

GRE Mathematics Practice Test 23

Upper-Intermediate Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

Intermediate reinforcement, combining standard applications with selected more demanding arguments. This is cycle 4 of 19, position 3 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Let C be the unit circle $x^2 + y^2 = 1$, $z = 4$, counterclockwise viewed from above. Evaluate $\int_C -5y(x^2 + y^2)^3 dx + (5x(x^2 + y^2)^3) dy + 4z dz$.

- (A) -10π
- (B) 20π
- (C) 5π
- (D) 10π
- (E) $1 + 10\pi$

2. Let $V = \begin{bmatrix} 1 & 19 & 361 \\ 1 & 21 & 441 \\ 1 & 24 & 576 \end{bmatrix}$. Find $\det(V^T V)$.

- (A) 902
- (B) 899
- (C) 900
- (D) 901
- (E) 898

3. If $y' = 10xy$ and $y(0) = 1$, find the coefficient of x^8 in the Maclaurin series of y .

- (A) $\frac{625}{48}$
- (B) $\frac{649}{24}$
- (C) $-\frac{625}{24}$
- (D) $\frac{625}{12}$
- (E) $\frac{625}{24}$

Questions 4–6

4. A uniformly random subset of 10 objects is selected from a collection with 11 red and 12 blue objects. If X is the number of red objects selected, find $E[X]$.

- (A) $\frac{55}{23}$
- (B) $\frac{220}{23}$
- (C) $\frac{156}{23}$
- (D) $\frac{110}{23}$
- (E) $\frac{133}{23}$

5. Evaluate $\int_0^{\pi/2} \sin^9 t \cos^{11} t \, dt$.

- (A) $\frac{4! 5!}{2 \cdot 10!}$
- (B) $2 \left(\frac{4! 5!}{2 \cdot 10!} \right)$
- (C) $10 \left(\frac{4! 5!}{2 \cdot 10!} \right)$
- (D) $\frac{1}{2} \left(\frac{4! 5!}{2 \cdot 10!} \right)$
- (E) $126 \left(\frac{4! 5!}{2 \cdot 10!} \right)$

6. Let $u = (1, 2, \dots, 25)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $\det B$.

- (A) 325
- (B) 327
- (C) 328
- (D) 324
- (E) 326

Questions 7–9

7. Find the coefficient of x^6 in $\cos(5x)/(1 - 3x^2)$.
- (A) $-\frac{4187}{144}$
(B) $\frac{4187}{144}$
(C) -20935
(D) $-\frac{3125}{144}$
(E) $-\frac{4331}{144}$
8. X, Y are independent Poisson variables with means 10 and 11. Given $X + Y = 12$, find $\text{Var}(X | X + Y)$.
- (A) $\frac{220}{147}$
(B) $\frac{880}{147}$
(C) $\frac{734}{147}$
(D) $\frac{440}{147}$
(E) $\frac{587}{147}$
9. Find the arc length of $r(t) = (5(t - t^3/3), 5t^2)$ for $0 \leq t \leq 1/4$.
- (A) $\frac{245}{192}$
(B) $-\frac{245}{192}$
(C) $\frac{245}{384}$
(D) $\frac{437}{192}$
(E) $\frac{245}{96}$

Questions 10–12

10. Let $D = \text{diag}(\underbrace{1, \dots, 1}_3, \underbrace{2, \dots, 2}_{22})$. Find the dimension of the space of real matrices X with $XD = DX$.

- (A) 495
- (B) 494
- (C) 492
- (D) 493
- (E) 491

11. Two solutions of $y'' + 5x^4y' + 4y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) e^{-1}
- (B) $e^{-1} + 1$
- (C) $\frac{2}{e}$
- (D) $\frac{1}{2e}$
- (E) $-\frac{1}{e}$

12. X, Y are independent, with variances 10 and 11. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 4
- (B) 6
- (C) 8
- (D) 10
- (E) 12

Questions 13–15

13. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{5t} \cos(4t)$, and $y = e^{5t} \sin(4t)$, find d^2f/dt^2 at $t = 1$.

