) dx + (4x + 2y) dy$, where C is the line segment from $(0, 0)$ to $(6, 1)$.

- (A) 122
- (B) -61
- (C) 61
- (D) 62
- (E) $\frac{61}{2}$

32. On $[0, 1]$, f is 7 on rationals and $1/7$ on irrationals. Find its upper Darboux integral minus its lower Darboux integral.

- (A) $\frac{48}{7}$
- (B) $\frac{62}{7}$
- (C) $\frac{24}{7}$
- (D) $\frac{55}{7}$
- (E) $\frac{96}{7}$

33. Evaluate $\lim_{x \rightarrow 0} (1 + 4x)^{1/(1x)}$.

- (A) $e^{\frac{1}{4}}$
- (B) e^5
- (C) e^{16}
- (D) 4
- (E) e^4

Questions 34–36

34. Let A be the 22×22 matrix whose diagonal entries are 19 and whose off-diagonal entries are 1. What is its smallest eigenvalue?

- (A) 19
- (B) 17
- (C) 20
- (D) 16
- (E) 18

35. The region $0 \leq x \leq 1$, $0 \leq y \leq 4x^2$ is revolved about the x -axis. Find its volume.

- (A) $\frac{32\pi}{5}$
- (B) $1 + \frac{16\pi}{5}$
- (C) $\frac{8\pi}{5}$
- (D) $-\frac{16\pi}{5}$
- (E) $\frac{16\pi}{5}$

36. A set with 7 elements has the discrete topology. How many of its subsets are simultaneously open and closed?

- (A) 2^6
- (B) 2^9
- (C) 2^{10}
- (D) 2^7
- (E) 2^8

Questions 37–39

37. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(4+2j)(6+2j)}$.

- (A) $\frac{1}{6}$
- (B) $\frac{1}{8}$
- (C) $\frac{1}{4}$
- (D) $\frac{1}{2}$
- (E) $\frac{1}{16}$

38. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 48$ and $x + y = 19$. Find xy .

- (A) 47
- (B) 46
- (C) 50
- (D) 48
- (E) 49

39. Find the coefficient of x^4 in $\cos(4x)/(1 - 1x^2)$.

- (A) $\frac{11}{3}$
- (B) $-\frac{11}{3}$
- (C) $\frac{32}{3}$
- (D) 88
- (E) $\frac{8}{3}$

Questions 40–42

40. Let $\zeta = e^{2\pi i/9}$. Find $\sum_{k=0}^8 \zeta^k$.

- (A) 3
- (B) 2
- (C) 1
- (D) 4
- (E) 0

41. If $y' + 4y = 2e^{-4x}$ and $y(0) = 6$, find $y(1)$.

- (A) $\frac{8}{e^4}$
- (B) $\frac{8}{e^4} + 1$
- (C) $-\frac{8}{e^4}$
- (D) $\frac{16}{e^4}$
- (E) $\frac{4}{e^4}$

42. A sequence satisfies $u_0 = 16$ and $u_{k+1} = 1u_k + 3$ for all $k \geq 0$. Find u_5 .

- (A) 33
- (B) 29
- (C) 30
- (D) 31
- (E) 32

Questions 43–45

43. Find the arc length of $r(t) = (4t, 2t)$ for $0 \leq t \leq 1$.

- (A) $-2\sqrt{5}$
- (B) $2\sqrt{5}$
- (C) $1 + 2\sqrt{5}$
- (D) $\sqrt{5}$
- (E) $4\sqrt{5}$

44. An analytic function has real part $u(x, y) = x^2 - y^2 + 7x$ and satisfies $f(0) = 0$. Find $\text{Im } f(1 + i)$.

- (A) 6
- (B) 12
- (C) 3
- (D) 15
- (E) 9

45. For $f(x, y) = x^2 + 4xy + 2y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) 1
- (B) -2
- (C) 4
- (D) 0
- (E) 2

Questions 46–48

46. A linear map $T : \mathbb{R}^{23} \rightarrow \mathbb{R}^{25}$ has rank 4. What is its nullity?

- (A) 18
- (B) 17
- (C) 21
- (D) 19
- (E) 20

47. For $D = \{(x, y) : x^2 + y^2 \leq 4\}$, evaluate $\iint_D (x^2 + y^2)^1 dA$.

- (A) $\frac{16\pi}{3}$
- (B) 8π
- (C) 16π
- (D) 4π
- (E) 2π

48. What is the coefficient of x^2 in $(1 + 2x)^7$?

- (A) 84
- (B) 56
- (C) 112
- (D) 28
- (E) 140

Questions 49–51

49. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^4 y^1 dV$.

