

GRE Mathematics Practice Test 26

Advanced Challenge

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

The most demanding form in this five-test cycle. Emphasis on connected ideas, non-routine reasoning, and careful conceptual distinctions. This is cycle 5 of 19, position 1 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 6z^2 = x + 6y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) -12
- (B) 72
- (C) -36
- (D) -6
- (E) -72

2. Let $u = (1, 2, \dots, 28)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $v^T B^{-1}u$.

- (A) $\frac{2033}{814}$
- (B) $\frac{405}{814}$
- (C) $\frac{406}{407}$
- (D) $\frac{1219}{814}$
- (E) $\frac{813}{407}$

3. The solution of $y' = 6x^6y^3$ has $y(0) = 1/7$. Find $y(1)$.

- (A) $\frac{\sqrt{2317}}{662}$
- (B) $\frac{2\sqrt{2317}}{331}$
- (C) $-\frac{\sqrt{2317}}{331}$
- (D) $\frac{\sqrt{2317}}{331} + 1$
- (E) $\frac{\sqrt{2317}}{331}$

Questions 4–6

4. $X_1, \dots, X_8$ are independent with common mean and variance 10. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 24
- (B) 47
- (C) 116
- (D) 93
- (E) 70

5. Let $f(x) = \log(1 + 6x)$. Find $f^{(6)}(0)$.

- (A) $6 \left((-1)^5 5! 6^6 \right)$
- (B) $\frac{1}{6} \left((-1)^5 5! 6^6 \right)$
- (C) $2 \left((-1)^5 5! 6^6 \right)$
- (D) $-(-1)^5 5! 6^6$
- (E) $(-1)^5 5! 6^6$

6. Let $D = \text{diag}(\underbrace{1, \dots, 1}_4, \underbrace{2, \dots, 2}_{24})$. Find the dimension of the space of real matrices X with $XD = DX$ and $\text{tr } X = 0$.

- (A) 591
- (B) 593
- (C) 590
- (D) 592
- (E) 589

Questions 7–9

7. Let C be $x^2/36 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-2y^3) dx + (\frac{x^3}{3}) dy$.
- (A) 126π
(B) $1 + 63\pi$
(C) 63π
(D) -63π
(E) $\frac{63\pi}{2}$
8. A permutation of 10 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z^2]$.
- (A) 0
(B) 3
(C) 4
(D) 2
(E) 1
9. Evaluate $\iiint_{x^2+y^2+z^2 \leq 6} (x^2 + y^2 + z^2)^5 dV$.
- (A) $\frac{1}{3} \left(\frac{4\pi 6^{\frac{13}{2}}}{13} \right)$
(B) $\frac{13}{66} \left(\frac{4\pi 6^{\frac{13}{2}}}{13} \right)$
(C) $\frac{1}{2} \left(\frac{4\pi 6^{\frac{13}{2}}}{13} \right)$
(D) $\frac{4\pi 6^{\frac{13}{2}}}{13}$
(E) $13 \left(\frac{4\pi 6^{\frac{13}{2}}}{13} \right)$

Questions 10–12

10. When $x^3 + 1$ is divided by $(x - 23)(x - 24)$, the remainder is $ux + v$. Find u .
- (A) 1657
(B) 1658
(C) 1655
(D) 1656
(E) 1659
11. Under $x = u + 6v$, $y = u - 6v$, what is the area of the image of $0 \leq u, v \leq 1$?
- (A) 144
(B) 12
(C) $\frac{1}{12}$
(D) 0
(E) 36
12. A coin success probability p has uniform prior on $[0, 1]$. After observing 8 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?
- (A) $\frac{1}{4}$
(B) $\frac{11}{12}$
(C) $\frac{7}{8}$
(D) $\frac{3}{4}$
(E) $\frac{3}{8}$

Questions 13–15

13. Evaluate $\int_0^1 \int_x^1 6y^6 \, dy \, dx$.

- (A) $\frac{3}{28}$
- (B) 1
- (C) $\frac{3}{4}$
- (D) 3
- (E) $\frac{6}{7}$

14. Let $P = \text{diag}(\underbrace{1, \dots, 1}_3, \underbrace{0, \dots, 0}_{26})$. What is the dimension of the space of 29×29 real matrices X satisfying $PX = X$ and $XP = 0$?

