

GRE Mathematics Practice Test 31

Advanced Challenge

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

The most demanding form in this five-test cycle. Emphasis on connected ideas, non-routine reasoning, and careful conceptual distinctions. This is cycle 6 of 19, position 1 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j(x-7)^j}{j^{\frac{5}{3}}(\log j)^m}$ converge absolutely at $x = 6$? Give the least such m , or 0 if none exists.

- (A) 4
- (B) 2
- (C) 1
- (D) 0
- (E) 3

2. Let $A = \begin{bmatrix} 1 & 27 & 0 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 3)$ entry of A^3 ?

- (A) 567
- (B) 565
- (C) 569
- (D) 566
- (E) 568

3. Solve $y' + 7y = x^6 e^{-7x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{2}{7e^7}$
- (B) $\frac{1}{6e^7}$
- (C) $\frac{e^7}{7}$
- (D) $\frac{1}{7e^7}$
- (E) e^{-7}

Questions 4–6

4. Let $X_1, \dots, X_{13}$ be independent uniform random variables on $[0, 1]$. What is $E[(\max_i X_i)^2]$?

- (A) $\frac{2}{15}$
- (B) $\frac{14}{15}$
- (C) $\frac{13}{15}$
- (D) $\frac{15}{13}$
- (E) $\frac{30}{13}$

5. Two solutions of $y'' + 7x^6y' + 6y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $-\frac{1}{e}$
- (B) e^{-1}
- (C) $\frac{2}{e}$
- (D) $e^{-1} + 1$
- (E) $\frac{1}{2e}$

6. An invertible 33×33 matrix A has determinant 3. Find $\det(\text{adj } A)$.

- (A) $3^{32} \cdot 5$
- (B) $2 \cdot 3^{32}$
- (C) 3^{32}
- (D) 3^{33}
- (E) 3^{31}

Questions 7–9

7. If $y'' + 36y = 7$ and $y(0) = y'(0) = 0$, find $y(\pi/6)$.

- (A) $-\frac{7}{18}$
- (B) $\frac{7}{18}$
- (C) $\frac{25}{18}$
- (D) $\frac{7}{9}$
- (E) $\frac{7}{36}$

8. Independent trials have success probability $1/13$. Let T count trials up to and including the first success. Find $E[T^2]$.

- (A) 217
- (B) 109
- (C) 433
- (D) 325
- (E) 541

9. Evaluate $\iiint_B 7z^2(x^2 + y^2 + z^2)^6 dV$, where B is the unit ball.

- (A) $\frac{14\pi}{51}$
- (B) $-\frac{28\pi}{51}$
- (C) $\frac{28\pi}{51}$
- (D) $\frac{56\pi}{51}$
- (E) $1 + \frac{28\pi}{51}$

Questions 10–12

10. On P_{34} , the real polynomials of degree at most 34, define $T(p) = x^2p'' - 8xp' + 20p$. Find its rank.

- (A) 33
- (B) 34
- (C) 32
- (D) 35
- (E) 31

11. For $\mathbf{F} = (7x^6y, y^6z, xz^6)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) 283
- (B) -282
- (C) 564
- (D) 141
- (E) 282

12. A uniformly random subset of 13 objects is selected from a collection with 14 red and 15 blue objects. If X is the number of red objects selected, find $\text{Var}(X)$.

- (A) $\frac{3120}{841}$
- (B) $\frac{2401}{841}$
- (C) $\frac{3242}{841}$
- (D) $\frac{780}{841}$
- (E) $\frac{1560}{841}$

Questions 13–15

13. Find the directional derivative of $f(x, y, z) = x^2y + 6xz^2 + 7y^2z$ at $(1, 1, 1)$ in the direction $(1, 2, 0)$.

- (A) $1 + \frac{38\sqrt{5}}{5}$
- (B) $-\frac{38\sqrt{5}}{5}$
- (C) $\frac{76\sqrt{5}}{5}$
- (D) $\frac{19\sqrt{5}}{5}$
- (E) $\frac{38\sqrt{5}}{5}$

14. Subspaces U, V of $\mathbb{R}^{35}$ have dimensions 6 and 7. What is the smallest possible value of $\dim(U \cap V)$?

