

GRE Mathematics Practice Test 28

Upper-Intermediate Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

Intermediate reinforcement, combining standard applications with selected more demanding arguments. This is cycle 5 of 19, position 3 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. For $f(x, y) = x^2 + 6xy + 4y^2$, what is its maximum directional derivative at $(1, -1)$?
- (A) -2
(B) $2\sqrt{5}$
(C) -4
(D) 20
(E) 6
2. Fix a nonzero vector $u \in \mathbb{R}^{30}$. What is the dimension of the space of skew-symmetric 30×30 matrices A satisfying $Au = 0$?
- (A) 406
(B) 407
(C) 408
(D) 404
(E) 405
3. For $x > 0$, $x^2y'' + (1 - 10)xy' + 24y = 0$, $y(1) = 0$, $y'(1) = 6$. Find $y(e)$.
- (A) $-e^4 + e^6$
(B) $3(1 - e^2)e^4$
(C) $\frac{e^6}{10}$
(D) $e^4 + e^6$
(E) $-3(1 - e^2)e^4$

Questions 4–6

4. $X_1, \dots, X_{10}$ are independent with common mean and variance 12. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 72
- (B) 36
- (C) 180
- (D) 144
- (E) 108

5. Let $\mathbf{F} = (6y^4, z^4, x^4)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (1, 1, 4)/\sqrt{18}$.

- (A) $\frac{52\sqrt{2}}{3}$
- (B) $-\frac{104\sqrt{2}}{3}$
- (C) $-\frac{52\sqrt{2}}{3}$
- (D) $-\frac{26\sqrt{2}}{3}$
- (E) $1 - \frac{52\sqrt{2}}{3}$

6. Let $A = \begin{bmatrix} 1 & 24 & 0 \\ 0 & 1 & 8 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 3)$ entry of A^4 ?

- (A) 1154
- (B) 1150
- (C) 1151
- (D) 1152
- (E) 1153

Questions 7–9

7. Let $f(x, y) = x^2 + 12xy + (40)y^2$. What is the product of the eigenvalues of the Hessian of f ?
- (A) 4
(B) 160
(C) 16
(D) 8
(E) 40
8. A permutation of 12 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z^2]$.
- (A) 2
(B) 0
(C) 4
(D) 3
(E) 1
9. If $y' + 6y = 4x^4e^{-6x}$ and $y(0) = 4$, find $y(1)$.
- (A) $\frac{48}{5e^6}$
(B) $\frac{24}{5e^6} + 1$
(C) $\frac{12}{5e^6}$
(D) $-\frac{24}{5e^6}$
(E) $\frac{24}{5e^6}$

Questions 10–12

10. An invertible 30×30 matrix A has determinant 3. Find $\det(\text{adj } A)$.

- (A) 3^{29}
- (B) $3^{29} \cdot 5$
- (C) 3^{30}
- (D) 3^{28}
- (E) $2 \cdot 3^{29}$

11. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(6+4j)(10+4j)}$.

- (A) $\frac{2}{3}$
- (B) $\frac{1}{10}$
- (C) $\frac{1}{6}$
- (D) $\frac{1}{4}$
- (E) $\frac{1}{24}$

12. A coin success probability p has uniform prior on $[0, 1]$. After observing 10 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?

- (A) $\frac{11}{14}$
- (B) $\frac{11}{28}$
- (C) $\frac{3}{14}$
- (D) $\frac{11}{42}$
- (E) $\frac{25}{28}$

Questions 13–15

13. Evaluate $\lim_{x \rightarrow 0} \frac{e^{6x} - \sum_{j=0}^3 (6x)^j / j!}{x^4}$.

- (A) $\frac{6^4}{4!}$
- (B) $\frac{1}{6} \left(\frac{6^4}{4!} \right)$
- (C) $4 \left(\frac{6^4}{4!} \right)$
- (D) $-\frac{6^4}{4!}$
- (E) $24 \left(\frac{6^4}{4!} \right)$

14. On P_{31} , the real polynomials of degree at most 31, define $T(p) = x^2 p'' - 6p$. Find its rank.

- (A) 31
- (B) 29
- (C) 32
- (D) 33
- (E) 30

15. Evaluate $\int_0^1 x^5 (\log x)^3 dx$.

- (A) $36 \left(\frac{(-1)^3 3!}{6^4} \right)$
- (B) $\frac{(-1)^3 3!}{6^4}$
- (C) $-\frac{(-1)^3 3!}{6^4}$
- (D) $\frac{1}{4} \left(\frac{(-1)^3 3!}{6^4} \right)$
- (E) $6 \left(\frac{(-1)^3 3!}{6^4} \right)$

Questions 16–18

16. A random variable has mean 0 and variance 10. What upper bound does the Chebyshev inequality give for $P(|X| \geq 20)$?