- (A) 8
- (B) 10
- (C) 100
- (D) 82
- (E) 0

14. When $x^3 + 1$ is divided by $(x - 20)(x - 21)$, the remainder is $ux + v$. Find u .

- (A) 1259
- (B) 1263
- (C) 1262
- (D) 1260
- (E) 1261

15. Find the coefficient of x^4 in the Taylor series of $(1 - 5x)^{-2}$ at zero.

- (A) $5 \cdot 5^4$
- (B) $\frac{4}{5} (5 \cdot 5^4)$
- (C) $2 (5 \cdot 5^4)$
- (D) $\frac{1}{5} (5 \cdot 5^4)$
- (E) $\frac{1}{2} (5 \cdot 5^4)$

Questions 16–18

16. X has density cx^{10} on $0 < x < 1$ and zero elsewhere. Find the normalizing constant c .

- (A) 8
- (B) 14
- (C) 5
- (D) 17
- (E) 11

17. Evaluate $\int_0^1 x^5(1-x)^3 dx$.

- (A) $21 \left(\frac{5!3!}{9!} \right)$
- (B) $\frac{5!3!}{9!}$
- (C) $\frac{1}{4} \left(\frac{5!3!}{9!} \right)$
- (D) $6 \left(\frac{5!3!}{9!} \right)$
- (E) $9 \left(\frac{5!3!}{9!} \right)$

18. Let $P = \text{diag}(\underbrace{1, \dots, 1}_3, \underbrace{0, \dots, 0}_{23})$. What is the dimension of the space of 26×26 real matrices X satisfying $PX = X$ and $XP = 0$?

- (A) 71
- (B) 68
- (C) 69
- (D) 67
- (E) 70

Questions 19–21

19. Evaluate $\int_0^1 \int_x^1 5y^4 dy dx$.

- (A) 1
- (B) $\frac{1}{6}$
- (C) $\frac{5}{4}$
- (D) $\frac{5}{6}$
- (E) $\frac{5}{2}$

20. Independent exponential waiting times X, Y have rates 10 and 12. Find $P(X < Y)$.

- (A) $\frac{8}{11}$
- (B) $\frac{6}{11}$
- (C) $\frac{5}{11}$
- (D) $\frac{5}{33}$
- (E) $\frac{33}{52}$

21. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-5)^j}{j^1 (\log j)^m}$ converge absolutely at $x = 4$? Give the least such m , or 0 if none exists.

- (A) 4
- (B) 2
- (C) 3
- (D) 1
- (E) 0

Questions 22–24

22. A 3×3 matrix A has characteristic polynomial $(t - 19)(t + 1)(t - 2)$. What is $\text{tr}(A^3)$?

- (A) 6864
- (B) 6867
- (C) 6865
- (D) 6866
- (E) 6868

23. Let $f(x) = \log(1 + 5x)$. Find $f^{(4)}(0)$.

- (A) $\frac{1}{5} \left((-1)^3 3! 5^4 \right)$
- (B) $(-1)^3 3! 5^4$
- (C) $4 \left((-1)^3 3! 5^4 \right)$
- (D) $\frac{1}{4} \left((-1)^3 3! 5^4 \right)$
- (E) $-(-1)^3 3! 5^4$

24. For which real a does $f_m(x) = m^a x^{10} (1 - x)^m$ converge uniformly to zero on $[0, 1]$?

- (A) $a \leq 10$
- (B) All real a
- (C) $a < 10$
- (D) $a > 10$
- (E) $a < 0$

Questions 25–27

25. For $f(x) = x^4 e^{5x}$, find $f^{(6)}(0)$.

- (A) $\frac{1}{360} \left(\frac{6! 5^2}{2} \right)$
- (B) $\frac{2}{5} \left(\frac{6! 5^2}{2} \right)$
- (C) $2 \left(\frac{6! 5^2}{2} \right)$
- (D) $\frac{1}{30} \left(\frac{6! 5^2}{2} \right)$
- (E) $\frac{6! 5^2}{2}$

26. Let N be the 25×25 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\dim(\ker N^4 / \ker N^2)$.

- (A) 0
- (B) 4
- (C) 3
- (D) 2
- (E) 1

27. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(5x) - \sum_{j=0}^2 (-1)^j (5x)^{2j+1} / (2j+1)!}{x^7}$.

