

GRE Mathematics Practice Test 22

Advanced Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A demanding practice form with frequent multi-step arguments and a mix of computational and conceptual questions. This is cycle 4 of 19, position 2 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^5 y^4 dV$.

- (A) $\frac{5! 4!}{12!}$
- (B) $12 \left(\frac{5! 4!}{12!} \right)$
- (C) $\frac{1}{30}$
- (D) $6 \left(\frac{5! 4!}{12!} \right)$
- (E) $\frac{1}{6} \left(\frac{5! 4!}{12!} \right)$

2. Let $u = (4, 5)^T$ and $v = (1, 0)^T$. Find the squared distance from $(4, 0)^T$ to $\text{span}(u)$.

- (A) $\frac{841}{82}$
- (B) $\frac{359}{41}$
- (C) $\frac{441}{41}$
- (D) $\frac{400}{41}$
- (E) $\frac{759}{82}$

3. Let $F(x) = \int_0^x (x - t)^4 e^{5t} dt$. Find $F^{(5)}(0)$.

- (A) 120
- (B) 48
- (C) 0
- (D) 24
- (E) $\frac{1}{24}$

Questions 4–6

4. A uniformly random subset of 9 objects is selected from a collection with 10 red and 11 blue objects. If X is the number of red objects selected, find $E[X]$.

- (A) $\frac{15}{7}$
- (B) $\frac{60}{7}$
- (C) $\frac{44}{7}$
- (D) $\frac{30}{7}$
- (E) $\frac{37}{7}$

5. Evaluate $\int_0^{\log 2} e^x(e^x + 5)^5 dx$.

- (A) $7^5 - 6^5$
- (B) $-\frac{7^6 - 6^6}{6}$
- (C) $\frac{7^6}{6}$
- (D) $6 \left(\frac{7^6 - 6^6}{6} \right)$
- (E) $\frac{7^6 - 6^6}{6}$

6. An idempotent 24×24 real matrix P has rank 1. What is $\det(I + P)$?

- (A) 4
- (B) 2
- (C) 3
- (D) 0
- (E) 1

Questions 7–9

7. Find the directional derivative of $f(x, y, z) = x^2y + 5xz^2 + 5y^2z$ at $(1, 1, 1)$ in the direction $(1, 3, 0)$.

- (A) $1 + 4\sqrt{10}$
- (B) $-4\sqrt{10}$
- (C) $4\sqrt{10}$
- (D) $2\sqrt{10}$
- (E) $8\sqrt{10}$

8. X, Y are independent Poisson variables with means 9 and 10. Given $X + Y = 11$, find $\text{Var}(X \mid X + Y)$.

- (A) $\frac{1351}{361}$
- (B) $\frac{1980}{361}$
- (C) $\frac{990}{361}$
- (D) $\frac{1712}{361}$
- (E) $\frac{495}{361}$

9. For $D = \{(x, y) : x^2 + y^2 \leq 5\}$, evaluate $\iint_D (x^2 + y^2)^4 dA$.

- (A) $\frac{3125\pi}{9}$
- (B) 125π
- (C) 3125π
- (D) 1250π
- (E) 625π

Questions 10–12

10. For $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 18 \\ 4 \\ 22 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find the minimum squared residual.
- (A) $\frac{1021}{6}$
(B) $\frac{512}{3}$
(C) $\frac{1027}{6}$
(D) $\frac{509}{3}$
(E) $\frac{515}{3}$
11. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-5)^j}{j^{\frac{4}{3}} (\log j)^m}$ converge absolutely at $x = 4$? Give the least such m , or 0 if none exists.
- (A) 1
(B) 0
(C) 3
(D) 4
(E) 2
12. X, Y are independent, with variances 9 and 10. Find $\text{Cov}(X + 2Y, 3X - Y)$.
- (A) 11
(B) 7
(C) 9
(D) 5
(E) 3

Questions 13–15

13. Under $x = u + 5v$, $y = u - 5v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) 100
- (B) 25
- (C) 0
- (D) $\frac{1}{10}$
- (E) 10