- (A) 76
- (B) 77
- (C) 78
- (D) 79
- (E) 80

15. The region $0 \leq x \leq 1$, $0 \leq y \leq 6x^6$ is revolved about the line $x = -2$. Find its volume.

- (A) $1 + \frac{69\pi}{14}$
- (B) $\frac{69\pi}{28}$
- (C) $\frac{69\pi}{7}$
- (D) $\frac{69\pi}{14}$
- (E) $-\frac{69\pi}{14}$

Questions 16–18

16. A random variable has mean 0 and variance 8. What upper bound does the Chebyshev inequality give for $P(|X| \geq 16)$?

- (A) $\frac{31}{32}$
- (B) $\frac{33}{64}$
- (C) $\frac{1}{96}$
- (D) $\frac{1}{64}$
- (E) $\frac{1}{32}$

17. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 6x$ and has density $\rho(x, y) = x^4y$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{7}{8}$
- (B) $\frac{7}{16}$
- (C) $-\frac{7}{8}$
- (D) $\frac{15}{8}$
- (E) $\frac{7}{4}$

18. A 3×3 matrix A has characteristic polynomial $(t - 22)(t + 1)(t - 2)$. What is $\text{tr}(A^3 - 2A + I)$?

- (A) 10613
- (B) 10614
- (C) 10610
- (D) 10611
- (E) 10612

Questions 19–21

19. The local inverse $y(x)$ of $x = 6y + y^3$ satisfies $y(0) = 0$. Find $y^{(7)}(0)$.

- (A) $-\frac{60480}{6^{10}}$
- (B) $\frac{60480}{6^{10}}$
- (C) $\frac{1}{5040} \left(-\frac{60480}{6^{10}} \right)$
- (D) $6 \left(-\frac{60480}{6^{10}} \right)$
- (E) $\frac{1}{6} \left(-\frac{60480}{6^{10}} \right)$

20. A box is chosen uniformly from two boxes. Box A contains 13 red and 15 blue balls; box B contains 15 red and 13 blue balls. Two balls are drawn without replacement and both are red. What is the conditional probability that A was chosen?

- (A) $\frac{26}{61}$
- (B) $\frac{13}{61}$
- (C) $\frac{87}{122}$
- (D) $\frac{35}{61}$
- (E) $\frac{26}{183}$

21. If $y'' - 12y' + 36y = 6e^{6x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $1 + 3e^6$
- (B) $6e^6$
- (C) $3e^6$
- (D) $\frac{3e^6}{2}$
- (E) $-3e^6$

Questions 22–24

22. Let N be the 28×28 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\dim(\ker N^6 / \ker N^3)$.

- (A) 1
- (B) 3
- (C) 2
- (D) 5
- (E) 4

23. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-6)^j}{j^{\frac{5}{3}} (\log j)^m}$ converge absolutely at $x = 5$? Give the least such m , or 0 if none exists.

- (A) 3
- (B) 0
- (C) 1
- (D) 4
- (E) 2

24. Which subset of $\mathbb{R}$ is compact, with 8 fixed as displayed?

- (A) $\{8/k : k \geq 1\}$
- (B) $8\mathbb{Z}$
- (C) $\{0\} \cup \{8/k : k \geq 1\}$
- (D) $(0, 8)$
- (E) $\mathbb{Q} \cap [0, 8]$

Questions 25–27

25. For $\mathbf{F} = (6x^6y, y^6z, xz^6)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) 135
- (B) 270
- (C) 540
- (D) 271
- (E) -270

26. Let A be the 28×28 matrix whose diagonal entries are 25 and whose off-diagonal entries are 1. What is its largest eigenvalue?

