

GRE Mathematics Practice Test 25

Standard-Level Target

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A mixed standard-level target, intended to approximate public GRE Mathematics practice in scope and overall demand. This is cycle 4 of 19, position 5 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Evaluate $\iiint_{x^2+y^2+z^2 \leq 5} (x^2 + y^2 + z^2)^1 dV$.

- (A) $\frac{4\pi 5^{\frac{5}{2}}}{5}$
- (B) $\frac{1}{3} \left(\frac{4\pi 5^{\frac{5}{2}}}{5} \right)$
- (C) $\frac{1}{2} \left(\frac{4\pi 5^{\frac{5}{2}}}{5} \right)$
- (D) $5 \left(\frac{4\pi 5^{\frac{5}{2}}}{5} \right)$
- (E) $2 \left(\frac{4\pi 5^{\frac{5}{2}}}{5} \right)$

2. A linear map $T : \mathbb{R}^{28} \rightarrow \mathbb{R}^{30}$ has rank 3. What is its nullity?

- (A) 23
- (B) 25
- (C) 26
- (D) 24
- (E) 27

3. Evaluate $\int_0^1 \int_0^x x^5 y^2 dy dx$.

- (A) $\frac{1}{36}$
- (B) $\frac{1}{24}$
- (C) $\frac{1}{27}$
- (D) $\frac{1}{9}$
- (E) $\frac{1}{18}$

Questions 4–6

4. A uniformly random subset of 12 objects is selected from a collection with 13 red and 14 blue objects. If X is the number of red objects selected, find $E[X]$.

- (A) $\frac{61}{9}$
- (B) $\frac{70}{9}$
- (C) $\frac{104}{9}$
- (D) $\frac{52}{9}$
- (E) $\frac{26}{9}$

5. If $y' + 5y = 2e^{-5x}$ and $y(0) = 6$, find $y(1)$.

- (A) $\frac{8}{e^5} + 1$
- (B) $-\frac{8}{e^5}$
- (C) $\frac{16}{e^5}$
- (D) $\frac{8}{e^5}$
- (E) $\frac{4}{e^5}$

6. What is the dimension of the real vector space of 27×27 symmetric matrices?

- (A) 380
- (B) 377
- (C) 379
- (D) 376
- (E) 378

Questions 7–9

7. For $y' = 5y(1 - y/2)$, what is the derivative of the right-hand side with respect to y at the positive equilibrium?

- (A) 5
- (B) -5
- (C) -10
- (D) -4
- (E) $-\frac{5}{2}$

8. X, Y are independent Poisson variables with means 12 and 13. Given $X+Y = 14$, find $E[X \mid X+Y]$.

- (A) $\frac{336}{25}$
- (B) $\frac{84}{25}$
- (C) $\frac{193}{25}$
- (D) $\frac{168}{25}$
- (E) $\frac{218}{25}$

9. Solve $y'' - 12y' + 35y = 0$ with $y(0) = 1$, $y'(0) = 0$. Find $y(1)$.

- (A) $\frac{(7 - 5e^2)e^5}{4}$
- (B) $\frac{(7 - 5e^2)e^5}{2}$
- (C) $\frac{(7 - 5e^2)e^5}{2} + 1$
- (D) $(7 - 5e^2)e^5$
- (E) $\frac{(-7 + 5e^2)e^5}{2}$

Questions 10–12

10. Let $A = \begin{bmatrix} 1 & 21 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 2)$ entry of A^5 ?

- (A) 103
- (B) 104
- (C) 106
- (D) 105
- (E) 107

11. If $y'' + 4y = 0$, $y(0) = 5$, and $y'(0) = 6$, find $y(\pi/4)$.

- (A) -3
- (B) 3
- (C) 4
- (D) 6
- (E) $\frac{3}{2}$

12. X, Y are independent, with variances 12 and 13. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 7
- (B) 16
- (C) 4
- (D) 10
- (E) 13

Questions 13–15

13. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(5j)!}{(j!)^5} x^{2j}$?