14. What is the coefficient of x^7 in $(x + 4)^7(1 - x)^3$?

- (A) -1315
- (B) -1317
- (C) -1314
- (D) -1316
- (E) -1313

15. Find the upward flux of $\mathbf{F} = (x, y, z)$ through $z = 5x^2 + 5y^2$, $0 \leq x, y \leq 1$.

- (A) $\frac{10}{3}$
- (B) $-\frac{5}{3}$
- (C) $-\frac{7}{3}$
- (D) $-\frac{10}{3}$
- (E) $-\frac{20}{3}$

Questions 16–18

16. X has density cx^9 on $0 < x < 1$ and zero elsewhere. Find $E[X^2]$.

- (A) $\frac{11}{12}$
- (B) $\frac{5}{18}$
- (C) $\frac{5}{12}$
- (D) $\frac{1}{6}$
- (E) $\frac{5}{6}$

17. If $y' = y(1 - y/5)$ and $y(0) = 5/2$, find $y(-\log 5)$.

- (A) $\frac{5}{3}$
- (B) $\frac{6}{5}$
- (C) $\frac{12}{5}$
- (D) $-\frac{5}{6}$
- (E) $\frac{11}{6}$

18. A matrix A satisfies $(A - 19I)(A - 21I) = 0$, and its minimal polynomial has degree two. If $A^5 = uA + vI$, find u .

- (A) 804001
- (B) 804002
- (C) 804003
- (D) 804000
- (E) 803999

Questions 19–21

19. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{5t} \cos(5t)$, and $y = e^{5t} \sin(5t)$, find d^2f/dt^2 at $t = 1$.

- (A) -1
- (B) 1
- (C) 100
- (D) 0
- (E) 10

20. Independent exponential waiting times X, Y have rates 9 and 11. Find $P(X < Y)$.

- (A) $\frac{3}{20}$
- (B) $\frac{11}{20}$
- (C) $\frac{9}{20}$
- (D) $\frac{29}{40}$
- (E) $\frac{9}{40}$

21. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 5x)^{1/(4x)} - e^{5/4}}{x}$.

- (A) $-\frac{25e^{\frac{5}{4}}}{4}$
- (B) $\frac{25e^{\frac{5}{4}}}{8}$
- (C) $-\frac{25e^{\frac{5}{4}}}{8}$
- (D) $-\frac{5e^{\frac{5}{4}}}{8}$
- (E) $e^{\frac{5}{4}}$

Questions 22–24

22. Let $V = \begin{bmatrix} 1 & 18 & 324 \\ 1 & 20 & 400 \\ 1 & 23 & 529 \end{bmatrix}$. Find $\det(V^T V)$.

- (A) 901
- (B) 899
- (C) 900
- (D) 898
- (E) 902

23. Find the coefficient of x^4 in $e^{4x} \log(1 + 5x)$.

- (A) $-\frac{145}{4}$
- (B) $-\frac{125}{4}$
- (C) $-\frac{625}{4}$
- (D) -870
- (E) $\frac{145}{4}$

24. For which real a does $f_m(x) = m^a x^9 (1 - x)^m$ converge uniformly to zero on $[0, 1]$?

- (A) All real a
- (B) $a > 9$
- (C) $a < 0$
- (D) $a < 9$
- (E) $a \leq 9$

Questions 25–27

25. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(5x) - \sum_{j=0}^3 (-1)^j (5x)^{2j+1} / (2j+1)!}{x^9}$.
- (A) $\frac{1}{5} \left(\frac{(-1)^4 5^9}{9!} \right)$
(B) $-\frac{(-1)^4 5^9}{9!}$
(C) $5 \left(\frac{(-1)^4 5^9}{9!} \right)$
(D) $9 \left(\frac{(-1)^4 5^9}{9!} \right)$
(E) $\frac{(-1)^4 5^9}{9!}$
26. Let $u = (1, 2, \dots, 24)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $v^T B^{-1} u$.
- (A) $\frac{901}{602}$
(B) $\frac{1503}{602}$
(C) $\frac{601}{301}$
(D) $\frac{301}{300}$
(E) $\frac{301}{299}$
27. What is the maximum of $x^5 y^5$ over $x, y \geq 0$ subject to $x + y = 1$?
- (A) $10 \left(\left(\frac{5}{10} \right)^5 \left(\frac{5}{10} \right)^5 \right)$
(B) $\left(\frac{5}{10} \right)^5 \left(\frac{5}{10} \right)^5$
(C) $256 \left(\left(\frac{5}{10} \right)^5 \left(\frac{5}{10} \right)^5 \right)$
(D) $2 \left(\left(\frac{5}{10} \right)^5 \left(\frac{5}{10} \right)^5 \right)$
(E) $\frac{1}{2} \left(\left(\frac{5}{10} \right)^5 \left(\frac{5}{10} \right)^5 \right)$

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{9p}]$ converge?