- (A) 52
- (B) 50
- (C) 53
- (D) 51
- (E) 54

27. Evaluate $\int_0^{\log 2} e^x(e^x + 6)^6 dx$.

- (A) $8^6 - 7^6$
- (B) $\frac{8^7 - 7^7}{7}$
- (C) $\frac{8^7}{7}$
- (D) $-\frac{8^7 - 7^7}{7}$
- (E) $7 \left(\frac{8^7 - 7^7}{7} \right)$

Questions 28–30

28. For $a_k = (-1)^k + 8/k$, find $\limsup_{k \rightarrow \infty} a_k - \liminf_{k \rightarrow \infty} a_k$.

- (A) 2
- (B) 0
- (C) 1
- (D) 4
- (E) 3

29. If $y' = y(1 - y/6)$ and $y(0) = 3$, find $y(-\log 6)$.

- (A) $-\frac{6}{7}$
- (B) $\frac{12}{7}$
- (C) $\frac{6}{7}$
- (D) $\frac{13}{7}$
- (E) $\frac{3}{7}$

30. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 154$ and $x + y = 29$. Find $x^2 + y^2$.

- (A) 532
- (B) 534
- (C) 535
- (D) 533
- (E) 531

Questions 31–33

31. Evaluate $\lim_{x \rightarrow 0} \frac{e^{6x} - \sum_{j=0}^5 (6x)^j / j!}{x^6}$.

- (A) $-\frac{6^6}{6!}$
- (B) $720 \left(\frac{6^6}{6!} \right)$
- (C) $\frac{1}{6} \left(\frac{6^6}{6!} \right)$
- (D) $\frac{6^6}{6!}$
- (E) $6 \left(\frac{6^6}{6!} \right)$

32. For which real a is $f(x) = x^a$ in $L^8(0, 1)$?

- (A) $a \geq -1/8$
- (B) $a > 0$
- (C) $a < 1/8$
- (D) All real a
- (E) $a > -1/8$

33. Two solutions of $y'' + 6x^6y' + 6y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $\frac{2}{e^{\frac{6}{7}}}$
- (B) $\frac{1}{2e^{\frac{6}{7}}}$
- (C) $-\frac{1}{e^{\frac{6}{7}}}$
- (D) $e^{-\frac{6}{7}} + 1$
- (E) $e^{-\frac{6}{7}}$

Questions 34–36

34. A sequence satisfies $u_0 = 22$ and $u_{k+1} = 2u_k + 7$ for all $k \geq 0$. Find u_8 .

- (A) 7416
- (B) 7418
- (C) 7419
- (D) 7415
- (E) 7417

35. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^6 y^5 dV$.

- (A) $\frac{1}{6} \left(\frac{6! 5!}{14!} \right)$
- (B) $6 \left(\frac{6! 5!}{14!} \right)$
- (C) $14 \left(\frac{6! 5!}{14!} \right)$
- (D) $\frac{6! 5!}{14!}$
- (E) $\frac{1}{42}$

36. $T : \mathbb{R} \rightarrow \mathbb{R}$ has Lipschitz constant at most $1/8$. If $x_1 = T(x_0)$ and $|x_1 - x_0| \leq 1$, what is the sharp upper bound on $|T^3(x_0) - x_*|$, where x_* is its fixed point?

- (A) $\frac{447}{448}$
- (B) $\frac{1}{448}$
- (C) $\frac{1}{896}$
- (D) $\frac{1}{1344}$
- (E) $\frac{449}{896}$

Questions 37–39

37. For the curve $x = t^6$, $y = t^7 + 6t$, find d^2y/dx^2 at $t = 1$.

- (A) -1
- (B) $\frac{13}{6}$
- (C) $\frac{37}{36}$
- (D) $-\frac{23}{36}$
- (E) $-\frac{23}{6}$

38. Linear maps $T : \mathbb{R}^{29} \rightarrow \mathbb{R}^{29}$ and $U : \mathbb{R}^{29} \rightarrow \mathbb{R}^{29}$ have ranks 3 and 3. What is the largest possible rank of UT ?