- (A) $\frac{(-1)^3 5^7}{7!}$
- (B) $7 \left(\frac{(-1)^3 5^7}{7!} \right)$
- (C) $\frac{1}{5} \left(\frac{(-1)^3 5^7}{7!} \right)$
- (D) $5 \left(\frac{(-1)^3 5^7}{7!} \right)$
- (E) $-\frac{(-1)^3 5^7}{7!}$

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{10p}]$ converge?

- (A) All real p
- (B) $p > 1/10$
- (C) $p < 1/10$
- (D) $p > 0$
- (E) $p \geq 1/10$

29. The solution of $y' = 5x^4y^3$ has $y(0) = 1/6$. Find $y(1)$.

- (A) $-\frac{\sqrt{34}}{34}$
- (B) $\frac{\sqrt{34}}{34} + 1$
- (C) $\frac{\sqrt{34}}{34}$
- (D) $\frac{\sqrt{34}}{68}$
- (E) $\frac{\sqrt{34}}{17}$

30. Let A be the 25×25 matrix whose diagonal entries are 22 and whose off-diagonal entries are 1. What is its largest eigenvalue?

- (A) 44
- (B) 46
- (C) 45
- (D) 48
- (E) 47

Questions 31–33

31. Evaluate $\int_0^1 x^4(\log x)^3 dx$.

- (A) $5 \left(\frac{(-1)^3 3!}{5^4} \right)$
- (B) $\frac{(-1)^3 3!}{5^4}$
- (C) $\frac{125}{6} \left(\frac{(-1)^3 3!}{5^4} \right)$
- (D) $\frac{1}{4} \left(\frac{(-1)^3 3!}{5^4} \right)$
- (E) $-\frac{(-1)^3 3!}{5^4}$

32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{10 + 1/k : k \geq 1\}$ have?

- (A) 3
- (B) 1
- (C) 0
- (D) 2
- (E) 4

33. For $x > 0$, $x^2 y'' + (-13)xy' + 45y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $-\frac{(1 - e^4) e^5}{8}$
- (B) $-\frac{e^5}{4} + 1 + \frac{e^9}{4}$
- (C) $\frac{(1 - e^4) e^5}{4}$
- (D) $-\frac{(1 - e^4) e^5}{4}$
- (E) $-\frac{(1 - e^4) e^5}{2}$

Questions 34–36

34. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 133$ and $x + y = 26$. Find $x^2 + y^2$.

- (A) 410
- (B) 408
- (C) 409
- (D) 411
- (E) 412

35. What is the maximum of $x^4 y^5$ over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\frac{20}{81}$
- (B) $\left(\frac{4}{9}\right)^4 \left(\frac{5}{9}\right)^5$
- (C) $\frac{1}{2^9}$
- (D) $9 \left(\left(\frac{4}{9}\right)^4 \left(\frac{5}{9}\right)^5 \right)$
- (E) $\left(\frac{4}{9}\right)^5 \left(\frac{5}{9}\right)^4$

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 10\}$ have?

- (A) 17
- (B) 5
- (C) 8
- (D) 14
- (E) 11

Questions 37–39

37. Evaluate $\lim_{x \rightarrow 0} \frac{e^{5x} - \sum_{j=0}^3 (5x)^j / j!}{x^4}$.

- (A) $\frac{5^4}{4!}$
- (B) $24 \left(\frac{5^4}{4!} \right)$
- (C) $4 \left(\frac{5^4}{4!} \right)$
- (D) $-\frac{5^4}{4!}$
- (E) $\frac{1}{5} \left(\frac{5^4}{4!} \right)$

38. A sequence satisfies $u_0 = 19$ and $u_{k+1} = 2u_k + 7$ for all $k \geq 0$. Find u_5 .

- (A) 824
- (B) 827
- (C) 825
- (D) 823
- (E) 826

39. If $y'' - 10y' + 25y = 4e^{5x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $1 + 2e^5$
- (B) $4e^5$
- (C) $-2e^5$
- (D) $2e^5$
- (E) e^5