- (A) $p < 1/9$
- (B) $p > 0$
- (C) All real p
- (D) $p > 1/9$
- (E) $p \geq 1/9$

29. Let C be the unit circle $x^2 + y^2 = 1$, $z = 5$, counterclockwise viewed from above. Evaluate $\int_C -5y(x^2 + y^2)^4 dx + (5x(x^2 + y^2)^4) dy + 3z dz$.

- (A) $1 + 10\pi$
- (B) 20π
- (C) -10π
- (D) 5π
- (E) 10π

30. Let $D = \text{diag}(\underbrace{1, \dots, 1}_3, \underbrace{2, \dots, 2}_{21})$. Find the dimension of the space of real matrices X with $XD = DX$.

- (A) 449
- (B) 451
- (C) 452
- (D) 448
- (E) 450

Questions 31–33

31. For $\mathbf{F} = (5x^5y, y^5z, xz^5)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) 270
- (B) $\frac{135}{2}$
- (C) 136
- (D) 135
- (E) -135

32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{9 + 1/k : k \geq 1\}$ have?

- (A) 1
- (B) 4
- (C) 2
- (D) 0
- (E) 3

33. Evaluate $\int_C (5x^4 + 5y) dx + (5x + 5y^4) dy$, where C is $r(t) = (t, 3t^2)$, $0 \leq t \leq 1$.

- (A) 259
- (B) 260
- (C) -259
- (D) $\frac{259}{2}$
- (E) 518

Questions 34–36

34. When $x^3 + 1$ is divided by $(x - 19)(x - 20)$, the remainder is $ux + v$. Find u .

- (A) 1143
- (B) 1142
- (C) 1141
- (D) 1140
- (E) 1139

35. Let $f(x) = \log(1 + 5x)$. Find $f^{(5)}(0)$.

- (A) $\frac{1}{5} \left((-1)^4 4! 5^5 \right)$
- (B) $5 \left((-1)^4 4! 5^5 \right)$
- (C) $(-1)^4 4! 5^5$
- (D) $-(-1)^4 4! 5^5$
- (E) $2 \left((-1)^4 4! 5^5 \right)$

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 9\}$ have?

- (A) 7
- (B) 10
- (C) 13
- (D) 16
- (E) 4

Questions 37–39

37. Evaluate $\int_0^1 x^4(\log x)^4 dx$.

- (A) $\frac{(-1)^4 4!}{5^5}$
- (B) $\frac{1}{5} \left(\frac{(-1)^4 4!}{5^5} \right)$
- (C) $-\frac{(-1)^4 4!}{5^5}$
- (D) $5 \left(\frac{(-1)^4 4!}{5^5} \right)$
- (E) $\frac{625}{24} \left(\frac{(-1)^4 4!}{5^5} \right)$

38. Let $P = \text{diag}(\underbrace{1, \dots, 1}_2, \underbrace{0, \dots, 0}_{23})$. What is the dimension of the space of 25×25 real matrices X satisfying $PX = X$ and $XP = 0$?

- (A) 48
- (B) 45
- (C) 44
- (D) 46
- (E) 47

39. Evaluate $\int_0^1 \frac{x^4}{1 + 5x^5} dx$.

- (A) $\frac{\log(6)}{25}$
- (B) $\log(6)$
- (C) $\frac{\log(6)}{5}$
- (D) $-\frac{\log(6)}{25}$
- (E) $\frac{2 \log(6)}{25}$

Questions 40–42

40. What is the coefficient of z^{-1} in the Laurent expansion of e^{4z}/z^3 about zero?

- (A) 8
- (B) 4
- (C) 10
- (D) 6
- (E) 12

41. Evaluate $\int_0^{\pi/2} \sin^{11} t \cos^{11} t \, dt$.

- (A) $2 \left(\frac{5! 5!}{2 \cdot 11!} \right)$
- (B) $252 \left(\frac{5! 5!}{2 \cdot 11!} \right)$
- (C) $11 \left(\frac{5! 5!}{2 \cdot 11!} \right)$
- (D) $\frac{1}{2} \left(\frac{5! 5!}{2 \cdot 11!} \right)$
- (E) $\frac{5! 5!}{2 \cdot 11!}$