- (A) 5
- (B) 2
- (C) 1
- (D) 4
- (E) 3

39. Evaluate $\int_0^1 x^5(\log x)^5 dx$.

- (A) $\frac{324}{5} \left(\frac{(-1)^5 5!}{6^6} \right)$
- (B) $\frac{(-1)^5 5!}{6^6}$
- (C) $-\frac{(-1)^5 5!}{6^6}$
- (D) $\frac{1}{6} \left(\frac{(-1)^5 5!}{6^6} \right)$
- (E) $6 \left(\frac{(-1)^5 5!}{6^6} \right)$

Questions 40–42

40. The circle $|z| = 2$ is traversed counterclockwise. What is $\frac{1}{2\pi i} \int_{|z|=2} \frac{e^z}{z^9} dz$?

- (A) $1/16!$
- (B) $1/8!$
- (C) $8!$
- (D) $1/7!$
- (E) $1/9!$

41. Let $F(x) = \int_0^{x^6} e^{6t} dt$. Find $F'(1)$.

- (A) $\frac{e^6}{6}$
- (B) e^{36}
- (C) e^6
- (D) $36e^6$
- (E) $6e^6$

42. Let $R = (\mathbb{Z}/28\mathbb{Z})/([7])$, where $([7])$ is the ideal generated by $[7]$. How many units does R have?

- (A) 4
- (B) 7
- (C) 8
- (D) 5
- (E) 6

Questions 43–45

43. Solve $y' + 6y = x^6e^{-6x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{6}{7e^6}$
- (B) e^{-6}
- (C) $\frac{1}{7e^6}$
- (D) $\frac{1}{6e^6}$
- (E) $\frac{e^6}{7}$

44. Find the residue of $z^8/(z-1)^3$ at $z = 1$.

- (A) 10
- (B) 19
- (C) 46
- (D) 28
- (E) 37

45. Let $\mathbf{F} = (6y^6, z^6, x^6)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (1, 1, 2)/\sqrt{6}$.

- (A) $-7\sqrt{6}$
- (B) $-14\sqrt{6}$
- (C) $-28\sqrt{6}$
- (D) $14\sqrt{6}$
- (E) $1 - 14\sqrt{6}$

Questions 46–48

46. What is the largest integer k such that 3^k divides $31!$?

- (A) 14
- (B) 15
- (C) 16
- (D) 12
- (E) 13

47. Let $F(x) = \int_0^x (x - t)^5 e^{6t} dt$. Find $F^{(6)}(0)$.

- (A) 720
- (B) 0
- (C) 120
- (D) $\frac{1}{120}$
- (E) 240

48. What is the least positive multiplicative inverse of 2 modulo 17?

- (A) 9
- (B) 3
- (C) 12
- (D) 6
- (E) 15

Questions 49–51

49. For $x > 0$, $x^2y'' + (1 - 12)xy' + 36y = 0$, $y(1) = 0$, $y'(1) = 6$. Find $y(e)$.
- (A) 0
 - (B) $\frac{e^6}{12}$
 - (C) $-6e^6$
 - (D) $6e^6$
 - (E) $2e^6$
50. In the ring $\mathbb{Z}/156\mathbb{Z}$, how many elements satisfy $x^2 = 1$?
- (A) 7
 - (B) 10
 - (C) 8
 - (D) 9
 - (E) 6
51. For $f(x, y) = x^2 + 6xy + 6y^2$, what is its maximum directional derivative at $(1, -1)$?
- (A) -6
 - (B) -4
 - (C) 10
 - (D) $2\sqrt{13}$
 - (E) 52

Questions 52–54

52. How many truth assignments to $p, q, r_1, \dots, r_6$ make $p \Rightarrow q$ false?

- (A) 2^7
- (B) 2^8
- (C) 2^5
- (D) 2^6
- (E) 2^9

53. Evaluate $\int_0^1 x^6(1-x)^5 dx$.

- (A) $\frac{6!5!}{12!}$
- (B) $12 \left(\frac{6!5!}{12!} \right)$
- (C) $7 \left(\frac{6!5!}{12!} \right)$
- (D) $\frac{1}{6} \left(\frac{6!5!}{12!} \right)$
- (E) $132 \left(\frac{6!5!}{12!} \right)$

54. A permutation consists of disjoint cycles of lengths 4 and 10, together with a disjoint transposition when the sum of these two lengths is odd. It fixes all other points. What is its order?