Questions 40–42

40. What is the coefficient of z^{-1} in the Laurent expansion of e^{5z}/z^3 about zero?
- (A) $\frac{27}{2}$
(B) $\frac{25}{2}$
(C) $\frac{25}{4}$
(D) 25
(E) $\frac{29}{2}$
41. For $f(x, y) = x^2 + 5xy + 4y^2$, what is its maximum directional derivative at $(1, -1)$?
- (A) -3
(B) 18
(C) 6
(D) $3\sqrt{2}$
(E) $6\sqrt{2}$
42. How many maximal proper subgroups does the cyclic group of order 276 have?
- (A) 5
(B) 1
(C) 4
(D) 2
(E) 3

Questions 43–45

43. For $D = \{(x, y) : x^2 + y^2 \leq 5\}$, evaluate $\iint_D (x^2 + y^2)^3 dA$.
- (A) $\frac{625\pi}{2}$
(B) $\frac{625\pi}{7}$
(C) $\frac{625\pi}{4}$
(D) 625π
(E) $\frac{125\pi}{4}$
44. An entire function satisfies $|f(z)| \leq C(1 + |z|^5)$ for some $C > 0$. What is the largest possible degree of f ?
- (A) 6
(B) 5
(C) 7
(D) 4
(E) 3
45. For the curve $x = t^4$, $y = t^5 + 5t$, find d^2y/dx^2 at $t = 1$.
- (A) $\frac{5}{2}$
(B) $-\frac{5}{2}$
(C) $\frac{5}{4}$
(D) $-\frac{5}{4}$
(E) $-\frac{5}{8}$

Questions 46–48

46. How many residues x modulo 48 satisfy $4x \equiv 24 \pmod{48}$ and $2x \equiv 0 \pmod{48}$?

- (A) 2
- (B) 1
- (C) 3
- (D) 4
- (E) 0

47. Evaluate $\iiint_B 5z^2(x^2 + y^2 + z^2)^4 dV$, where B is the unit ball.

- (A) $\frac{20\pi}{39}$
- (B) $\frac{10\pi}{39}$
- (C) $\frac{40\pi}{39}$
- (D) $1 + \frac{20\pi}{39}$
- (E) $-\frac{20\pi}{39}$

48. A connected planar graph has 13 vertices and 21 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 16
- (B) 4
- (C) 7
- (D) 10
- (E) 13

Questions 49–51

49. Evaluate $\int_C (4x^3 + 5y) dx + (5x + 4y^3) dy$, where C is $r(t) = (t, 4t^2)$, $0 \leq t \leq 1$.

- (A) 278
- (B) -277
- (C) 277
- (D) 554
- (E) $\frac{277}{2}$

50. The field K has 5^5 elements. Find the largest possible multiplicative order of a nonzero element of K .

- (A) $5^5 - 1$
- (B) $5^5 + 1$
- (C) $5^6 - 1$
- (D) $5^4 - 1$
- (E) 5^5

51. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 5x$ and has density $\rho(x, y) = x^2y$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{5}{6}$
- (B) $\frac{11}{6}$
- (C) $\frac{5}{3}$
- (D) $-\frac{5}{6}$
- (E) $\frac{5}{12}$

Questions 52–54

52. How many integers in $\{1, \dots, 60\}$ are divisible by 2 or 3?

- (A) 40
- (B) 66
- (C) 14
- (D) 27
- (E) 53

53. Evaluate $\int_0^\infty x^4 e^{-5x} dx$.

- (A) $\frac{1}{5} \left(\frac{4!}{5^5} \right)$
- (B) $\frac{4!}{5^5}$
- (C) $5 \left(\frac{4!}{5^5} \right)$
- (D) $\frac{1}{4} \left(\frac{4!}{5^5} \right)$
- (E) $3125 \left(\frac{4!}{5^5} \right)$

54. Let $G = \mathbb{Z}/144\mathbb{Z} \times \mathbb{Z}/100\mathbb{Z}$. Find the order of G/H , where $H = \{g \in G : 4g = 0\}$.