42. In $R = \mathbb{F}_7[x]/(x^5)$, how many elements are units?

- (A) $7^7 - 7^6$
- (B) $7^6 - 7^5$
- (C) $7^5 - 7^0$
- (D) $7^5 - 7^4$
- (E) $7^6 - 7^4$

Questions 43–45

43. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(5j)!}{(j!)^5} x^{5j}$?
- (A) $2(5^{-1})$
(B) 5^{-5}
(C) 5^1
(D) $\frac{1}{2}(5^{-1})$
(E) 5^{-1}
44. An entire function satisfies $|f(z)| \leq C(1 + |z|^4)$ for some $C > 0$. What is the largest possible degree of f ?
- (A) 3
(B) 2
(C) 5
(D) 4
(E) 6
45. Solve $y' + 5y = x^5 e^{-5x}$, $y(0) = 0$, and find $y(1)$.
- (A) e^{-5}
(B) $\frac{1}{6e^5}$
(C) $\frac{5}{6e^5}$
(D) $\frac{1}{5e^5}$
(E) $\frac{e^5}{6}$

Questions 46–48

46. Integers x satisfy $x \equiv 4 \pmod{7}$ and $x \equiv 6 \pmod{9}$. Find the sum of all such x with $0 \leq x < 126$.

- (A) 181
- (B) 184
- (C) 185
- (D) 182
- (E) 183

47. Evaluate $\int_0^1 x^5(1-x)^4 dx$.

- (A) $10 \left(\frac{5!4!}{10!} \right)$
- (B) $\frac{5!4!}{10!}$
- (C) $6 \left(\frac{5!4!}{10!} \right)$
- (D) $\frac{1}{5} \left(\frac{5!4!}{10!} \right)$
- (E) $42 \left(\frac{5!4!}{10!} \right)$

48. A connected planar graph has 12 vertices and 19 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 3
- (B) 9
- (C) 12
- (D) 15
- (E) 6

Questions 49–51

49. Find the total outward flux of $\mathbf{F} = (5x^5y, y^5z, xz^5)$ through the boundary of $[0, 1]^3$.
- (A) $\frac{7}{4}$
(B) $\frac{7}{2}$
(C) $\frac{9}{2}$
(D) $-\frac{7}{2}$
(E) 7
50. The symmetric group S_{24} acts on the two-element subsets of $\{1, \dots, 24\}$. What is the index of the setwise stabilizer of $\{1, 2\}$?
- (A) 278
(B) 275
(C) 274
(D) 277
(E) 276
51. The region $0 \leq x \leq 1$, $0 \leq y \leq 5x^5$ is revolved about the line $x = -3$. Find its volume.
- (A) $\frac{45\pi}{14}$
(B) $\frac{45\pi}{7}$
(C) $1 + \frac{45\pi}{7}$
(D) $\frac{90\pi}{7}$
(E) $-\frac{45\pi}{7}$

Questions 52–54

52. How many integers in $\{1, \dots, 54\}$ are divisible by 2 or 3?

- (A) 24
- (B) 36
- (C) 12
- (D) 48
- (E) 60

53. Evaluate $\int_0^\infty x^5 e^{-5x} dx$.

- (A) $\frac{1}{5} \left(\frac{5!}{5^6} \right)$
- (B) $5 \left(\frac{5!}{5^6} \right)$
- (C) $15625 \left(\frac{5!}{5^6} \right)$
- (D) $\frac{5!}{5^6}$
- (E) $2 \left(\frac{5!}{5^6} \right)$

54. In $\mathbb{Z}/498\mathbb{Z}$, how many residues are not idempotent ($x^2 \neq x$)?

- (A) 491
- (B) 488
- (C) 492
- (D) 489
- (E) 490

Questions 55–57

55. If $y'' + 25y = 5$ and $y(0) = y'(0) = 0$, find $y(\pi/5)$.

- (A) $\frac{1}{5}$
- (B) $\frac{4}{5}$
- (C) $\frac{2}{5}$
- (D) $\frac{7}{5}$
- (E) $-\frac{2}{5}$

56. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 9 have no adjacent 1s?