- (A) 5^{-5}
- (B) $5^{-\frac{5}{2}}$
- (C) $5^{-\frac{2}{5}}$
- (D) $\frac{1}{5}$
- (E) $5^{\frac{5}{2}}$

14. An invertible 27×27 matrix A has determinant 3. Find $\det(A^T A)$.

- (A) 7
- (B) 11
- (C) 9
- (D) 10
- (E) 8

15. What is the maximum of $x^2 y^5$ over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\left(\frac{2}{7}\right)^5 \left(\frac{5}{7}\right)^2$
- (B) $\frac{1}{2^7}$
- (C) $\left(\frac{2}{7}\right)^2 \left(\frac{5}{7}\right)^5$
- (D) $\frac{16807}{1250} \left(\left(\frac{2}{7}\right)^2 \left(\frac{5}{7}\right)^5\right)$
- (E) $7 \left(\left(\frac{2}{7}\right)^2 \left(\frac{5}{7}\right)^5\right)$

Questions 16–18

16. X has density cx^{12} on $0 < x < 1$ and zero elsewhere. Find the normalizing constant c .
- (A) 21
(B) 17
(C) 5
(D) 9
(E) 13
17. Let C be the unit circle $x^2 + y^2 = 1$, $z = 2$, counterclockwise viewed from above. Evaluate $\int_C -5y(x^2 + y^2) dx + (5x(x^2 + y^2)) dy$.
- (A) $1 + 10\pi$
(B) 20π
(C) 5π
(D) 10π
(E) -10π
18. On the space P_{28} of real polynomials of degree at most 28, define $T(p) = p'$. What is the rank of T ?
- (A) 28
(B) 30
(C) 29
(D) 27
(E) 26

Questions 19–21

19. For $x > 0$, $xy' = 5y$ and $y(1) = 1$. Find $y(e^2)$.

- (A) $1 + e^{10}$
- (B) e^{10}
- (C) $-e^{10}$
- (D) $2e^{10}$
- (E) $\frac{e^{10}}{2}$

20. Independent exponential waiting times X, Y have rates 12 and 14. Find $P(X < Y)$.

- (A) $\frac{3}{13}$
- (B) $\frac{2}{13}$
- (C) $\frac{19}{26}$
- (D) $\frac{6}{13}$
- (E) $\frac{7}{13}$

21. Evaluate $\int_0^1 x^4(\log x)^1 dx$.

- (A) $\frac{(-1)^1 1!}{5^2}$
- (B) $-\frac{(-1)^1 1!}{5^2}$
- (C) $2 \left(\frac{(-1)^1 1!}{5^2} \right)$
- (D) $5 \left(\frac{(-1)^1 1!}{5^2} \right)$
- (E) $\frac{1}{2} \left(\frac{(-1)^1 1!}{5^2} \right)$

Questions 22–24

22. Subspaces U, V of $\mathbb{R}^{29}$ have dimensions 2 and 3. What is the largest possible value of $\dim(U \cap V)$?
- (A) 0
(B) 3
(C) 4
(D) 2
(E) 1
23. For $\mathbf{F} = (5x^2, y^2, z^2)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.
- (A) 32
(B) 8
(C) -16
(D) 16
(E) 17
24. For which real a does $f_m(x) = x^{12m}/m^a$ converge uniformly to zero on $[0, 1]$?
- (A) $a > 1$
(B) All real a
(C) $a > 0$
(D) $a < 0$
(E) $a = 0$

Questions 25–27

25. Let C be $x^2/4 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-y) dx + (5x) dy$.
- (A) 24π
(B) -12π
(C) 6π
(D) $1 + 12\pi$
(E) 12π
26. Let V consist of real polynomials of degree at most 8 that vanish at 21, 22, and 24. Find $\dim V$.
- (A) 4
(B) 6
(C) 8
(D) 5
(E) 7
27. Two solutions of $y'' + 5x^1y' + 2y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.
- (A) $-\frac{1}{e^{\frac{5}{2}}}$
(B) $\frac{1}{2e^{\frac{5}{2}}}$
(C) $\frac{-5}{e^{\frac{5}{2}}}$
(D) $e^{-\frac{5}{2}} + 1$
(E) $e^{-\frac{5}{2}}$

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{12p}]$ converge?

