

GRE Mathematics Practice Test 24

Bridge to Standard

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A bridge toward standard practice, with routine multi-step questions and a smaller number of challenging concepts. This is cycle 4 of 19, position 4 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. For $y' = 5y(1 - y/3)$, what is the derivative of the right-hand side with respect to y at the positive equilibrium?

- (A) -5
- (B) -10
- (C) -4
- (D) 5
- (E) $-\frac{5}{2}$

2. A 3×3 matrix A has characteristic polynomial $(t - 20)(t + 1)(t - 2)$. What is $\text{tr}(A^2)$?

- (A) 407
- (B) 403
- (C) 405
- (D) 406
- (E) 404

3. The region $0 \leq x \leq 1$, $0 \leq y \leq 5x^3$ is revolved about the x -axis. Find its volume.

- (A) $1 + \frac{25\pi}{7}$
- (B) $\frac{50\pi}{7}$
- (C) $\frac{25\pi}{7}$
- (D) $-\frac{25\pi}{7}$
- (E) $\frac{25\pi}{14}$

Questions 4–6

4. A uniformly random subset of 11 objects is selected from a collection with 12 red and 13 blue objects. If X is the number of red objects selected, find $E[X]$.

- (A) $\frac{182}{25}$
- (B) $\frac{264}{25}$
- (C) $\frac{157}{25}$
- (D) $\frac{66}{25}$
- (E) $\frac{132}{25}$

5. Evaluate $\int_0^1 x^4(\log x)^2 dx$.

- (A) $\frac{(-1)^2 2!}{5^3}$
- (B) $-\frac{(-1)^2 2!}{5^3}$
- (C) $\frac{1}{3} \left(\frac{(-1)^2 2!}{5^3} \right)$
- (D) $5 \left(\frac{(-1)^2 2!}{5^3} \right)$
- (E) $\frac{25}{2} \left(\frac{(-1)^2 2!}{5^3} \right)$

6. Let N be the 26×26 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\text{rank } N^2$.

- (A) 23
- (B) 22
- (C) 26
- (D) 24
- (E) 25

Questions 7–9

7. Evaluate $\int_0^1 x^5(1-x)^2 dx$.

- (A) $8 \left(\frac{5! 2!}{8!} \right)$
- (B) $\frac{5! 2!}{8!}$
- (C) $6 \left(\frac{5! 2!}{8!} \right)$
- (D) $\frac{1}{3} \left(\frac{5! 2!}{8!} \right)$
- (E) $\frac{28}{3} \left(\frac{5! 2!}{8!} \right)$

8. X, Y are independent Poisson variables with means 11 and 12. Given $X + Y = 13$, find $\text{Var}(X | X + Y)$.

- (A) $\frac{858}{529}$
- (B) $\frac{2245}{529}$
- (C) $\frac{2774}{529}$
- (D) $\frac{1716}{529}$
- (E) $\frac{3432}{529}$

9. Evaluate $\int_0^\infty x^3 e^{-5x} dx$.

- (A) $5 \left(\frac{3!}{5^4} \right)$
- (B) $\frac{3!}{5^4}$
- (C) $625 \left(\frac{3!}{5^4} \right)$
- (D) $\frac{1}{3} \left(\frac{3!}{5^4} \right)$
- (E) $\frac{1}{5} \left(\frac{3!}{5^4} \right)$

Questions 10–12

10. Let A be the 26×26 matrix whose diagonal entries are 23 and whose off-diagonal entries are 1. What is its smallest eigenvalue?

- (A) 21
- (B) 24
- (C) 20
- (D) 23
- (E) 22

11. Evaluate $\int_0^{\pi/2} \sin^7 t \cos^{11} t \, dt$.

- (A) $9 \left(\frac{3! 5!}{2 \cdot 9!} \right)$
- (B) $\frac{3! 5!}{2 \cdot 9!}$
- (C) $\frac{1}{2} \left(\frac{3! 5!}{2 \cdot 9!} \right)$
- (D) $2 \left(\frac{3! 5!}{2 \cdot 9!} \right)$
- (E) $56 \left(\frac{3! 5!}{2 \cdot 9!} \right)$

12. X, Y are independent, with variances 11 and 12. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 9
- (B) 6
- (C) 15
- (D) 3
- (E) 12

Questions 13–15

13. Let $f(x, y) = x^2 + 10xy + (28)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 12
- (B) 112
- (C) 28
- (D) 3
- (E) 6

14. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 100$ and $x + y = 25$. Find xy .

- (A) 98
- (B) 101
- (C) 100
- (D) 99
- (E) 102

15. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(5j)!}{(j!)^5} x^{3j}$?