- (A) F_{13}
- (B) F_{12}
- (C) F_{10}
- (D) F_9
- (E) F_{11}

57. If $y'' - 10y' + 25y = 5e^{5x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $\frac{5e^5}{4}$
- (B) $\frac{5e^5}{2}$
- (C) $-\frac{5e^5}{2}$
- (D) $1 + \frac{5e^5}{2}$
- (E) $5e^5$

Questions 58–60

58. In the additive group $\mathbb{Z}/42\mathbb{Z}$, find the index of the subgroup generated by $[3]$.

- (A) 3
- (B) 1
- (C) 2
- (D) 4
- (E) 5

59. If $y' = 10xy$ and $y(0) = 1$, find the coefficient of x^{10} in the Maclaurin series of y .

- (A) $-\frac{625}{24}$
- (B) $\frac{625}{48}$
- (C) $\frac{625}{12}$
- (D) $\frac{649}{24}$
- (E) $\frac{625}{24}$

60. For Newton iteration on $f(x) = x^2 - 81$ near the root $r = 9$, find $\lim_{k \rightarrow \infty} (x_{k+1} - r)/(x_k - r)^2$ for nonterminating convergent iterates.

- (A) $\frac{1}{54}$
- (B) $\frac{19}{36}$
- (C) $\frac{1}{18}$
- (D) $\frac{17}{18}$
- (E) $\frac{1}{36}$

Questions 61–63

61. For $x > 0$, $x^2y'' + (1 - 10)xy' + 25y = 0$, $y(1) = 0$, $y'(1) = 5$. Find $y(e)$.

- (A) $5e^5$
- (B) $2e^5$
- (C) $-5e^5$
- (D) $\frac{e^5}{10}$
- (E) 0

62. How many invertible 3×3 matrices over $\mathbb{F}_7$ are there with determinant 1?

- (A) $\frac{(7^3 - 7^0)(7^3 - 7^1)}{6}$
- (B) $\frac{(7^3 - 7^0)(7^3 - 7^0)(7^3 - 7^0)}{6}$
- (C) $\frac{(7^3 - 7^0)(7^3 - 7^1)(7^3 - 7^2)}{6}$
- (D) $\frac{(7^3 - 7^1)(7^3 - 7^2)(7^3 - 7^3)}{6}$
- (E) $\frac{7^3 7^3 7^3}{6}$

63. Evaluate $\lim_{x \rightarrow 0} \frac{e^{5x} - \sum_{j=0}^4 (5x)^j / j!}{x^5}$.

- (A) $120 \left(\frac{5^5}{5!} \right)$
- (B) $-\frac{5^5}{5!}$
- (C) $\frac{5^5}{5!}$
- (D) $\frac{1}{5} \left(\frac{5^5}{5!} \right)$
- (E) $5 \left(\frac{5^5}{5!} \right)$

Questions 64–66

64. Bisection starts with an interval of length 2^9 . After 11 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{1}{8}$
- (B) $\frac{9}{16}$
- (C) $\frac{7}{8}$
- (D) $\frac{1}{16}$
- (E) $\frac{1}{24}$

65. Let $f(x, y) = x^2 + 10xy + (30)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 20
- (B) 5
- (C) 10
- (D) 120
- (E) 30

66. How many permutations in S_{24} consist of one 4-cycle and 20 fixed points?

- (A) 63755
- (B) 63754
- (C) 63756
- (D) 63757
- (E) 63758

Answer Key

Practice Test 22 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	A	23	A	45	B
2	D	24	D	46	E
3	D	25	E	47	B
4	D	26	D	48	B
5	E	27	B	49	B
6	B	28	D	50	E
7	C	29	E	51	B
8	C	30	E	52	B
9	E	31	D	53	D
10	B	32	C	54	E
11	A	33	A	55	C
12	B	34	C	56	E
13	E	35	C	57	B
14	A	36	B	58	A
15	D	37	A	59	E
16	E	38	D	60	C
17	B	39	A	61	A
18	A	40	A	62	C
19	D	41	E	63	C
20	C	42	D	64	A
21	C	43	E	65	A
22	C	44	D	66	C

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