- (A) 3
- (B) 2
- (C) 0
- (D) 1
- (E) 4

15. Find the coefficient of x^4 in $e^{5x} \log(1 + 7x)$.

- (A) -189
- (B) $-\frac{2401}{4}$
- (C) -4536
- (D) 189
- (E) -182

Questions 16–18

16. X, Y are independent Poisson variables with means 13 and 14. Given $X + Y = 15$, find $\text{Var}(X | X + Y)$.

- (A) $\frac{1820}{243}$
- (B) $\frac{455}{243}$
- (C) $\frac{1396}{243}$
- (D) $\frac{910}{243}$
- (E) $\frac{1153}{243}$

17. The local inverse $y(x)$ of $x = 7y + y^3$ satisfies $y(0) = 0$. Find $y^{(7)}(0)$.

- (A) $-\frac{60480}{7^{10}}$
- (B) $\frac{60480}{7^{10}}$
- (C) $\frac{1}{5040} \left(-\frac{60480}{7^{10}} \right)$
- (D) $7 \left(-\frac{60480}{7^{10}} \right)$
- (E) $\frac{1}{7} \left(-\frac{60480}{7^{10}} \right)$

18. Let V consist of real polynomials of degree at most 14 that have both value and first derivative zero at 27, 28, and 30. Find $\dim V$.

- (A) 7
- (B) 10
- (C) 11
- (D) 8
- (E) 9

Questions 19–21

19. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(7x) - \sum_{j=0}^4 (-1)^j (7x)^{2j+1} / (2j+1)!}{x^{11}}$.
- (A) $\frac{1}{7} \left(\frac{(-1)^5 7^{11}}{11!} \right)$
(B) $11 \left(\frac{(-1)^5 7^{11}}{11!} \right)$
(C) $-\frac{(-1)^5 7^{11}}{11!}$
(D) $7 \left(\frac{(-1)^5 7^{11}}{11!} \right)$
(E) $\frac{(-1)^5 7^{11}}{11!}$
20. X, Y are independent, with variances 13 and 14. Find $\text{Cov}(X + 2Y, 3X - Y)$.
- (A) 5
(B) 14
(C) 8
(D) 11
(E) 17
21. The region $0 \leq x \leq 1, 0 \leq y \leq 7x^6$ is revolved about the line $x = -2$. Find its volume.
- (A) $-\frac{23\pi}{4}$
(B) $\frac{23\pi}{4}$
(C) $1 + \frac{23\pi}{4}$
(D) $\frac{23\pi}{8}$
(E) $\frac{23\pi}{2}$

Questions 22–24

22. The roots r, s of $x^2 - 34x + 189 = 0$ are counted with multiplicity. Find $r^4 + s^4$.

- (A) $34^4 - 4(189)(34^2) + 2(189^2)$
- (B) $34^4 + 2(189^2)$
- (C) $34^4 - 2(189)(34^2) + 2(189^2)$
- (D) $34^4 - 4(189)(34^2) + 189^2$
- (E) $34^4 - 4(189)(34^2)$

23. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{7t} \cos(6t)$, and $y = e^{7t} \sin(6t)$, find d^2f/dt^2 at $t = 1$.

- (A) 12
- (B) 196
- (C) 14
- (D) 170
- (E) 0

24. A continuous $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(q) = q^8$ for every rational q . Find $f(2)$.

- (A) 2^{11}
- (B) 2^7
- (C) 2^{10}
- (D) 2^8
- (E) 2^9

Questions 25–27

25. Let C be the unit circle $x^2 + y^2 = 1$, $z = 6$, counterclockwise viewed from above. Evaluate $\int_C -7y(x^2 + y^2)^5 dx + (7x(x^2 + y^2)^5) dy + 2z dz$.

- (A) 14π
- (B) -14π
- (C) 7π
- (D) $1 + 14\pi$
- (E) 28π

26. For real t , let $M(t) = \begin{pmatrix} t-27 & 7 & 0 \\ 0 & t-27 & 1 \\ 0 & 0 & t-27 \end{pmatrix}$. At $t = 27$, what is $\dim \ker(M(t)^2) - \dim \ker M(t)$?