- (A) $p < 1/12$
- (B) All real p
- (C) $p > 1/12$
- (D) $p \geq 1/12$
- (E) $p > 0$

29. For $f(x, y) = x^2 + 5xy + 2y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) 10
- (B) 4
- (C) -3
- (D) 1
- (E) $\sqrt{10}$

30. The roots r, s of $x^2 - 24x + 63 = 0$ are counted with multiplicity. Find $r^2 + s^2$.

- (A) 450
- (B) 449
- (C) 451
- (D) 448
- (E) 452

Questions 31–33

31. For $f(x) = x^2e^{5x}$, find $f^{(4)}(0)$.

- (A) $\frac{1}{2} \left(\frac{4! 5^2}{2} \right)$
- (B) $2 \left(\frac{4! 5^2}{2} \right)$
- (C) $\frac{2}{5} \left(\frac{4! 5^2}{2} \right)$
- (D) $\frac{1}{12} \left(\frac{4! 5^2}{2} \right)$
- (E) $\frac{4! 5^2}{2}$

32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{12 + 1/k : k \geq 1\}$ have?

- (A) 2
- (B) 0
- (C) 1
- (D) 4
- (E) 3

33. If $y' = 5y$, $y(0) = 1$, what is the coefficient of x^2 in the Maclaurin series of y ?

- (A) 25
- (B) $\frac{27}{2}$
- (C) $\frac{25}{2}$
- (D) $\frac{25}{4}$
- (E) $-\frac{25}{2}$

Questions 34–36

34. For real t , let $M(t) = \begin{pmatrix} t-21 & 3 & 0 \\ 0 & t-21 & 1 \\ 0 & 0 & t-21 \end{pmatrix}$. At $t = 21$, what is $\text{rank } M(t)$?

- (A) 1
- (B) 3
- (C) 4
- (D) 2
- (E) 0

35. Find the directional derivative of $f(x, y, z) = x^2 + 5y^2 + 2z^2$ at $(1, 1, 1)$ in the direction $(1, 6, 0)$.

- (A) $1 + \frac{62\sqrt{37}}{37}$
- (B) $\frac{124\sqrt{37}}{37}$
- (C) $-\frac{62\sqrt{37}}{37}$
- (D) $\frac{62\sqrt{37}}{37}$
- (E) $\frac{31\sqrt{37}}{37}$

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 12\}$ have?

- (A) 13
- (B) 17
- (C) 5
- (D) 9
- (E) 21

Questions 37–39

37. For $D = \{(x, y) : x^2 + y^2 \leq 5\}$, evaluate $\iint_D (x^2 + y^2)^1 dA$.

- (A) $\frac{25\pi}{2}$
- (B) 25π
- (C) $\frac{5\pi}{2}$
- (D) $\frac{25\pi}{3}$
- (E) $\frac{25\pi}{4}$

38. Let $u = (4, 2)^T$ and $v = (1, 0)^T$. Find the squared norm of the orthogonal projection of v onto $\text{span}(u)$.

- (A) $\frac{23}{10}$
- (B) $\frac{9}{5}$
- (C) $\frac{13}{10}$
- (D) $\frac{3}{10}$
- (E) $\frac{4}{5}$

39. Find the coefficient of x^2 in $e^{1x} \log(1 + 5x)$.

- (A) $-\frac{5}{2}$
- (B) -15
- (C) $-\frac{15}{2}$
- (D) $-\frac{25}{2}$
- (E) $\frac{15}{2}$

Questions 40–42

40. What is the coefficient of z^{-1} in the Laurent expansion of e^{7z}/z^3 about zero?

- (A) $\frac{51}{2}$
- (B) 49
- (C) $\frac{49}{2}$
- (D) $\frac{53}{2}$
- (E) $\frac{49}{4}$

