

GRE Mathematics Practice Test 21

Advanced Challenge

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

The most demanding form in this five-test cycle. Emphasis on connected ideas, non-routine reasoning, and careful conceptual distinctions. This is cycle 4 of 19, position 1 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. The solution of $y' = 5x^6y^3$ has $y(0) = 1/6$. Find $y(1)$.

- (A) $\frac{\sqrt{14}}{22}$
- (B) $-\frac{\sqrt{14}}{22}$
- (C) $\frac{\sqrt{14}}{22} + 1$
- (D) $\frac{\sqrt{14}}{11}$
- (E) $\frac{\sqrt{14}}{44}$

2. On P_{24} , the real polynomials of degree at most 24, define $T(p) = x^2p'' - 4xp' + 6p$. Find its rank.

- (A) 22
- (B) 23
- (C) 21
- (D) 25
- (E) 24

3. Evaluate $\lim_{x \rightarrow 0} \frac{e^{5x} - \sum_{j=0}^5 (5x)^j / j!}{x^6}$.

- (A) $\frac{5^6}{6!}$
- (B) $6 \left(\frac{5^6}{6!} \right)$
- (C) $720 \left(\frac{5^6}{6!} \right)$
- (D) $\frac{1}{5} \left(\frac{5^6}{6!} \right)$
- (E) $-\frac{5^6}{6!}$

Questions 4–6

4. A uniformly random subset of 8 objects is selected from a collection with 9 red and 10 blue objects. If X is the number of red objects selected, find $\text{Var}(X)$.

- (A) $\frac{1162}{361}$
- (B) $\frac{440}{361}$
- (C) $\frac{880}{361}$
- (D) $\frac{220}{361}$
- (E) $\frac{801}{361}$

5. Let $F(x) = \int_0^x (x-t)^5 e^{5t} dt$. Find $F^{(6)}(0)$.

- (A) 0
- (B) $\frac{1}{120}$
- (C) 600
- (D) 720
- (E) 120

6. Subspaces U, V of $\mathbb{R}^{25}$ have dimensions 4 and 5. What is the smallest possible value of $\dim(U \cap V)$?

- (A) 2
- (B) 3
- (C) 0
- (D) 1
- (E) 4

Questions 7–9

7. Under $x = u + 5v$, $y = u - 6v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) 1
- (B) 121
- (C) $\frac{1}{11}$
- (D) 30
- (E) 11

8. X, Y are independent Poisson variables with means 8 and 9. Given $X + Y = 10$, find $\text{Var}(X \mid X + Y)$.

- (A) $\frac{1009}{289}$
- (B) $\frac{1298}{289}$
- (C) $\frac{360}{289}$
- (D) $\frac{1440}{289}$
- (E) $\frac{720}{289}$

9. The region $0 \leq x \leq 1$, $0 \leq y \leq 5x^6$ is revolved about the line $x = -2$. Find its volume.

- (A) $\frac{115\pi}{14}$
- (B) $\frac{115\pi}{28}$
- (C) $-\frac{115\pi}{28}$
- (D) $\frac{115\pi}{56}$
- (E) $1 + \frac{115\pi}{28}$

Questions 10–12

10. Let V consist of real polynomials of degree at most 12 that have both value and first derivative zero at 17, 18, and 20. Find $\dim V$.

- (A) 5
- (B) 8
- (C) 9
- (D) 7
- (E) 6

11. The local inverse $y(x)$ of $x = 5y + y^3$ satisfies $y(0) = 0$. Find $y^{(7)}(0)$.

- (A) $5 \left(-\frac{60480}{5^{10}} \right)$
- (B) $\frac{1}{5040} \left(-\frac{60480}{5^{10}} \right)$
- (C) $-\frac{60480}{5^{10}}$
- (D) $\frac{60480}{5^{10}}$
- (E) $\frac{1}{5} \left(-\frac{60480}{5^{10}} \right)$

12. X, Y are independent, with variances 8 and 9. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 10
- (B) 6
- (C) 2
- (D) 8
- (E) 4

Questions 13–15

13. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(5j)!}{(j!)^5} x^{6j}$?