- (A) 4
- (B) 1
- (C) 3
- (D) 0
- (E) 2

27. If $y' = 14xy$ and $y(0) = 1$, find the coefficient of x^{12} in the Maclaurin series of y .

- (A) $\frac{117649}{360}$
- (B) $-\frac{117649}{720}$
- (C) $\frac{118369}{720}$
- (D) $\frac{117649}{720}$
- (E) $\frac{117649}{1440}$

Questions 28–30

28. Find the radius of convergence of $\sum_{k \geq 1} k^2 z^{2k} / 8^{2k} + \sum_{k \geq 0} z^{2k+1} / (9)^{2k+1}$.

- (A) 12
- (B) 8
- (C) 10
- (D) 6
- (E) 4

29. Evaluate $\int_0^{\pi/2} \sin^{13} t \cos^{15} t \, dt$.

- (A) $14 \left(\frac{6! 7!}{2 \cdot 14!} \right)$
- (B) $\frac{6! 7!}{2 \cdot 14!}$
- (C) $\frac{1}{2} \left(\frac{6! 7!}{2 \cdot 14!} \right)$
- (D) $1716 \left(\frac{6! 7!}{2 \cdot 14!} \right)$
- (E) $2 \left(\frac{6! 7!}{2 \cdot 14!} \right)$

30. Let $u = (6, 6)^T$ and $v = (1, 0)^T$. Find the squared distance from $(6, 0)^T$ to $\text{span}(u)$.

- (A) 18
- (B) 20
- (C) 17
- (D) 19
- (E) 16

Questions 31–33

31. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 7z^2 = x + 6y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) 84
- (B) -6
- (C) -42
- (D) -14
- (E) -84

32. On $[0, 1]$, f is 8 on rationals and $1/8$ on irrationals. Find its upper Darboux integral minus its lower Darboux integral.

- (A) $\frac{71}{8}$
- (B) $\frac{63}{4}$
- (C) $\frac{79}{8}$
- (D) $\frac{63}{8}$
- (E) $\frac{63}{16}$

33. Evaluate $\int_0^1 \int_x^1 7y^6 \, dy \, dx$.

- (A) $\frac{7}{2}$
- (B) $\frac{7}{6}$
- (C) $\frac{7}{8}$
- (D) 1
- (E) $\frac{1}{8}$

Questions 34–36

34. An idempotent 33×33 real matrix P has rank 2. What is $\det(I + P)$?

- (A) 5
- (B) 4
- (C) 3
- (D) 6
- (E) 2

35. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(7+6j)(13+6j)}$.

- (A) $\frac{1}{6}$
- (B) $\frac{1}{13}$
- (C) $\frac{6}{7}$
- (D) $\frac{1}{42}$
- (E) $\frac{1}{7}$

36. A set with 8 elements has the discrete topology. How many of its subsets are simultaneously open and closed?

- (A) 2^7
- (B) 2^8
- (C) 2^9
- (D) 2^{10}
- (E) 2^{11}

Questions 37–39

37. Evaluate $\int_0^\infty x^6 e^{-7x} dx$.

- (A) $7 \left(\frac{6!}{7^7} \right)$
- (B) $\frac{1}{7} \left(\frac{6!}{7^7} \right)$
- (C) 720
- (D) $\frac{1}{6} \left(\frac{6!}{7^7} \right)$
- (E) $\frac{6!}{7^7}$

38. For $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 27 \\ 7 \\ 34 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find the minimum squared residual.

- (A) $\frac{1106}{3}$
- (B) $\frac{2215}{6}$
- (C) $\frac{2209}{6}$
- (D) $\frac{2203}{6}$
- (E) $\frac{1103}{3}$

39. For the curve $x = t^6$, $y = t^7 + 7t$, find d^2y/dx^2 at $t = 1$.

- (A) $-\frac{14}{3}$
- (B) $\frac{7}{6}$
- (C) $-\frac{7}{9}$
- (D) $-\frac{7}{6}$
- (E) $\frac{7}{3}$