41. Evaluate $\int_0^{\pi/2} \sin^5 t \cos^{11} t \, dt$.

- (A) $21 \left(\frac{2! 5!}{2 \cdot 8!} \right)$
- (B) $8 \left(\frac{2! 5!}{2 \cdot 8!} \right)$
- (C) $\frac{2! 5!}{2 \cdot 8!}$
- (D) $2 \left(\frac{2! 5!}{2 \cdot 8!} \right)$
- (E) $\frac{1}{2} \left(\frac{2! 5!}{2 \cdot 8!} \right)$

42. An idempotent 27×27 real matrix P has rank 4. What is $\text{tr } P$?

- (A) 4
- (B) 3
- (C) 2
- (D) 5
- (E) 6

Questions 43–45

43. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(5+2j)(7+2j)}$.

- (A) $\frac{1}{10}$
- (B) $\frac{1}{7}$
- (C) $\frac{1}{5}$
- (D) $\frac{2}{5}$
- (E) $\frac{1}{2}$

44. An entire function satisfies $|f(z)| \leq 7$ for all complex z . What is $f'(1)$?

- (A) 4
- (B) 1
- (C) 3
- (D) 2
- (E) 0

45. Find the total outward flux of $\mathbf{F} = (5x^2, y^2, z^2)$ through the boundary of $[0, 1]^3$.

- (A) $\frac{7}{2}$
- (B) 14
- (C) -7
- (D) 7
- (E) 8

Questions 46–48

46. For $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $b = \begin{bmatrix} 21 \\ 3 \\ 24 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find x_2 .

- (A) 4
- (B) 3
- (C) 5
- (D) 1
- (E) 2

47. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-5)^j}{j^{\frac{1}{3}} (\log j)^m}$ converge absolutely at $x = 4$? Give the least such m , or 0 if none exists.

- (A) 2
- (B) 4
- (C) 0
- (D) 3
- (E) 1

48. A connected planar graph has 15 vertices and 25 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 12
- (B) 16
- (C) 4
- (D) 8
- (E) 20

Questions 49–51

49. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(5x) - \sum_{j=0}^0 (-1)^j (5x)^{2j+1} / (2j+1)!}{x^3}$.
- (A) $\frac{1}{5} \left(\frac{(-1)^1 5^3}{3!} \right)$
(B) $5 \left(\frac{(-1)^1 5^3}{3!} \right)$
(C) $3 \left(\frac{(-1)^1 5^3}{3!} \right)$
(D) $-\frac{(-1)^1 5^3}{3!}$
(E) $\frac{(-1)^1 5^3}{3!}$
50. Integers x satisfy $x \equiv 0 \pmod{7}$ and $x \equiv 0 \pmod{9}$. Find the least nonnegative solution.
- (A) 0
(B) 4
(C) 3
(D) 1
(E) 2
51. Find the upward flux of $\mathbf{F} = (0, 0, z)$ through $z = 5x^2 + 2y^2$, $0 \leq x, y \leq 1$.
- (A) $-\frac{7}{3}$
(B) $\frac{7}{3}$
(C) $\frac{7}{6}$
(D) $\frac{14}{3}$
(E) $\frac{10}{3}$

Questions 52–54

- 52.** How many integers in $\{1, \dots, 72\}$ are divisible by 2 or 3?
- (A) 80
 - (B) 64
 - (C) 48
 - (D) 32
 - (E) 16
- 53.** Evaluate $\int_C (2x + 5y) dx + (5x + 2y) dy$, where C is the line segment from $(0, 0)$ to $(6, 1)$.
- (A) 67
 - (B) -67
 - (C) 134
 - (D) $\frac{67}{2}$
 - (E) 68
- 54.** The symmetric group S_{27} acts on $\{1, \dots, 27\}$. What is the index of the stabilizer of 1?
- (A) 27
 - (B) 29
 - (C) 25
 - (D) 28
 - (E) 26