- (A) $5^{-\frac{5}{6}}$
- (B) $\frac{1}{5}$
- (C) $5^{-\frac{6}{5}}$
- (D) $5^{\frac{5}{6}}$
- (E) 5^{-5}

14. The roots r, s of $x^2 - 23x + 102 = 0$ are counted with multiplicity. Find $r^4 + s^4$.

- (A) $23^4 - 4(102)(23^2) + 102^2$
- (B) $23^4 - 4(102)(23^2)$
- (C) $23^4 + 2(102^2)$
- (D) $23^4 - 4(102)(23^2) + 2(102^2)$
- (E) $23^4 - 2(102)(23^2) + 2(102^2)$

15. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(5+6j)(11+6j)}$.

- (A) $\frac{1}{11}$
- (B) $\frac{1}{5}$
- (C) $\frac{1}{30}$
- (D) $\frac{6}{5}$
- (E) $\frac{1}{6}$

Questions 16–18

16. X has density cx^8 on $0 < x < 1$ and zero elsewhere. Find $E[X^2]$.

- (A) $\frac{9}{11}$
- (B) $\frac{2}{11}$
- (C) $\frac{3}{11}$
- (D) $\frac{9}{22}$
- (E) $\frac{10}{11}$

17. If $y' = y(1 - y/6)$ and $y(0) = 3$, find $y(-\log 5)$.

- (A) -1
- (B) 1
- (C) $\frac{1}{2}$
- (D) 0
- (E) 2

18. For real t , let $M(t) = \begin{pmatrix} t-17 & 6 & 0 \\ 0 & t-17 & 1 \\ 0 & 0 & t-17 \end{pmatrix}$. At $t = 17$, what is $\dim \ker(M(t)^2) - \dim \ker M(t)$?

- (A) 2
- (B) 3
- (C) 0
- (D) 4
- (E) 1

Questions 19–21

19. Find the coefficient of x^6 in $\cos(5x)/(1 - 5x^2)$.

- (A) $\frac{11375}{144}$
- (B) $-\frac{56875}{144}$
- (C) $-\frac{11375}{144}$
- (D) $-\frac{3125}{144}$
- (E) $-\frac{11519}{144}$

20. Independent exponential waiting times X, Y have rates 8 and 10. Find $E[\min(X, Y)]$.

- (A) $\frac{19}{36}$
- (B) $\frac{17}{18}$
- (C) $\frac{1}{36}$
- (D) $\frac{1}{18}$
- (E) $\frac{1}{54}$

21. Find the directional derivative of $f(x, y, z) = x^2y + 6xz^2 + 5y^2z$ at $(1, 1, 1)$ in the direction $(1, 2, 0)$.

- (A) $1 + 6\sqrt{5}$
- (B) $6\sqrt{5}$
- (C) $12\sqrt{5}$
- (D) $3\sqrt{5}$
- (E) $-6\sqrt{5}$

Questions 22–24

22. Let $u = (4, 6)^T$ and $v = (1, 0)^T$. Find the squared distance from $(4, 0)^T$ to $\text{span}(u)$.

- (A) $\frac{301}{26}$
- (B) $\frac{131}{13}$
- (C) $\frac{157}{13}$
- (D) $\frac{144}{13}$
- (E) $\frac{275}{26}$

23. If $y'' + 36y = 5$ and $y(0) = y'(0) = 0$, find $y(\pi/6)$.

- (A) $\frac{5}{18}$
- (B) $-\frac{5}{18}$
- (C) $\frac{23}{18}$
- (D) $\frac{5}{9}$
- (E) $\frac{5}{36}$

24. For which real a does $f_m(x) = m^a x^8 (1 - x)^m$ converge uniformly to zero on $[0, 1]$?

- (A) $a \leq 8$
- (B) $a < 8$
- (C) All real a
- (D) $a < 0$
- (E) $a > 8$

Questions 25–27

25. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 5x$ and has density $\rho(x, y) = x^4y$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{7}{16}$
- (B) $-\frac{7}{8}$
- (C) $\frac{15}{8}$
- (D) $\frac{7}{8}$
- (E) $\frac{7}{4}$

26. An idempotent 23×23 real matrix P has rank 5. What is $\det(I + P)$?

- (A) 30
- (B) 33
- (C) 34
- (D) 32
- (E) 31

27. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 5z^2 = x + 6y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) -60
- (B) -10
- (C) -6
- (D) -30
- (E) 60

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{8p}]$ converge?