- (A) 902
- (B) 900
- (C) 901
- (D) 899
- (E) 898

Questions 55–57

55. For $x > 0$, $x^2y'' + (1 - 9)xy' + 20y = 0$, $y(1) = 0$, $y'(1) = 5$. Find $y(e)$.

- (A) $-e^4 + e^5$
- (B) $\frac{e^5}{9}$
- (C) $-5(1 - e)e^4$
- (D) $e^4 + e^5$
- (E) $5(1 - e)e^4$

56. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 10 have no adjacent 1s?

- (A) F_{11}
- (B) F_{14}
- (C) F_{13}
- (D) F_{10}
- (E) F_{12}

57. Under $x = u + 5v$, $y = u - 4v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) $\frac{1}{9}$
- (B) 81
- (C) 9
- (D) 1
- (E) 20

Questions 58–60

58. How many group homomorphisms $\mathbb{Z}/144\mathbb{Z} \rightarrow \mathbb{Z}/100\mathbb{Z}$ are there?

- (A) 3
- (B) 5
- (C) 2
- (D) 6
- (E) 4

59. Let $F(x) = \int_0^x (x-t)^3 e^{5t} dt$. Find $F^{(4)}(0)$.

- (A) $\frac{1}{6}$
- (B) 30
- (C) 24
- (D) 0
- (E) 6

60. Apply one Newton iteration to solve $x^2 - 10 = 0$, starting at $x_0 = 1$. What is x_1 ?

- (A) $\frac{11}{2}$
- (B) $\frac{15}{2}$
- (C) $\frac{13}{2}$
- (D) 11
- (E) $\frac{11}{4}$

Questions 61–63

61. Let $F(x) = \int_0^{x^4} e^{5t} dt$. Find $F'(1)$.

- (A) e^{20}
- (B) $4e^5$
- (C) e^5
- (D) $20e^5$
- (E) $\frac{e^5}{4}$

62. How many 1-dimensional subspaces does $\mathbb{F}_7^5$ have?

- (A) $\frac{7^6 - 1}{7 - 1}$
- (B) $\frac{7^7 - 1}{7 - 1}$
- (C) $\frac{7^5 - 1}{7 - 1}$
- (D) $\frac{7^4 - 1}{7 - 1}$
- (E) $\frac{7^8 - 1}{7 - 1}$

63. Let $\mathbf{F} = (5y^4, z^4, x^4)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (1, 1, 4)/\sqrt{18}$.

- (A) $1 - \frac{44\sqrt{2}}{3}$
- (B) $-\frac{44\sqrt{2}}{3}$
- (C) $\frac{44\sqrt{2}}{3}$
- (D) $-\frac{22\sqrt{2}}{3}$
- (E) $-\frac{88\sqrt{2}}{3}$

Questions 64–66

64. Bisection starts with an interval of length 2^{10} . After 12 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{1}{24}$
- (B) $\frac{9}{16}$
- (C) $\frac{7}{8}$
- (D) $\frac{1}{8}$
- (E) $\frac{1}{16}$

65. Find the curvature of the helix $r(t) = (5 \cos t, 5 \sin t, 4t)$.

- (A) $\frac{46}{41}$
- (B) $-\frac{5}{41}$
- (C) $\frac{5}{41}$
- (D) $\frac{10}{41}$
- (E) $\frac{5}{82}$

66. How many monic irreducible polynomials of degree 3 are there over $\mathbb{F}_7$?

- (A) 114
- (B) 113
- (C) 112
- (D) 110
- (E) 111

Answer Key

Practice Test 23 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	D	23	B	45	E
2	C	24	C	46	E
3	E	25	E	47	A
4	D	26	D	48	D
5	A	27	A	49	C
6	E	28	B	50	A
7	A	29	C	51	A
8	D	30	B	52	A
9	A	31	B	53	B
10	D	32	D	54	B
11	A	33	D	55	C
12	C	34	A	56	E
13	E	35	B	57	C
14	E	36	E	58	E
15	A	37	A	59	E
16	E	38	C	60	A
17	B	39	D	61	B
18	C	40	B	62	C
19	D	41	D	63	B
20	C	42	E	64	D
21	B	43	C	65	C
22	D	44	B	66	C

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