Questions 55–57

55. The solution of $y' = 5x^1y^2$, $y(0) = 1$, has its first positive singularity at which x ?
- (A) $\frac{2}{5}$
(B) $\frac{2\sqrt{10}}{5}$
(C) $\frac{\sqrt{5}}{5}$
(D) $\frac{\sqrt{10}}{2}$
(E) $\frac{\sqrt{10}}{5}$
56. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 12 have no adjacent 1s?
- (A) F_{12}
(B) F_{13}
(C) F_{14}
(D) F_{16}
(E) F_{15}
57. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 5x$ and has density $\rho(x, y) = 1$. Find its centroid coordinate $\bar{x}$.
- (A) $\frac{1}{3}$
(B) $\frac{5}{3}$
(C) $-\frac{2}{3}$
(D) $\frac{2}{3}$
(E) $\frac{4}{3}$

Questions 58–60

58. In $\mathbb{Z}/606\mathbb{Z}$, how many residues are idempotent ($x^2 = x$)?

- (A) 8
- (B) 7
- (C) 10
- (D) 6
- (E) 9

59. Solve $y' + 5y = x^2e^{-5x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{1}{2e^5}$
- (B) e^{-5}
- (C) $\frac{e^5}{3}$
- (D) $\frac{5}{3e^5}$
- (E) $\frac{1}{3e^5}$

60. Apply one Newton iteration to solve $x^2 - 12 = 0$, starting at $x_0 = 1$. What is x_1 ?

- (A) $\frac{13}{4}$
- (B) 13
- (C) $\frac{17}{2}$
- (D) $\frac{15}{2}$
- (E) $\frac{13}{2}$

Questions 61–63

61. For $x > 0$, $x^2 y'' + (1 - 7)xy' + 10y = 0$, $y(1) = 0$, $y'(1) = 5$. Find $y(e)$.

- (A) $\frac{e^5}{7}$
- (B) $e^2 + e^5$
- (C) $-\frac{5(1 - e^3)e^2}{3}$
- (D) $\frac{5(1 - e^3)e^2}{3}$
- (E) $-e^2 + e^5$

62. In the additive group $\mathbb{Z}/36\mathbb{Z}$, find the order of $[3]$.

- (A) 13
- (B) 12
- (C) 10
- (D) 14
- (E) 11

63. Evaluate $\int_0^{\log 2} e^x (e^x + 5)^2 dx$.

- (A) $-\frac{7^3 - 6^3}{3}$
- (B) $\frac{7^3 - 6^3}{3}$
- (C) $7^2 - 6^2$
- (D) $\frac{7^3}{3}$
- (E) $3\left(\frac{7^3 - 6^3}{3}\right)$

Questions 64–66

64. Bisection starts with an interval of length 2^{12} . After 14 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{7}{8}$
- (B) $\frac{1}{8}$
- (C) $\frac{1}{16}$
- (D) $\frac{1}{24}$
- (E) $\frac{9}{16}$

65. Let $F(x) = \int_0^x (x-t)^1 e^{5t} dt$. Find $F^{(2)}(0)$.

- (A) 0
- (B) $\frac{1}{2}$
- (C) 5
- (D) 2
- (E) 1

66. How many invertible 2×2 matrices over $\mathbb{F}_7$ are there?

- (A) $(7^2 - 7^1)(7^2 - 7^2)$
- (B) $(7^2 - 7^0)(7^2 - 7^1)$
- (C) $(7^2 - 7^0)$
- (D) $(7^2 - 7^0)(7^2 - 7^0)$
- (E) $7^2 7^2$

Answer Key

Practice Test 25 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	A	23	D	45	D
2	B	24	C	46	B
3	C	25	E	47	C
4	D	26	B	48	A
5	D	27	E	49	E
6	E	28	C	50	A
7	B	29	E	51	B
8	D	30	A	52	C
9	B	31	E	53	A
10	D	32	A	54	A
11	B	33	C	55	E
12	D	34	D	56	C
13	B	35	D	57	D
14	C	36	A	58	A
15	C	37	A	59	E
16	E	38	E	60	E
17	D	39	C	61	C
18	A	40	C	62	B
19	B	41	C	63	B
20	D	42	A	64	B
21	A	43	A	65	E
22	D	44	E	66	B

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