Questions 40–42

40. How many zeros, counted with multiplicity, does $z^8 + 11z + 1$ have in $|z| < 1$?
- (A) 0
(B) 2
(C) 3
(D) 1
(E) 4
41. Under $x = u + 7v$, $y = u - 6v$, what is the area of the image of $0 \leq u, v \leq 1$?
- (A) 13
(B) 169
(C) $\frac{1}{13}$
(D) 42
(E) 1
42. A permutation consists of disjoint cycles of lengths 3 and 8, together with a disjoint transposition when the sum of these two lengths is odd. It fixes all other points. What is its order?
- (A) 26
(B) 23
(C) 25
(D) 22
(E) 24

Questions 43–45

43. Evaluate $\int_0^{\log 2} e^x (e^x + 7)^6 dx$.

- (A) $-\frac{9^7 - 8^7}{7}$
- (B) $9^6 - 8^6$
- (C) $7 \left(\frac{9^7 - 8^7}{7} \right)$
- (D) $\frac{9^7}{7}$
- (E) $\frac{9^7 - 8^7}{7}$

44. A meromorphic function has 10 zeros and 2 poles inside a positively oriented simple closed contour, counted with multiplicity, and no zeros or poles on it. Find $\frac{1}{2\pi i} \int f'(z)/f(z) dz$.

- (A) 10
- (B) 12
- (C) 6
- (D) 8
- (E) 4

45. For $x > 0$, $x^2 y'' + (-19)xy' + 91y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $-\frac{e^7}{6} + 1 + \frac{e^{13}}{6}$
- (B) $-\frac{(1 - e^6) e^7}{6}$
- (C) $-\frac{(1 - e^6) e^7}{12}$
- (D) $-\frac{(1 - e^6) e^7}{3}$
- (E) $\frac{(1 - e^6) e^7}{6}$

Questions 46–48

46. In $R = \mathbb{F}_{11}[x]/(x^7)$, how many elements are units?

- (A) $11^7 - 11^0$
- (B) $11^9 - 11^8$
- (C) $11^8 - 11^7$
- (D) $11^7 - 11^6$
- (E) $11^8 - 11^6$

47. Evaluate $\int_0^1 \frac{x^5}{1+7x^6} dx$.

- (A) $\frac{\log(2)}{2}$
- (B) $-\frac{\log(2)}{14}$
- (C) $\log(8)$
- (D) $\frac{3 \log(2)}{7}$
- (E) $\frac{\log(2)}{14}$

48. How many nonnegative integer triples sum to 26, if each coordinate is at most 13?

- (A) 105
- (B) 175
- (C) 140
- (D) 35
- (E) 70

Questions 49–51

49. Let $f(x, y) = x^2 + 14xy + (55)y^2$. What is the product of the eigenvalues of the Hessian of f ?
- (A) 12
(B) 55
(C) 220
(D) 6
(E) 24
50. Integers x satisfy $x \equiv 1 \pmod{7}$ and $x \equiv 4 \pmod{9}$. Find the sum of all such x with $0 \leq x < 126$.
- (A) 107
(B) 109
(C) 108
(D) 105
(E) 106
51. If $y' = y(1 - y/6)$ and $y(0) = 3$, find $y(-\log 7)$.
- (A) $\frac{3}{4}$
(B) $\frac{7}{4}$
(C) $-\frac{3}{4}$
(D) $\frac{3}{2}$
(E) $\frac{3}{8}$

Questions 52–54

52. A tree has 13 vertices of degree 3; every other vertex has degree 1. How many leaves does it have?

- (A) 5
- (B) 25
- (C) 20
- (D) 10
- (E) 15

53. Find the coefficient of x^6 in $\cos(7x)/(1 - 5x^2)$.

- (A) -108499
- (B) $-\frac{117649}{720}$
- (C) $-\frac{108499}{720}$
- (D) $\frac{108499}{720}$
- (E) $-\frac{109219}{720}$

54. The symmetric group S_{33} acts on the two-element subsets of $\{1, \dots, 33\}$. What is the index of the setwise stabilizer of $\{1, 2\}$?