- (A) $5^{-\frac{3}{5}}$
- (B) $5^{\frac{5}{3}}$
- (C) $5^{-\frac{5}{3}}$
- (D) $\frac{1}{5}$
- (E) 5^{-5}

Questions 16–18

16. X has density cx^{11} on $0 < x < 1$ and zero elsewhere. Find the normalizing constant c .

- (A) 8
- (B) 4
- (C) 20
- (D) 12
- (E) 16

17. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 5z^2 = x + 3y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) -30
- (B) -15
- (C) -3
- (D) 30
- (E) -10

18. A sequence satisfies $u_0 = 20$ and $u_{k+1} = 1u_k + 5$ for all $k \geq 0$. Find u_4 .

- (A) 41
- (B) 42
- (C) 38
- (D) 39
- (E) 40

Questions 19–21

19. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-5)^j}{j^{\frac{2}{3}} (\log j)^m}$ converge absolutely at $x = 4$? Give the least such m , or 0 if none exists.

- (A) 0
- (B) 3
- (C) 1
- (D) 2
- (E) 4

20. Independent exponential waiting times X, Y have rates 11 and 13. Find $P(X < Y)$.

- (A) $\frac{11}{48}$
- (B) $\frac{11}{72}$
- (C) $\frac{35}{48}$
- (D) $\frac{11}{24}$
- (E) $\frac{13}{24}$

21. For $f(x, y) = x^2 + 5xy + 3y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) 4
- (B) $\sqrt{10}$
- (C) -3
- (D) 10
- (E) -1

Questions 22–24

- 22.** A linear map $T : \mathbb{R}^{27} \rightarrow \mathbb{R}^{29}$ has rank 2. What is its nullity?
- (A) 24
 - (B) 27
 - (C) 26
 - (D) 25
 - (E) 23
- 23.** Under $x = u + 5v$, $y = u - 3v$, what is the area of the image of $0 \leq u, v \leq 1$?
- (A) 64
 - (B) 15
 - (C) 2
 - (D) $\frac{1}{8}$
 - (E) 8
- 24.** For which real a does $f_m(x) = x^{11m}/m^a$ converge uniformly to zero on $[0, 1]$?
- (A) $a > 1$
 - (B) $a > 0$
 - (C) All real a
 - (D) $a < 0$
 - (E) $a = 0$

Questions 25–27

25. For the curve $x = t^3$, $y = t^4 + 5t$, find d^2y/dx^2 at $t = 1$.

- (A) -2
- (B) $-\frac{5}{3}$
- (C) $\frac{14}{9}$
- (D) $-\frac{2}{3}$
- (E) 3

26. What is the dimension of the real vector space of 26×26 symmetric matrices?

- (A) 352
- (B) 353
- (C) 351
- (D) 350
- (E) 349

27. For $x > 0$, $x^2y'' + (1 - 8)xy' + 15y = 0$, $y(1) = 0$, $y'(1) = 5$. Find $y(e)$.

- (A) $\frac{e^5}{8}$
- (B) $-\frac{5(1 - e^2)e^3}{2}$
- (C) $-e^3 + e^5$
- (D) $\frac{5(1 - e^2)e^3}{2}$
- (E) $e^3 + e^5$

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{11p}]$ converge?

- (A) $p > 1/11$
- (B) $p > 0$
- (C) $p \geq 1/11$
- (D) $p < 1/11$
- (E) All real p

29. If $y' + 5y = 3e^{-5x}$ and $y(0) = 5$, find $y(1)$.

- (A) $\frac{8}{e^5} + 1$
- (B) $\frac{16}{e^5}$
- (C) $\frac{8}{e^5}$
- (D) $-\frac{8}{e^5}$
- (E) $\frac{4}{e^5}$

30. Let $A = \begin{bmatrix} 1 & 20 & 0 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 2)$ entry of A^4 ?

- (A) 81
- (B) 78
- (C) 82
- (D) 79
- (E) 80

Questions 31–33

31. Find the directional derivative of $f(x, y, z) = x^2 + 5y^2 + 3z^2$ at $(1, 1, 1)$ in the direction $(1, 5, 0)$.

- (A) $4\sqrt{26}$
- (B) $1 + 2\sqrt{26}$
- (C) $-2\sqrt{26}$
- (D) $2\sqrt{26}$
- (E) $\sqrt{26}$

32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{11 + 1/k : k \geq 1\}$ have?

- (A) 4
- (B) 0
- (C) 2
- (D) 1
- (E) 3

33. Evaluate $\int_0^1 \int_0^x x^5 y^3 dy dx$.

- (A) $\frac{1}{10}$
- (B) $\frac{1}{36}$
- (C) $\frac{1}{40}$
- (D) $\frac{1}{50}$
- (E) $\frac{1}{24}$

Questions 34–36

34. An invertible 26×26 matrix A has determinant 2. Find $\det(A^T A)$.

- (A) 6
- (B) 2
- (C) 3
- (D) 5
- (E) 4

35. Evaluate $\int_0^1 \int_x^1 5y^3 \, dy \, dx$.

- (A) $\frac{5}{2}$
- (B) $\frac{5}{4}$
- (C) $\frac{1}{4}$
- (D) $\frac{5}{3}$
- (E) 1

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 11\}$ have?