- (A) $\frac{41}{80}$
- (B) $\frac{1}{80}$
- (C) $\frac{1}{40}$
- (D) $\frac{1}{120}$
- (E) $\frac{39}{40}$

17. Find the coefficient of x^4 in $e^{3x} \log(1 + 6x)$.

- (A) -324
- (B) -3888
- (C) 162
- (D) -156
- (E) -162

18. Subspaces U, V of $\mathbb{R}^{32}$ have dimensions 4 and 5. What is the smallest possible value of $\dim(U \cap V)$?

- (A) 2
- (B) 0
- (C) 1
- (D) 4
- (E) 3

Questions 19–21

19. Let C be the unit circle $x^2 + y^2 = 1$, $z = 4$, counterclockwise viewed from above. Evaluate $\int_C -6y(x^2 + y^2)^3 dx + (6x(x^2 + y^2)^3) dy + 4z dz$.

- (A) $1 + 12\pi$
- (B) 24π
- (C) 6π
- (D) -12π
- (E) 12π

20. A box is chosen uniformly from two boxes. Box A contains 15 red and 17 blue balls; box B contains 17 red and 15 blue balls. Two balls are drawn without replacement and both are red. What is the conditional probability that A was chosen?

- (A) $\frac{136}{241}$
- (B) $\frac{35}{241}$
- (C) $\frac{105}{482}$
- (D) $\frac{105}{241}$
- (E) $\frac{173}{241}$

21. The solution of $y' = 6x^4y^3$ has $y(0) = 1/7$. Find $y(1)$.

- (A) $\frac{\sqrt{1165}}{466}$
- (B) $\frac{\sqrt{1165}}{233} + 1$
- (C) $-\frac{\sqrt{1165}}{233}$
- (D) $\frac{\sqrt{1165}}{233}$
- (E) $\frac{2\sqrt{1165}}{233}$

Questions 22–24

22. Let V consist of real polynomials of degree at most 11 that vanish at 24, 25, and 27. Find $\dim V$.

- (A) 7
- (B) 9
- (C) 8
- (D) 10
- (E) 11

23. Evaluate $\int_0^{\log 2} e^x (e^x + 6)^4 dx$.

- (A) $5 \left(\frac{8^5 - 7^5}{5} \right)$
- (B) $\frac{8^5}{5}$
- (C) $8^4 - 7^4$
- (D) $\frac{8^5 - 7^5}{5}$
- (E) $-\frac{8^5 - 7^5}{5}$

24. Which subset of $\mathbb{R}$ is compact, with 10 fixed as displayed?

- (A) $\{10/k : k \geq 1\}$
- (B) $10\mathbb{Z}$
- (C) $\{0\} \cup \{10/k : k \geq 1\}$
- (D) $(0, 10)$
- (E) $\mathbb{Q} \cap [0, 10]$

Questions 25–27

25. Evaluate $\int_0^1 \int_0^x x^6 y^4 dy dx$.

- (A) $\frac{1}{12}$
- (B) $\frac{1}{35}$
- (C) $\frac{1}{60}$
- (D) $\frac{1}{55}$
- (E) $\frac{1}{72}$

26. The roots r, s of $x^2 - 32x + 192 = 0$ are counted with multiplicity. Find $r^3 + s^3$.

- (A) 14338
- (B) 14336
- (C) 14337
- (D) 14335
- (E) 14334

27. Let $F(x) = \int_0^{x^4} e^{6t} dt$. Find $F'(1)$.

- (A) $24e^6$
- (B) e^{24}
- (C) $\frac{e^6}{4}$
- (D) $4e^6$
- (E) e^6

Questions 28–30

28. For $a_k = (-1)^k + 10/k$, find $\limsup_{k \rightarrow \infty} a_k - \liminf_{k \rightarrow \infty} a_k$.

- (A) 2
- (B) 0
- (C) 1
- (D) 3
- (E) 4

29. Under $x = u + 6v$, $y = u - 4v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) 100
- (B) $\frac{1}{10}$
- (C) 10
- (D) 24
- (E) 2

30. For real t , let $M(t) = \begin{pmatrix} t-24 & 8 & 0 \\ 0 & t-24 & 1 \\ 0 & 0 & t-24 \end{pmatrix}$. At $t = 24$, what is $\dim \ker(M(t)^2) - \dim \ker M(t)$?