- (A) $p < 1/8$
- (B) $p \geq 1/8$
- (C) $p > 0$
- (D) $p > 1/8$
- (E) All real p

29. Evaluate $\int_0^1 \int_x^1 5y^6 dy dx$.

- (A) $\frac{5}{2}$
- (B) $\frac{5}{6}$
- (C) $\frac{5}{7}$
- (D) $\frac{5}{56}$
- (E) $\frac{5}{8}$

30. For $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 17 \\ 6 \\ 23 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find the minimum squared residual.

- (A) $\frac{787}{6}$
- (B) $\frac{395}{3}$
- (C) $\frac{392}{3}$
- (D) $\frac{781}{6}$
- (E) $\frac{389}{3}$

Questions 31–33

31. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(5x) - \sum_{j=0}^4 (-1)^j (5x)^{2j+1} / (2j+1)!}{x^{11}}$.
- (A) $5 \left(\frac{(-1)^5 5^{11}}{11!} \right)$
(B) $-\frac{(-1)^5 5^{11}}{11!}$
(C) $11 \left(\frac{(-1)^5 5^{11}}{11!} \right)$
(D) $\frac{1}{5} \left(\frac{(-1)^5 5^{11}}{11!} \right)$
(E) $\frac{(-1)^5 5^{11}}{11!}$
32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{8 + 1/k : k \geq 1\}$ have?
- (A) 1
(B) 0
(C) 4
(D) 3
(E) 2
33. The solution of $y' = 5x^5 y^2$, $y(0) = 1$, has its first positive singularity at which x ?
- (A) $\frac{6}{5}$
(B) $\frac{\sqrt[6]{3} \cdot 5^{\frac{5}{6}}}{5}$
(C) $\frac{5^{\frac{5}{6}}}{5}$
(D) $\frac{\sqrt[6]{5} \cdot 6^{\frac{5}{6}}}{6}$
(E) $\frac{5^{\frac{5}{6}} \sqrt[6]{6}}{5}$

Questions 34–36

34. What is the coefficient of x^8 in $(x + 4)^8(1 - x)^3$?

- (A) -2336
- (B) -2335
- (C) -2337
- (D) -2333
- (E) -2334

35. Find the coefficient of x^4 in $e^{5x} \log(1 + 5x)$.

- (A) $-\frac{625}{4}$
- (B) 5
- (C) 0
- (D) 1
- (E) -1

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 8\}$ have?

- (A) 12
- (B) 3
- (C) 6
- (D) 15
- (E) 9

Questions 37–39

37. Evaluate $\int_0^1 x^4(\log x)^5 dx$.

- (A) $\frac{(-1)^5 5!}{5^6}$
- (B) $\frac{625}{24} \left(\frac{(-1)^5 5!}{5^6} \right)$
- (C) $5 \left(\frac{(-1)^5 5!}{5^6} \right)$
- (D) $-\frac{(-1)^5 5!}{5^6}$
- (E) $\frac{1}{6} \left(\frac{(-1)^5 5!}{5^6} \right)$

38. A matrix A satisfies $(A - 18I)(A - 20I) = 0$, and its minimal polynomial has degree two. If $A^5 = uA + vI$, find u .

- (A) 655215
- (B) 655214
- (C) 655217
- (D) 655216
- (E) 655218

39. What is the maximum of $x^6 y^5$ over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\left(\frac{6}{11}\right)^6 \left(\frac{5}{11}\right)^5$
- (B) $\frac{30}{121}$
- (C) $\frac{1}{2^{11}}$
- (D) $11 \left(\left(\frac{6}{11}\right)^6 \left(\frac{5}{11}\right)^5 \right)$
- (E) $\left(\frac{6}{11}\right)^5 \left(\frac{5}{11}\right)^6$

