

GRE Mathematics Practice Test 29

Bridge to Standard

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A bridge toward standard practice, with routine multi-step questions and a smaller number of challenging concepts. This is cycle 5 of 19, position 4 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. For $y' = 6y(1 - y/3)$, what is the derivative of the right-hand side with respect to y at the positive equilibrium?

- (A) -5
- (B) -6
- (C) -12
- (D) 6
- (E) -3

2. Let V consist of real polynomials of degree at most 10 that vanish at 25, 26, and 28. Find $\dim V$.

- (A) 6
- (B) 10
- (C) 8
- (D) 7
- (E) 9

3. If $y' + 6y = 3e^{-6x}$ and $y(0) = 5$, find $y(1)$.

- (A) $\frac{8}{e^6}$
- (B) $\frac{8}{e^6} + 1$
- (C) $\frac{4}{e^6}$
- (D) $-\frac{8}{e^6}$
- (E) $\frac{16}{e^6}$

Questions 4–6

4. $X_1, \dots, X_{11}$ are independent with common mean and variance 13. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 130
- (B) 87
- (C) 173
- (D) 44
- (E) 216

5. Evaluate $\int_0^1 \frac{x^2}{1+6x^3} dx$.

- (A) $\frac{\log(7)}{3}$
- (B) $\frac{\log(7)}{6}$
- (C) $\frac{\log(7)}{18}$
- (D) $-\frac{\log(7)}{18}$
- (E) $\log(7)$

6. The roots r, s of $x^2 - 31x + 150 = 0$ are counted with multiplicity. Find $r^2 + s^2$.

- (A) 663
- (B) 661
- (C) 660
- (D) 662
- (E) 659

Questions 7–9

7. The function $y(x)$ satisfies $y^3 + 6y = 2x$ and $y(0) = 0$. Find $y'(0)$.

- (A) $\frac{2}{9}$
- (B) $-\frac{1}{3}$
- (C) 6
- (D) 0
- (E) $\frac{1}{3}$

8. A permutation of 13 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z]$.

- (A) 4
- (B) 0
- (C) 3
- (D) 2
- (E) 1

9. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-6)^j}{j^{\frac{2}{3}} (\log j)^m}$ converge absolutely at $x = 5$? Give the least such m , or 0 if none exists.

- (A) 2
- (B) 1
- (C) 4
- (D) 0
- (E) 3

Questions 10–12

10. For real t , let $M(t) = \begin{pmatrix} t-25 & 6 & 0 \\ 0 & t-25 & 1 \\ 0 & 0 & t-25 \end{pmatrix}$. At $t = 25$, what is $\text{rank } M(t)$?

- (A) 4
- (B) 3
- (C) 1
- (D) 0
- (E) 2

11. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 6z^2 = x + 3y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) -36
- (B) -12
- (C) -3
- (D) 36
- (E) -18

12. A coin success probability p has uniform prior on $[0, 1]$. After observing 11 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?

- (A) $\frac{4}{5}$
- (B) $\frac{2}{5}$
- (C) $\frac{4}{15}$
- (D) $\frac{9}{10}$
- (E) $\frac{1}{5}$

Questions 13–15

13. Evaluate $\int_C (3x^2 + 6y) dx + (6x + 3y^2) dy$, where C is the line segment from $(0, 0)$ to $(5, 1)$.

- (A) 157
- (B) 312
- (C) 156
- (D) 78
- (E) -156

14. Let $u = (5, 3)^T$ and $v = (1, 0)^T$. Find the squared norm of the orthogonal projection of v onto $\text{span}(u)$.

- (A) $\frac{59}{34}$
- (B) $\frac{4}{17}$
- (C) $\frac{38}{17}$
- (D) $\frac{21}{17}$
- (E) $\frac{25}{34}$

15. Let $F(x) = \int_0^x (x - t)^2 e^{6t} dt$. Find $F^{(3)}(0)$.

- (A) 2
- (B) 12
- (C) $\frac{1}{2}$
- (D) 0
- (E) 6

Questions 16–18

16. A random variable has mean 0 and variance 11. What upper bound does the Chebyshev inequality give for $P(|X| \geq 22)$?