- (A) $8 \left(\frac{4! 1!}{8!} \right)$
- (B) $\frac{4! 1!}{8!}$
- (C) $\frac{1}{6} \left(\frac{4! 1!}{8!} \right)$
- (D) $6 \left(\frac{4! 1!}{8!} \right)$
- (E) $\frac{1}{10}$

50. In the ring $\mathbb{Z}/120\mathbb{Z}$, how many elements are units?

- (A) 33
- (B) 34
- (C) 32
- (D) 30
- (E) 31

51. Let $f(x, y) = x^2 + 8xy + (18)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 18
- (B) 4
- (C) 2
- (D) 8
- (E) 72

Questions 52–54

52. How many edges does the complete bipartite graph $K_{7,9}$ have?

- (A) 84
- (B) 42
- (C) 21
- (D) 63
- (E) 105

53. Find the directional derivative of $f(x, y, z) = x^2 + 4y^2 + 2z^2$ at $(1, 1, 1)$ in the direction $(1, 6, 0)$.

- (A) $\frac{25\sqrt{37}}{37}$
- (B) $-\frac{50\sqrt{37}}{37}$
- (C) $1 + \frac{50\sqrt{37}}{37}$
- (D) $\frac{50\sqrt{37}}{37}$
- (E) $\frac{100\sqrt{37}}{37}$

54. A permutation consists of disjoint cycles of lengths 3 and 9 and fixes all other points. What is its order?

- (A) 8
- (B) 11
- (C) 7
- (D) 9
- (E) 10

Questions 55–57

55. Find the curvature at $t = 0$ of $r(t) = (2t, 4t^2)$.

- (A) -2
- (B) 1
- (C) 2
- (D) 3
- (E) 4

56. How many functions from a set of 12 elements onto a set of 2 elements exist?

- (A) 2^{11}
- (B) $2^{12} - 3$
- (C) $2^{12} - 2$
- (D) 2^{12}
- (E) $2^{12} - 1$

57. For the curve $x = t^2$, $y = t^3 + 4t$, find d^2y/dx^2 at $t = 1$.

- (A) $\frac{7}{4}$
- (B) -2
- (C) $-\frac{1}{2}$
- (D) $\frac{7}{2}$
- (E) $-\frac{1}{4}$

Questions 58–60

58. In $R = \mathbb{F}_2[x]/(x^4)$, how many elements are there?

- (A) 2^6
- (B) 2^5
- (C) 2^4
- (D) 2^7
- (E) 2^3

59. The function $y(x)$ satisfies $y^3 + 4y = 1x$ and $y(0) = 0$. Find $y'(0)$.

- (A) $-\frac{1}{4}$
- (B) $\frac{1}{4}$
- (C) 0
- (D) $\frac{1}{7}$
- (E) 4

60. How many nonnegative integer triples (a, b, c) satisfy $a + b + c = 12$?

- (A) 91
- (B) 121
- (C) 31
- (D) 151
- (E) 61

Questions 61–63

61. For $f(x) = x^2 e^{4x}$, find $f^{(4)}(0)$.

- (A) $\frac{1}{12} \left(\frac{4! 4^2}{2} \right)$
- (B) $-1 \left(\frac{4! 4^2}{2} \right)$
- (C) $\frac{1}{2} \left(\frac{4! 4^2}{2} \right)$
- (D) $\frac{4! 4^2}{2}$
- (E) $2 \left(\frac{4! 4^2}{2} \right)$

62. Integers x satisfy $x \equiv 0 \pmod{11}$ and $x \equiv 10 \pmod{16}$. Find the least nonnegative solution.

- (A) 156
- (B) 155
- (C) 154
- (D) 153
- (E) 152

63. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 4x$ and has density $\rho(x, y) = 1$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{1}{3}$
- (B) $\frac{4}{3}$
- (C) $\frac{2}{3}$
- (D) $-\frac{2}{3}$
- (E) $\frac{5}{3}$

Questions 64–66

64. A tree has 16 vertices. What is the sum of its vertex degrees?

- (A) 10
- (B) 20
- (C) 30
- (D) 50
- (E) 40

65. Let $F(x) = \int_0^x (x-t)^1 e^{4t} dt$. Find $F^{(2)}(0)$.

- (A) 4
- (B) 1
- (C) $\frac{1}{2}$
- (D) 2
- (E) 0

66. The symmetric group S_{22} acts on $\{1, \dots, 22\}$. What is the index of the stabilizer of 1?

- (A) 22
- (B) 23
- (C) 21
- (D) 24
- (E) 20

Answer Key

Practice Test 20 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	B	23	D	45	E
2	E	24	C	46	D
3	A	25	C	47	B
4	D	26	C	48	A
5	E	27	A	49	B
6	E	28	C	50	C
7	B	29	C	51	D
8	B	30	B	52	D
9	B	31	C	53	D
10	B	32	A	54	D
11	A	33	E	55	C
12	E	34	E	56	C
13	A	35	E	57	E
14	D	36	D	58	C
15	B	37	B	59	B
16	E	38	D	60	A
17	D	39	A	61	D
18	A	40	E	62	C
19	A	41	A	63	C
20	A	42	D	64	C
21	E	43	B	65	B
22	C	44	E	66	A

zarimion.com/gre

100 practice tests, complete answer keys, and future updates.
Please retain the source attribution when sharing this material.