- (A) 1
- (B) 3
- (C) 4
- (D) 2
- (E) 0

Questions 31–33

31. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(6j)!}{(j!)^6} x^{4j}$?
- (A) 6^{-6}
(B) $6^{\frac{3}{2}}$
(C) $6^{-\frac{2}{3}}$
(D) $6^{-\frac{3}{2}}$
(E) $\frac{1}{6}$
32. For which real a is $f(x) = x^a$ in $L^{10}(0, 1)$?
- (A) $a > -1/10$
(B) $a < 1/10$
(C) $a > 0$
(D) All real a
(E) $a \geq -1/10$
33. The region $0 \leq x \leq 1$, $0 \leq y \leq 6x^4$ is revolved about the line $x = -4$. Find its volume.
- (A) $\frac{116\pi}{5}$
(B) $1 + \frac{58\pi}{5}$
(C) $\frac{58\pi}{5}$
(D) $-\frac{58\pi}{5}$
(E) $\frac{29\pi}{5}$

Questions 34–36

34. Let $u = (5, 4)^T$ and $v = (1, 0)^T$. Find the squared distance from $(5, 0)^T$ to $\text{span}(u)$.
- (A) $\frac{841}{82}$
(B) $\frac{82}{759}$
(C) $\frac{359}{41}$
(D) $\frac{400}{41}$
(E) $\frac{441}{41}$
35. Let C be $x^2/16 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-2y^3) dx + (\frac{x^3}{3}) dy$.
- (A) 44π
(B) 22π
(C) 11π
(D) -22π
(E) $1 + 22\pi$
36. A map $T : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $|T(x) - T(y)| \leq |x - y|/10$. If $T(x_*) = x_*$ and $|x_0 - x_*| \leq 1$, what is the smallest guaranteed upper bound for $|T^3(x_0) - x_*|$?
- (A) $\frac{1}{3000}$
(B) $\frac{1}{1000}$
(C) $\frac{1000}{999}$
(D) $\frac{1000}{2000}$
(E) $\frac{1001}{2000}$

Questions 37–39

37. For the curve $x = t^4$, $y = t^5 + 6t$, find d^2y/dx^2 at $t = 1$.

- (A) $\frac{23}{16}$
- (B) $-\frac{13}{16}$
- (C) $\frac{11}{4}$
- (D) $-\frac{3}{2}$
- (E) $-\frac{13}{4}$

38. An idempotent 30×30 real matrix P has rank 3. What is $\det(I + P)$?

- (A) 10
- (B) 6
- (C) 9
- (D) 7
- (E) 8

39. Evaluate $\iiint_{x^2+y^2+z^2 \leq 6} (x^2 + y^2 + z^2)^3 dV$.

- (A) $\frac{1}{3} \left(\frac{4\pi 6^{\frac{9}{2}}}{9} \right)$
- (B) $9 \left(\frac{4\pi 6^{\frac{9}{2}}}{9} \right)$
- (C) $\frac{4\pi 6^{\frac{9}{2}}}{9}$
- (D) $\frac{1}{2} \left(\frac{4\pi 6^{\frac{9}{2}}}{9} \right)$
- (E) $\frac{3}{14} \left(\frac{4\pi 6^{\frac{9}{2}}}{9} \right)$

Questions 40–42

40. The circle $|z| = 2$ is traversed counterclockwise. What is $\frac{1}{2\pi i} \int_{|z|=2} \frac{e^z}{z^{11}} dz$?

- (A) $1/11!$
- (B) $1/10!$
- (C) $1/20!$
- (D) $1/9!$
- (E) $10!$

41. Find the coefficient of x^4 in the Taylor series of $(1 - 6x)^{-2}$ at zero.

- (A) $\frac{4}{5} (5 \cdot 6^4)$
- (B) $5 \cdot 6^4$
- (C) $\frac{1}{6} (5 \cdot 6^4)$
- (D) $2 (5 \cdot 6^4)$
- (E) $\frac{1}{5} (5 \cdot 6^4)$

42. Let $G = \mathbb{Z}/174\mathbb{Z} \times \mathbb{Z}/120\mathbb{Z}$. Find the order of G/H , where $H = \{g \in G : 6g = 0\}$.