- (A) $\frac{1}{132}$
- (B) $\frac{1}{88}$
- (C) $\frac{45}{88}$
- (D) $\frac{1}{44}$
- (E) $\frac{43}{44}$

17. Evaluate $\int_0^{\log 2} e^x(e^x + 6)^3 dx$.

- (A) $8^3 - 7^3$
- (B) $-\frac{8^4 - 7^4}{4}$
- (C) $4\left(\frac{8^4 - 7^4}{4}\right)$
- (D) $\frac{8^4}{4}$
- (E) $\frac{8^4 - 7^4}{4}$

18. An idempotent 31×31 real matrix P has rank 4. What is $\text{tr } P$?

- (A) 3
- (B) 4
- (C) 6
- (D) 5
- (E) 2

Questions 19–21

19. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{6t} \cos(3t)$, and $y = e^{6t} \sin(3t)$, find d^2f/dt^2 at $t = 1$.
- (A) 6
(B) 90
(C) 12
(D) 144
(E) 0
20. A box is chosen uniformly from two boxes. Box A contains 16 red and 18 blue balls; box B contains 18 red and 16 blue balls. Two balls are drawn without replacement and both are red. What is the conditional probability that A was chosen?
- (A) $\frac{40}{91}$
(B) $\frac{20}{91}$
(C) $\frac{131}{182}$
(D) $\frac{51}{91}$
(E) $\frac{40}{273}$
21. For $f(x, y) = x^2 + 6xy + 3y^2$, what is its maximum directional derivative at $(1, -1)$?
- (A) 8
(B) 0
(C) 4
(D) 16
(E) -4

Questions 22–24

22. For $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $b = \begin{bmatrix} 25 \\ 6 \\ 31 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find x_2 .

- (A) 5
- (B) 8
- (C) 4
- (D) 7
- (E) 6

23. Find the directional derivative of $f(x, y, z) = x^2 + 6y^2 + 3z^2$ at $(1, 1, 1)$ in the direction $(1, 5, 0)$.

- (A) $1 + \frac{31\sqrt{26}}{13}$
- (B) $-\frac{31\sqrt{26}}{13}$
- (C) $\frac{31\sqrt{26}}{13}$
- (D) $\frac{31\sqrt{26}}{26}$
- (E) $\frac{62\sqrt{26}}{13}$

24. Which subset of $\mathbb{R}$ is compact, with 11 fixed as displayed?

- (A) $\{11/k : k \geq 1\}$
- (B) $(0, 11)$
- (C) $11\mathbb{Z}$
- (D) $\{0\} \cup \{11/k : k \geq 1\}$
- (E) $\mathbb{Q} \cap [0, 11]$

Questions 25–27

25. Solve $y' + 6y = x^3e^{-6x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{e^6}{4}$
- (B) e^{-6}
- (C) $\frac{1}{4e^6}$
- (D) $\frac{1}{3e^6}$
- (E) $\frac{3}{2e^6}$

26. What is the coefficient of x^3 in $(x + 5)^5$?

- (A) 249
- (B) 252
- (C) 248
- (D) 251
- (E) 250

27. For $x > 0$, $xy' = 6y$ and $y(1) = 1$. Find $y(e^3)$.

- (A) $1 + e^{18}$
- (B) e^{18}
- (C) $\frac{e^{18}}{2}$
- (D) $2e^{18}$
- (E) $-e^{18}$

Questions 28–30

28. For $a_k = (-1)^k + 11/k$, find $\limsup_{k \rightarrow \infty} a_k - \liminf_{k \rightarrow \infty} a_k$.

- (A) 3
- (B) 1
- (C) 0
- (D) 2
- (E) 4

29. Let C be $x^2/9 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-y) dx + (6x) dy$.

- (A) $1 + 21\pi$
- (B) 42π
- (C) -21π
- (D) 21π
- (E) $\frac{21\pi}{2}$

30. A matrix A satisfies $(A - 26I)(A - 28I) = 0$, and its minimal polynomial has degree two. If $A^2 = uA + vI$, find u .