- (A) 527
- (B) 528
- (C) 529
- (D) 530
- (E) 526

Questions 55–57

55. If $y'' - 14y' + 49y = 6e^{7x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $3e^7$
- (B) $\frac{3e^7}{2}$
- (C) $-3e^7$
- (D) $6e^7$
- (E) $1 + 3e^7$

56. A connected planar graph has 16 vertices and 27 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 21
- (B) 17
- (C) 13
- (D) 5
- (E) 9

57. Let $f(x) = \log(1 + 7x)$. Find $f^{(6)}(0)$.

- (A) $\frac{1}{7} \left((-1)^5 5! 7^6 \right)$
- (B) $(-1)^5 5! 7^6$
- (C) $-(-1)^5 5! 7^6$
- (D) $\frac{1}{6} \left((-1)^5 5! 7^6 \right)$
- (E) $6 \left((-1)^5 5! 7^6 \right)$

Questions 58–60

58. In $\mathbb{Z}/786\mathbb{Z}$, how many residues are not idempotent ($x^2 \neq x$)?

- (A) 777
- (B) 778
- (C) 776
- (D) 780
- (E) 779

59. The solution of $y' = 7x^6y^3$ has $y(0) = 1/8$. Find $y(1)$.

- (A) $\frac{\sqrt{62}}{62}$
- (B) $\frac{\sqrt{62}}{31}$
- (C) $-\frac{\sqrt{62}}{62}$
- (D) $\frac{\sqrt{62}}{62} + 1$
- (E) $\frac{\sqrt{62}}{124}$

60. How many integers in $\{1, \dots, 78\}$ are divisible by 2 or 3?

- (A) 86
- (B) 69
- (C) 52
- (D) 35
- (E) 18

Questions 61–63

61. Find the curvature of the helix $r(t) = (7 \cos t, 7 \sin t, 6t)$.

- (A) $\frac{14}{85}$
- (B) $\frac{7}{85}$
- (C) $-\frac{7}{85}$
- (D) $\frac{92}{85}$
- (E) $\frac{7}{170}$

62. In the additive group $\mathbb{Z}/60\mathbb{Z}$, find the index of the subgroup generated by $[2]$.

- (A) 0
- (B) 4
- (C) 1
- (D) 2
- (E) 3

63. Let $F(x) = \int_0^x (x-t)^5 e^{7t} dt$. Find $F^{(6)}(0)$.

- (A) 720
- (B) 0
- (C) $\frac{1}{120}$
- (D) 120
- (E) 840

Questions 64–66

64. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 13 have no adjacent 1s?

- (A) F_{14}
- (B) F_{17}
- (C) F_{15}
- (D) F_{13}
- (E) F_{16}

65. Evaluate $\int_0^1 x^6(\log x)^5 dx$.

- (A) $\frac{16807}{120} \left(\frac{(-1)^5 5!}{7^6} \right)$
- (B) $\frac{(-1)^5 5!}{7^6}$
- (C) $-\frac{(-1)^5 5!}{7^6}$
- (D) $\frac{1}{6} \left(\frac{(-1)^5 5!}{7^6} \right)$
- (E) $7 \left(\frac{(-1)^5 5!}{7^6} \right)$

66. How many invertible 4×4 matrices over $\mathbb{F}_{13}$ are there with determinant 1?

- (A) $\frac{13^4 13^4 13^4 13^4}{12}$
- (B) $\frac{(13^4 - 13^0)(13^4 - 13^1)(13^4 - 13^2)}{12}$
- (C) $\frac{(13^4 - 13^0)(13^4 - 13^1)(13^4 - 13^2)(13^4 - 13^3)}{12}$
- (D) $\frac{(13^4 - 13^1)(13^4 - 13^2)(13^4 - 13^3)(13^4 - 13^4)}{12}$
- (E) $\frac{(13^4 - 13^0)(13^4 - 13^0)(13^4 - 13^0)(13^4 - 13^0)}{12}$

Answer Key

Practice Test 31 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	C	23	E	45	B
2	A	24	D	46	D
3	D	25	A	47	E
4	C	26	B	48	A
5	B	27	D	49	E
6	C	28	B	50	A
7	B	29	B	51	A
8	D	30	A	52	E
9	C	31	E	53	C
10	A	32	D	54	B
11	E	33	C	55	A
12	E	34	B	56	C
13	E	35	D	57	B
14	C	36	B	58	B
15	A	37	E	59	A
16	D	38	C	60	C
17	A	39	C	61	B
18	E	40	D	62	D
19	E	41	A	63	D
20	D	42	E	64	C
21	B	43	E	65	B
22	A	44	D	66	C

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