- (A) 580
- (B) 581
- (C) 579
- (D) 582
- (E) 578

Questions 43–45

43. The local inverse $y(x)$ of $x = 6y + y^3$ satisfies $y(0) = 0$. Find $y^{(3)}(0)$.

- (A) $6\left(-\frac{6}{6^4}\right)$
- (B) $-\frac{6}{6^4}$
- (C) $\frac{1}{6}\left(-\frac{6}{6^4}\right)$
- (D) $2\left(-\frac{6}{6^4}\right)$
- (E) $\frac{6}{6^4}$

44. Find the residue of $z^{10}/(z-1)^3$ at $z = 1$.

- (A) 45
- (B) 60
- (C) 30
- (D) 75
- (E) 15

45. What is the maximum of x^4y^6 over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\left(\frac{4}{10}\right)^6 \left(\frac{6}{10}\right)^4$
- (B) $\frac{6}{25}$
- (C) $\frac{1}{2^{10}}$
- (D) $10 \left(\left(\frac{4}{10}\right)^4 \left(\frac{6}{10}\right)^6 \right)$
- (E) $\left(\frac{4}{10}\right)^4 \left(\frac{6}{10}\right)^6$

Questions 46–48

46. How many group homomorphisms $\mathbb{Z}/174\mathbb{Z} \rightarrow \mathbb{Z}/120\mathbb{Z}$ are there?

- (A) 7
- (B) 5
- (C) 4
- (D) 6
- (E) 8

47. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 6x)^{1/(3x)} - e^{6/3}}{x}$.

- (A) $-e^2$
- (B) $-12e^2$
- (C) e^2
- (D) $-6e^2$
- (E) $6e^2$

48. What is the least positive multiplicative inverse of 2 modulo 21?

- (A) 8
- (B) 5
- (C) 17
- (D) 14
- (E) 11

Questions 49–51

49. Find the curvature of the helix $r(t) = (6 \cos t, 6 \sin t, 4t)$.

- (A) $\frac{3}{52}$
- (B) $-\frac{3}{26}$
- (C) $\frac{3}{26}$
- (D) $\frac{29}{26}$
- (E) $\frac{3}{13}$

50. How many 1-dimensional subspaces does $\mathbb{F}_{11}^5$ have?

- (A) $\frac{11^4 - 1}{11 - 1}$
- (B) $\frac{11^6 - 1}{11 - 1}$
- (C) $\frac{11^8 - 1}{11 - 1}$
- (D) $\frac{11^5 - 1}{11 - 1}$
- (E) $\frac{11^7 - 1}{11 - 1}$

51. For $x > 0$, $x^2 y'' + (-15)xy' + 60y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $-\frac{(1 - e^4) e^6}{8}$
- (B) $-\frac{(1 - e^4) e^6}{2}$
- (C) $-\frac{(1 - e^4) e^6}{4}$
- (D) $-\frac{e^6}{4} + 1 + \frac{e^{10}}{4}$
- (E) $\frac{(1 - e^4) e^6}{4}$

Questions 52–54

52. How many truth assignments to $p, q, r_1, \dots, r_8$ make $p \Rightarrow q$ false?

- (A) 2^{10}
- (B) 2^{11}
- (C) 2^9
- (D) 2^7
- (E) 2^8

53. Let $f(x) = \log(1 + 6x)$. Find $f^{(4)}(0)$.

- (A) $-(-1)^3 3! 6^4$
- (B) $4 \left((-1)^3 3! 6^4 \right)$
- (C) $\frac{1}{4} \left((-1)^3 3! 6^4 \right)$
- (D) $(-1)^3 3! 6^4$
- (E) $\frac{1}{6} \left((-1)^3 3! 6^4 \right)$