- (A) 52
- (B) 54
- (C) 56
- (D) 53
- (E) 55

Questions 31–33

31. Evaluate $\lim_{x \rightarrow 0} \frac{e^{6x} - \sum_{j=0}^2 (6x)^j / j!}{x^3}$.

- (A) $\frac{1}{6} \left(\frac{6^3}{3!} \right)$
- (B) $\frac{6^3}{3!}$
- (C) $-\frac{6^3}{3!}$
- (D) $6 \left(\frac{6^3}{3!} \right)$
- (E) $3 \left(\frac{6^3}{3!} \right)$

32. For which real a is $f(x) = x^a$ in $L^1(0, 1)$?

- (A) All real a
- (B) $a > 0$
- (C) $a \geq -1/11$
- (D) $a > -1/11$
- (E) $a < 1/11$

33. Find the coefficient of x^3 in the Taylor series of $(1 - 6x)^{-2}$ at zero.

- (A) $\frac{1}{6} (4 \cdot 6^3)$
- (B) $\frac{1}{4} (4 \cdot 6^3)$
- (C) $\frac{3}{4} (4 \cdot 6^3)$
- (D) $-4 \cdot 6^3$
- (E) $4 \cdot 6^3$

Questions 34–36

34. Let $V = \begin{bmatrix} 1 & 25 & 625 \\ 1 & 27 & 729 \\ 1 & 30 & 900 \end{bmatrix}$. Find $\det V$.

- (A) 28
- (B) 31
- (C) 29
- (D) 32
- (E) 30

35. The solution of $y' = 6x^3y^2$ has $y(0) = 1/7$. Find $y(1)$.

- (A) $\frac{2}{11}$
- (B) $-\frac{2}{11}$
- (C) $\frac{13}{11}$
- (D) $\frac{1}{11}$
- (E) $\frac{4}{11}$

36. A map $T : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $|T(x) - T(y)| \leq |x - y|/11$. If $T(x_*) = x_*$ and $|x_0 - x_*| \leq 1$, what is the smallest guaranteed upper bound for $|T^3(x_0) - x_*|$?

- (A) $\frac{1}{3993}$
- (B) $\frac{666}{1331}$
- (C) $\frac{1}{2662}$
- (D) $\frac{1330}{1331}$
- (E) $\frac{1}{1331}$

Questions 37–39

37. Evaluate $\int_0^1 \int_x^1 6y^3 \, dy \, dx$.

- (A) 2
- (B) $\frac{3}{2}$
- (C) $\frac{3}{10}$
- (D) $\frac{6}{5}$
- (E) 3

38. Let $u = (1, 2, \dots, 31)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $\det B$.

- (A) 495
- (B) 497
- (C) 498
- (D) 499
- (E) 496

39. Evaluate $\iiint_B 6(x^2 + y^2 + z^2)^{3/2} \, dV$, where B is the unit ball.

- (A) 8π
- (B) 2π
- (C) $1 + 4\pi$
- (D) 4π
- (E) -4π

Questions 40–42

40. The circle $|z| = 2$ is traversed counterclockwise. What is $\frac{1}{2\pi i} \int_{|z|=2} \frac{e^z}{z^{12}} dz$?

- (A) $11!$
- (B) $1/22!$
- (C) $1/10!$
- (D) $1/12!$
- (E) $1/11!$

41. Find the coefficient of x^6 in $\cos(6x)/(1 - 2x^2)$.

- (A) $-\frac{109}{5}$
- (B) -14976
- (C) $-\frac{104}{5}$
- (D) $-\frac{324}{5}$
- (E) $\frac{104}{5}$

42. Let D be a diagonal 31×31 real matrix with distinct diagonal entries. What is the dimension of the space of matrices X satisfying $XD = DX$?