- (A) 21
- (B) 20
- (C) 22
- (D) 19
- (E) 18

Questions 55–57

55. Find the total outward flux of $\mathbf{F} = (6x^6y, y^6z, xz^6)$ through the boundary of $[0, 1]^3$.

- (A) 4
- (B) 5
- (C) -4
- (D) 2
- (E) 8

56. Find the coefficient of x^3 in $(1 + x)^8(1 - x)^8(1 + 2x)^2$.

- (A) -22
- (B) -32
- (C) -42
- (D) -12
- (E) -52

57. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(6 + 6j)(12 + 6j)}$.

- (A) $\frac{1}{12}$
- (B) $\frac{1}{36}$
- (C) $\frac{1}{18}$
- (D) $\frac{1}{6}$
- (E) 1

Questions 58–60

58. In $R = \mathbb{F}_{11}[x]/(x^6)$, how many elements are units?

- (A) $11^7 - 11^5$
- (B) $11^8 - 11^7$
- (C) $11^6 - 11^0$
- (D) $11^7 - 11^6$
- (E) $11^6 - 11^5$

59. Find the coefficient of x^6 in the Taylor series of $(1 - 6x)^{-2}$ at zero.

- (A) $7 \cdot 6^6$
- (B) $\frac{6}{7} (7 \cdot 6^6)$
- (C) $\frac{1}{6} (7 \cdot 6^6)$
- (D) $\frac{1}{7} (7 \cdot 6^6)$
- (E) $2 (7 \cdot 6^6)$

60. How many distinct unoriented cycles of length 4 are in $K_{8,10}$?

- (A) 1260
- (B) 840
- (C) 2100
- (D) 420
- (E) 1680

Questions 61–63

61. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(6j)!}{(j!)^6} x^{6j}$?
- (A) $2(6^{-1})$
(B) 6^1
(C) 6^{-1}
(D) $\frac{1}{2}(6^{-1})$
(E) 6^{-6}
62. Integers x satisfy $x \equiv 8 \pmod{11}$ and $x \equiv 12 \pmod{16}$. Find the sum of all such x with $0 \leq x < 352$.
- (A) 456
(B) 458
(C) 454
(D) 455
(E) 457
63. Evaluate $\int_0^1 \frac{x^5}{1+6x^6} dx$.
- (A) $\frac{\log(7)}{18}$
(B) $-\frac{\log(7)}{36}$
(C) $\frac{\log(7)}{36}$
(D) $\frac{\log(7)}{6}$
(E) $\log(7)$

Questions 64–66

64. How many functions from a set of 13 elements onto a set of 3 elements exist?

- (A) $3^{13} - 3(2^{13}) - 3$
- (B) $3^{13} - 3(2^{13}) + 3$
- (C) 3^{13}
- (D) $3^{13} - 3(2^{13})$
- (E) $3^{13} - 2^{13}$

65. Let $f(x, y) = x^2 + 12xy + (42)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 42
- (B) 24
- (C) 12
- (D) 6
- (E) 168

66. The symmetric group S_{28} acts on the two-element subsets of $\{1, \dots, 28\}$. What is the index of the setwise stabilizer of $\{1, 2\}$?

- (A) 380
- (B) 376
- (C) 377
- (D) 378
- (E) 379

Answer Key

Practice Test 26 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	E	23	C	45	B
2	C	24	C	46	A
3	E	25	B	47	C
4	E	26	A	48	A
5	E	27	B	49	D
6	A	28	A	50	C
7	C	29	C	51	D
8	D	30	D	52	D
9	D	31	D	53	A
10	A	32	E	54	B
11	B	33	E	55	A
12	D	34	E	56	B
13	C	35	D	57	B
14	C	36	B	58	E
15	D	37	D	59	A
16	E	38	E	60	A
17	A	39	B	61	C
18	E	40	B	62	A
19	A	41	E	63	C
20	A	42	E	64	B
21	C	43	C	65	B
22	B	44	D	66	D

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