Questions 40–42

40. Find the coefficient of z^{-1} in $e^{3z}/[z^2(1-z)]$ for $0 < |z| < 1$.

- (A) 5
- (B) 3
- (C) 2
- (D) 4
- (E) 6

41. Evaluate $\int_C (6x^5 + 5y) dx + (5x + 6y^5) dy$, where C is $r(t) = (t, 2t^2)$, $0 \leq t \leq 1$.

- (A) $\frac{75}{2}$
- (B) 150
- (C) 75
- (D) -75
- (E) 76

42. How many monic irreducible polynomials of degree 5 are there over $\mathbb{F}_7$?

- (A) $\frac{3(7^5 - 7)}{5}$
- (B) $\frac{2(7^5 - 7)}{5}$
- (C) $2(7^5 - 7)$
- (D) $\frac{7^5 - 7}{5}$
- (E) $7^5 - 7$

Questions 43–45

43. Let $F(x) = \int_0^{x^6} e^{5t} dt$. Find $F'(1)$.

- (A) $6e^5$
- (B) $\frac{e^5}{6}$
- (C) e^{30}
- (D) e^5
- (E) $30e^5$

44. An entire function satisfies $|f(z)| \leq C(1 + |z|^3)$ for some $C > 0$. What is the largest possible degree of f ?

- (A) 4
- (B) 5
- (C) 1
- (D) 3
- (E) 2

45. Let C be $x^2/36 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C \left(-\frac{5y^3}{3}\right) dx + \left(\frac{x^3}{3}\right) dy$.

- (A) 123π
- (B) $-\frac{123\pi}{2}$
- (C) $\frac{123\pi}{2}$
- (D) $\frac{123\pi}{4}$
- (E) $1 + \frac{123\pi}{2}$

Questions 46–48

46. How many elements z of $\mathbb{F}_{11^5}$ satisfy $z^{121} = z$?

- (A) 12
- (B) 10
- (C) 13
- (D) 9
- (E) 11

47. For $x > 0$, $x^2 y'' + (-15)xy' + 55y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $-\frac{(1 - e^6)e^5}{3}$
- (B) $-\frac{e^5}{6} + 1 + \frac{e^{11}}{6}$
- (C) $\frac{(1 - e^6)e^5}{6}$
- (D) $-\frac{(1 - e^6)e^5}{12}$
- (E) $-\frac{(1 - e^6)e^5}{6}$

48. A connected planar graph has 11 vertices and 17 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 12
- (B) 8
- (C) 6
- (D) 4
- (E) 10

Questions 49–51

49. Let $\mathbf{F} = (5y^6, z^6, x^6)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (1, 1, 2)/\sqrt{6}$.

- (A) $-24\sqrt{6}$
- (B) $1 - 12\sqrt{6}$
- (C) $-6\sqrt{6}$
- (D) $-12\sqrt{6}$
- (E) $12\sqrt{6}$

50. How many generators does the cyclic group of order 117649 have?

- (A) $7^5(7 - 1)$
- (B) $7^4(7 - 1)$
- (C) $7^7(7 - 1)$
- (D) $7^6(7 - 1)$
- (E) $7^8(7 - 1)$

51. If $y' = 10xy$ and $y(0) = 1$, find the coefficient of x^{12} in the Maclaurin series of y .

- (A) $\frac{3269}{144}$
- (B) $-\frac{3125}{144}$
- (C) $\frac{3125}{144}$
- (D) $\frac{3125}{288}$
- (E) $\frac{3125}{72}$

Questions 52–54

52. How many integers in $\{1, \dots, 48\}$ are divisible by 2 or 3?

- (A) 22
- (B) 52
- (C) 32
- (D) 12
- (E) 42

53. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^5 y^5 dV$.

- (A) $6 \left(\frac{5! 5!}{13!} \right)$
- (B) $13 \left(\frac{5! 5!}{13!} \right)$
- (C) $\frac{5! 5!}{13!}$
- (D) $\frac{1}{6} \left(\frac{5! 5!}{13!} \right)$
- (E) $\frac{1}{36}$

54. Let $R = (\mathbb{Z}/30\mathbb{Z})/([6])$, where $([6])$ is the ideal generated by $[6]$. How many units does R have?