54. How many monic irreducible polynomials of degree 3 are there over $\mathbb{F}_{11}$?

- (A) 438
- (B) 439
- (C) 440
- (D) 442
- (E) 441

Questions 55–57

55. Evaluate $\int_0^\infty x^4 e^{-6x} dx$.

- (A) $\frac{1}{6} \left(\frac{4!}{6^5} \right)$
- (B) $\frac{1}{4} \left(\frac{4!}{6^5} \right)$
- (C) $\frac{4!}{6^5}$
- (D) $6 \left(\frac{4!}{6^5} \right)$
- (E) $7776 \left(\frac{4!}{6^5} \right)$

56. What is the coefficient of x^2 in $(1 + 2x)^{10}$?

- (A) 300
- (B) 60
- (C) 240
- (D) 120
- (E) 180

57. If $y'' - 12y' + 36y = 4e^{6x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $1 + 2e^6$
- (B) $-2e^6$
- (C) $4e^6$
- (D) e^6
- (E) $2e^6$

Questions 58–60

- 58.** How many elements z of $\mathbb{F}_{56}$ satisfy $z^{25} = z$?
- (A) 27
 - (B) 26
 - (C) 25
 - (D) 24
 - (E) 23
- 59.** Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 6z^2 = x + 4y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.
- (A) -12
 - (B) -4
 - (C) -24
 - (D) -48
 - (E) 48
- 60.** How many edges does the complete bipartite graph $K_{10,12}$ have?
- (A) 80
 - (B) 120
 - (C) 40
 - (D) 160
 - (E) 200

Questions 61–63

61. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(6x) - \sum_{j=0}^2 (-1)^j (6x)^{2j+1} / (2j+1)!}{x^7}$.
- (A) $\frac{1}{6} \left(\frac{(-1)^3 6^7}{7!} \right)$
(B) $7 \left(\frac{(-1)^3 6^7}{7!} \right)$
(C) $-\frac{(-1)^3 6^7}{7!}$
(D) $\frac{(-1)^3 6^7}{7!}$
(E) $6 \left(\frac{(-1)^3 6^7}{7!} \right)$
62. How many generators does the cyclic group of order 14641 have?
- (A) $11^3(11-1)$
(B) $11^6(11-1)$
(C) $11^4(11-1)$
(D) $11^5(11-1)$
(E) $11^2(11-1)$
63. Find the arc length of $r(t) = (6(t - t^3/3), 6t^2)$ for $0 \leq t \leq 1/4$.
- (A) $\frac{49}{64}$
(B) $-\frac{49}{32}$
(C) $\frac{49}{16}$
(D) $\frac{81}{32}$
(E) $\frac{49}{32}$

Questions 64–66

64. How many functions from a set of 15 elements onto a set of 3 elements exist?

- (A) $3^{15} - 2^{15}$
- (B) 3^{15}
- (C) $3^{15} - 3(2^{15})$
- (D) $3^{15} - 3(2^{15}) - 3$
- (E) $3^{15} - 3(2^{15}) + 3$

65. For $f(x) = x^4 e^{6x}$, find $f^{(6)}(0)$.

- (A) $\frac{6! 6^2}{2}$
- (B) $\frac{1}{360} \left(\frac{6! 6^2}{2} \right)$
- (C) $\frac{1}{3} \left(\frac{6! 6^2}{2} \right)$
- (D) $\frac{1}{30} \left(\frac{6! 6^2}{2} \right)$
- (E) $2 \left(\frac{6! 6^2}{2} \right)$

66. Let $R = (\mathbb{Z}/54\mathbb{Z})/([9])$, where $([9])$ is the ideal generated by $[9]$. How many units does R have?

- (A) 8
- (B) 6
- (C) 4
- (D) 7
- (E) 5

Answer Key

Practice Test 28 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	B	23	D	45	E
2	A	24	C	46	D
3	E	25	C	47	D
4	E	26	B	48	E
5	C	27	D	49	C
6	D	28	A	50	D
7	C	29	C	51	C
8	A	30	A	52	E
9	E	31	D	53	D
10	A	32	A	54	C
11	E	33	C	55	C
12	A	34	D	56	E
13	A	35	B	57	E
14	A	36	B	58	C
15	B	37	B	59	D
16	C	38	E	60	B
17	E	39	C	61	D
18	B	40	B	62	A
19	E	41	B	63	E
20	D	42	A	64	E
21	D	43	B	65	A
22	B	44	A	66	B

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