- (A) 16
- (B) 12
- (C) 20
- (D) 8
- (E) 4

Questions 37–39

37. The function $y(x)$ satisfies $y^3 + 5y = 2x$ and $y(0) = 0$. Find $y'(0)$.

- (A) $\frac{1}{4}$
- (B) 0
- (C) $-\frac{2}{5}$
- (D) $\frac{2}{5}$
- (E) 5

38. On the space P_{27} of real polynomials of degree at most 27, define $T(p) = p'$. What is the rank of T ?

- (A) 28
- (B) 27
- (C) 25
- (D) 26
- (E) 29

39. Solve $y' + 5y = x^3e^{-5x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{e^5}{4}$
- (B) $\frac{5}{4e^5}$
- (C) e^{-5}
- (D) $\frac{1}{4e^5}$
- (E) $\frac{1}{3e^5}$

Questions 40–42

40. What is the coefficient of z^{-1} in the Laurent expansion of e^{6z}/z^3 about zero?

- (A) 12
- (B) 24
- (C) 30
- (D) 18
- (E) 6

41. Find the coefficient of x^3 in $e^{2x} \log(1 + 5x)$.

- (A) $\frac{95}{3}$
- (B) $\frac{80}{3}$
- (C) $\frac{125}{3}$
- (D) 160
- (E) $-\frac{80}{3}$

42. Subspaces U, V of $\mathbb{R}^{28}$ have dimensions 3 and 4. What is the largest possible value of $\dim(U \cap V)$?

- (A) 4
- (B) 5
- (C) 1
- (D) 2
- (E) 3

Questions 43–45

43. Let $f(x, y) = e^{5x+3y}$. Find $\partial^3 f / \partial x^2 \partial y$ at $(0, 0)$.

- (A) $\frac{5}{3}(3 \cdot 5^2)$
- (B) $3 \cdot 5^2$
- (C) $\frac{1}{3}(3 \cdot 5^2)$
- (D) $\frac{9}{25}(3 \cdot 5^2)$
- (E) $\frac{3}{5}(3 \cdot 5^2)$

44. An entire function satisfies $|f(z)| \leq C(1 + |z|^6)$ for some $C > 0$. What is the largest possible degree of f ?

- (A) 2
- (B) 8
- (C) 10
- (D) 4
- (E) 6

45. If $y' = 5y$, $y(0) = 1$, what is the coefficient of x^3 in the Maclaurin series of y ?

- (A) $\frac{125}{3}$
- (B) $-\frac{125}{6}$
- (C) $\frac{131}{6}$
- (D) $\frac{125}{6}$
- (E) $\frac{6}{125}$

Questions 46–48

46. How many elements z of $\mathbb{F}_{3^5}$ satisfy $z^3 = z$?

- (A) 1
- (B) 2
- (C) 4
- (D) 5
- (E) 3

47. Evaluate $\lim_{x \rightarrow 0} (1 + 5x)^{1/(2x)}$.

- (A) 5
- (B) e^6
- (C) e^{25}
- (D) $e^{\frac{2}{5}}$
- (E) $e^{\frac{5}{2}}$

48. A connected planar graph has 14 vertices and 23 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 11
- (B) 14
- (C) 5
- (D) 8
- (E) 17

Questions 49–51

49. Find the coefficient of x^3 in the Taylor series of $(1 - 5x)^{-2}$ at zero.

- (A) $\frac{3}{4}(4 \cdot 5^3)$
- (B) $4 \cdot 5^3$
- (C) $-4 \cdot 5^3$
- (D) $\frac{1}{5}(4 \cdot 5^3)$
- (E) $\frac{1}{4}(4 \cdot 5^3)$

50. How many generators does the cyclic group of order 7 have?

- (A) 6
- (B) 7
- (C) 8
- (D) 5
- (E) 4

51. Let $F(x) = \int_0^x (x - t)^2 e^{5t} dt$. Find $F^{(3)}(0)$.

- (A) 10
- (B) $\frac{1}{2}$
- (C) 2
- (D) 0
- (E) 6

Questions 52–54

52. How many integers in $\{1, \dots, 66\}$ are divisible by 2 or 3?

- (A) 44
- (B) 58
- (C) 72
- (D) 16
- (E) 30

53. Let C be $x^2/9 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-y) dx + (5x) dy$.

- (A) 18π
- (B) 9π
- (C) 36π
- (D) $1 + 18\pi$
- (E) -18π

54. Let $R = (\mathbb{Z}/72\mathbb{Z})/([9])$, where $([9])$ is the ideal generated by $[9]$. How many elements does R have?