- (A) 0
- (B) 2
- (C) 3
- (D) 4
- (E) 1

Questions 55–57

55. If $y'' - 10y' + 25y = 6e^{5x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $\frac{3e^5}{2}$
- (B) $1 + 3e^5$
- (C) $3e^5$
- (D) $6e^5$
- (E) $-3e^5$

56. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 8 have no adjacent 1s?

- (A) F_9
- (B) F_{11}
- (C) F_{12}
- (D) F_8
- (E) F_{10}

57. Find the total outward flux of $\mathbf{F} = (5x^6y, y^6z, xz^6)$ through the boundary of $[0, 1]^3$.

- (A) $\frac{7}{2}$
- (B) $\frac{9}{2}$
- (C) 7
- (D) $-\frac{7}{2}$
- (E) $\frac{7}{4}$

Questions 58–60

58. What is the largest integer k such that 3^k divides $26!$?

- (A) 8
- (B) 11
- (C) 10
- (D) 9
- (E) 12

59. For $f(x, y) = x^2 + 5xy + 6y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) 58
- (B) $\sqrt{58}$
- (C) -7
- (D) -3
- (E) 10

60. For Newton iteration on $f(x) = x^2 - 64$ near the root $r = 8$, find $\lim_{k \rightarrow \infty} (x_{k+1} - r)/(x_k - r)^2$ for nonterminating convergent iterates.

- (A) $\frac{1}{16}$
- (B) $\frac{17}{32}$
- (C) $\frac{1}{32}$
- (D) $\frac{15}{16}$
- (E) $\frac{1}{48}$

Questions 61–63

61. For $x > 0$, $x^2 y'' + (1 - 11)xy' + 30y = 0$, $y(1) = 0$, $y'(1) = 5$. Find $y(e)$.

- (A) $\frac{e^5}{11}$
- (B) $-5(1 - e)e^5$
- (C) $5(1 - e)e^5$
- (D) $-e^6 + e^5$
- (E) $e^5 + e^6$

62. In the ring $\mathbb{Z}/126\mathbb{Z}$, how many elements satisfy $x^2 = 1$?

- (A) 6
- (B) 4
- (C) 3
- (D) 5
- (E) 2

63. Evaluate $\int_0^1 \int_0^x x^5 y^6 dy dx$.

- (A) $\frac{1}{104}$
- (B) $\frac{1}{42}$
- (C) $\frac{1}{84}$
- (D) $\frac{1}{13}$
- (E) $\frac{1}{91}$

Questions 64–66

64. Bisection starts with an interval of length 2^8 . After 10 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{1}{16}$
- (B) $\frac{9}{16}$
- (C) $\frac{1}{24}$
- (D) $\frac{1}{8}$
- (E) $\frac{7}{8}$

65. Evaluate $\int_0^{\pi/2} \sin^{13} t \cos^{11} t \, dt$.

- (A) $\frac{6! 5!}{2 \cdot 12!}$
- (B) $12 \left(\frac{6! 5!}{2 \cdot 12!} \right)$
- (C) $\frac{1}{2} \left(\frac{6! 5!}{2 \cdot 12!} \right)$
- (D) $2 \left(\frac{6! 5!}{2 \cdot 12!} \right)$
- (E) $462 \left(\frac{6! 5!}{2 \cdot 12!} \right)$

66. A permutation consists of disjoint cycles of lengths 5 and 3, together with a disjoint transposition when the sum of these two lengths is odd. It fixes all other points. What is its order?

- (A) 14
- (B) 16
- (C) 13
- (D) 17
- (E) 15

Answer Key

Practice Test 21 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	A	23	A	45	C
2	B	24	B	46	E
3	A	25	D	47	E
4	B	26	D	48	B
5	E	27	A	49	D
6	C	28	D	50	A
7	E	29	E	51	C
8	E	30	C	52	C
9	B	31	E	53	C
10	D	32	E	54	B
11	C	33	E	55	C
12	B	34	B	56	E
13	A	35	C	57	A
14	D	36	E	58	C
15	C	37	A	59	B
16	A	38	D	60	A
17	B	39	A	61	B
18	E	40	D	62	B
19	C	41	C	63	E
20	D	42	D	64	D
21	B	43	A	65	A
22	D	44	D	66	E

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