- (A) 33
- (B) 31
- (C) 30
- (D) 32
- (E) 29

Questions 43–45

43. Let $F(x) = \int_0^{x^3} e^{6t} dt$. Find $F'(1)$.

- (A) $18e^6$
- (B) e^{18}
- (C) $3e^6$
- (D) $\frac{e^6}{3}$
- (E) e^6

44. Find the residue of $z^{11}/(z-1)^2$ at $z=1$.

- (A) 8
- (B) 5
- (C) 17
- (D) 11
- (E) 14

45. Evaluate $\int_0^\infty x^3 e^{-6x} dx$.

- (A) $1296 \left(\frac{3!}{6^4} \right)$
- (B) $\frac{1}{3} \left(\frac{3!}{6^4} \right)$
- (C) $6 \left(\frac{3!}{6^4} \right)$
- (D) $\frac{3!}{6^4}$
- (E) $\frac{1}{6} \left(\frac{3!}{6^4} \right)$

Questions 46–48

46. What is the largest integer k such that 2^k divides $34!$?

- (A) 30
- (B) 31
- (C) 33
- (D) 34
- (E) 32

47. The region $0 \leq x \leq 1$, $0 \leq y \leq 6x^3$ is revolved about the x -axis. Find its volume.

- (A) $\frac{36\pi}{7}$
- (B) $\frac{18\pi}{7}$
- (C) $1 + \frac{36\pi}{7}$
- (D) $\frac{72\pi}{7}$
- (E) $-\frac{36\pi}{7}$

48. What is the least positive multiplicative inverse of 2 modulo 23?

- (A) 4
- (B) 8
- (C) 12
- (D) 16
- (E) 20

Questions 49–51

49. Evaluate $\int_0^1 x^6(1-x)^2 dx$.

- (A) $\frac{6! 2!}{9!}$
- (B) $12 \left(\frac{6! 2!}{9!} \right)$
- (C) $\frac{1}{3} \left(\frac{6! 2!}{9!} \right)$
- (D) $9 \left(\frac{6! 2!}{9!} \right)$
- (E) $7 \left(\frac{6! 2!}{9!} \right)$

50. In the ring $\mathbb{Z}/174\mathbb{Z}$, how many elements are units?

- (A) 56
- (B) 55
- (C) 58
- (D) 54
- (E) 57

51. Find the coefficient of x^3 in $e^{2x} \log(1+6x)$.

- (A) 288
- (B) 72
- (C) 48
- (D) -48
- (E) 54

Questions 52–54

52. How many truth assignments to $p, q, r_1, \dots, r_9$ make $p \Rightarrow q$ false?

- (A) 2^{11}
- (B) 2^{12}
- (C) 2^9
- (D) 2^{10}
- (E) 2^8

53. Evaluate $\iiint_{x^2+y^2+z^2 \leq 6} (x^2 + y^2 + z^2)^2 dV$.

- (A) $\frac{1}{3} \left(\frac{4\pi 6^{\frac{7}{2}}}{7} \right)$
- (B) $\frac{4\pi 6^{\frac{7}{2}}}{7}$
- (C) $\frac{1}{2} \left(\frac{4\pi 6^{\frac{7}{2}}}{7} \right)$
- (D) $\frac{7}{30} \left(\frac{4\pi 6^{\frac{7}{2}}}{7} \right)$
- (E) $7 \left(\frac{4\pi 6^{\frac{7}{2}}}{7} \right)$

54. A permutation consists of disjoint cycles of lengths 7 and 4 and fixes all other points. What is its order?

- (A) 26
- (B) 29
- (C) 28
- (D) 30
- (E) 27

Questions 55–57

55. For $x > 0$, $x^2 y'' + (1 - 9)xy' + 18y = 0$, $y(1) = 0$, $y'(1) = 6$. Find $y(e)$.

- (A) $e^3 + e^6$
- (B) $2(1 - e^3)e^3$
- (C) $\frac{e^6}{9}$
- (D) $-2(1 - e^3)e^3$
- (E) $-e^3 + e^6$

56. What is the coefficient of x^2 in $(1 + 2x)^{11}$?

- (A) 366
- (B) 147
- (C) 74
- (D) 220
- (E) 293

57. Let C be the unit circle $x^2 + y^2 = 1$, $z = 3$, counterclockwise viewed from above. Evaluate $\int_C -6y(x^2 + y^2)^2 dx + (6x(x^2 + y^2)^2) dy$.