- (A) 9
- (B) 7
- (C) 10
- (D) 8
- (E) 11

Questions 55–57

55. Evaluate $\iiint_{x^2+y^2+z^2 \leq 5} (x^2 + y^2 + z^2)^2 dV$.

- (A) $\frac{1}{2} \left(\frac{4\pi 5^{\frac{7}{2}}}{7} \right)$
- (B) $\frac{7}{25} \left(\frac{4\pi 5^{\frac{7}{2}}}{7} \right)$
- (C) $\frac{4\pi 5^{\frac{7}{2}}}{7}$
- (D) $7 \left(\frac{4\pi 5^{\frac{7}{2}}}{7} \right)$
- (E) $\frac{1}{3} \left(\frac{4\pi 5^{\frac{7}{2}}}{7} \right)$

56. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 11 have no adjacent 1s?

- (A) F_{12}
- (B) F_{15}
- (C) F_{13}
- (D) F_{11}
- (E) F_{14}

57. What is the maximum of x^3y^5 over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\frac{15}{64}$
- (B) $\left(\frac{3}{8}\right)^3 \left(\frac{5}{8}\right)^5$
- (C) $8 \left(\left(\frac{3}{8}\right)^3 \left(\frac{5}{8}\right)^5 \right)$
- (D) $\frac{1}{2^8}$
- (E) $\left(\frac{3}{8}\right)^5 \left(\frac{5}{8}\right)^3$

Questions 58–60

58. What is the largest integer k such that 2^k divides $29!$?

- (A) 23
- (B) 26
- (C) 25
- (D) 27
- (E) 24

59. Find the total outward flux of $\mathbf{F} = (5x^3, y^3, z^3)$ through the boundary of $[0, 1]^3$.

- (A) 7
- (B) 8
- (C) $\frac{7}{2}$
- (D) 14
- (E) -7

60. Apply one Newton iteration to solve $x^2 - 11 = 0$, starting at $x_0 = 1$. What is x_1 ?

- (A) 10
- (B) 2
- (C) 8
- (D) 4
- (E) 6

Questions 61–63

61. Solve $y'' - 13y' + 40y = 0$ with $y(0) = 1$, $y'(0) = 0$. Find $y(1)$.

- (A) $\frac{(8 - 5e^3)e^5}{3}$
- (B) $\frac{(8 - 5e^3)e^5}{3} + 1$
- (C) $\frac{(8 - 5e^3)e^5}{6}$
- (D) $\frac{(-8 + 5e^3)e^5}{3}$
- (E) $\frac{2(8 - 5e^3)e^5}{3}$

62. In the ring $\mathbb{Z}/144\mathbb{Z}$, how many elements are units?

- (A) 50
- (B) 47
- (C) 46
- (D) 48
- (E) 49

63. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 5x$ and has density $\rho(x, y) = x$. Find its centroid coordinate $\bar{x}$.

- (A) $-\frac{3}{4}$
- (B) $\frac{7}{4}$
- (C) $\frac{3}{8}$
- (D) $\frac{3}{2}$
- (E) $\frac{3}{4}$

Questions 64–66

64. Bisection starts with an interval of length 2^{11} . After 13 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{1}{24}$
- (B) $\frac{1}{8}$
- (C) $\frac{9}{16}$
- (D) $\frac{7}{8}$
- (E) $\frac{1}{16}$

65. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(5x) - \sum_{j=0}^1 (-1)^j (5x)^{2j+1} / (2j+1)!}{x^5}$.

- (A) $5 \left(\frac{(-1)^2 5^5}{5!} \right)$
- (B) $-\frac{(-1)^2 5^5}{5!}$
- (C) $2 \left(\frac{(-1)^2 5^5}{5!} \right)$
- (D) $\frac{(-1)^2 5^5}{5!}$
- (E) $\frac{1}{5} \left(\frac{(-1)^2 5^5}{5!} \right)$

66. A permutation consists of disjoint cycles of lengths 8 and 6 and fixes all other points. What is its order?

- (A) 23
- (B) 26
- (C) 24
- (D) 22
- (E) 25

Answer Key

Practice Test 24 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	A	23	E	45	D
2	C	24	B	46	E
3	C	25	D	47	E
4	E	26	C	48	A
5	A	27	B	49	B
6	D	28	A	50	A
7	B	29	C	51	C
8	D	30	E	52	A
9	B	31	D	53	A
10	E	32	C	54	A
11	B	33	C	55	C
12	A	34	E	56	C
13	A	35	E	57	B
14	C	36	B	58	C
15	C	37	D	59	A
16	D	38	B	60	E
17	A	39	D	61	A
18	E	40	D	62	D
19	A	41	B	63	E
20	D	42	E	64	B
21	B	43	B	65	D
22	D	44	E	66	C

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