- (A) $1 + 12\pi$
- (B) 6π
- (C) 12π
- (D) 24π
- (E) -12π

Questions 58–60

58. In $R = \mathbb{F}_3[x]/(x^6)$, how many elements are there?

- (A) 3^8
- (B) 3^6
- (C) 3^5
- (D) 3^7
- (E) 3^9

59. Evaluate $\lim_{x \rightarrow 0} (1 + 6x)^{1/(2x)}$.

- (A) $e^{\frac{1}{3}}$
- (B) 6
- (C) e^3
- (D) e^7
- (E) e^{36}

60. How many edges does the complete bipartite graph $K_{11,13}$ have?

- (A) 190
- (B) 143
- (C) 49
- (D) 237
- (E) 96

Questions 61–63

61. Evaluate $\int_0^1 x^5(\log x)^2 dx$.

- (A) $\frac{1}{3} \left(\frac{(-1)^2 2!}{6^3} \right)$
- (B) $\frac{(-1)^2 2!}{6^3}$
- (C) $-\frac{(-1)^2 2!}{6^3}$
- (D) $18 \left(\frac{(-1)^2 2!}{6^3} \right)$
- (E) $6 \left(\frac{(-1)^2 2!}{6^3} \right)$

62. Integers x satisfy $x \equiv 0 \pmod{11}$ and $x \equiv 15 \pmod{16}$. Find the least nonnegative solution.

- (A) 142
- (B) 143
- (C) 144
- (D) 141
- (E) 145

63. Two solutions of $y'' + 6x^1y' + 3y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) e^{-3}
- (B) $\frac{2}{e^3}$
- (C) $e^{-3} + 1$
- (D) $-\frac{1}{e^3}$
- (E) $\frac{1}{2e^3}$

Questions 64–66

64. How many functions from a set of 16 elements onto a set of 3 elements exist?

- (A) $3^{16} - 3(2^{16}) + 3$
- (B) $3^{16} - 2^{16}$
- (C) 3^{16}
- (D) $3^{16} - 3(2^{16}) - 3$
- (E) $3^{16} - 3(2^{16})$

65. Let $f(x) = \log(1 + 6x)$. Find $f^{(3)}(0)$.

- (A) $-(-1)^2 2! 6^3$
- (B) $\frac{1}{6} \left((-1)^2 2! 6^3 \right)$
- (C) $3 \left((-1)^2 2! 6^3 \right)$
- (D) $(-1)^2 2! 6^3$
- (E) $\frac{1}{3} \left((-1)^2 2! 6^3 \right)$

66. The symmetric group S_{31} acts on $\{1, \dots, 31\}$. What is the index of the stabilizer of 1?

- (A) 31
- (B) 33
- (C) 30
- (D) 32
- (E) 29

Answer Key

Practice Test 29 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	B	23	C	45	D
2	C	24	D	46	E
3	A	25	C	47	A
4	A	26	E	48	C
5	C	27	B	49	A
6	B	28	D	50	A
7	E	29	D	51	C
8	E	30	B	52	C
9	D	31	B	53	B
10	E	32	D	54	C
11	A	33	E	55	D
12	A	34	E	56	D
13	C	35	A	57	C
14	E	36	E	58	B
15	A	37	D	59	C
16	D	38	B	60	B
17	E	39	D	61	B
18	B	40	E	62	B
19	E	41	C	63	A
20	A	42	B	64	A
21	C	43	C	65	D
22	E	44	D	66	A

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