

GRE Mathematics Practice Test 32

Advanced Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A demanding practice form with frequent multi-step arguments and a mix of computational and conceptual questions. This is cycle 6 of 19, position 2 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. For $\mathbf{F} = (7x^5y, y^5z, xz^5)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) $\frac{155}{2}$
- (B) -155
- (C) 155
- (D) 310
- (E) 156

2. The roots r, s of $x^2 - 33x + 140 = 0$ are counted with multiplicity. Find $r^3 + s^3$.

- (A) 22075
- (B) 22078
- (C) 22077
- (D) 22076
- (E) 22079

3. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(7+5j)(12+5j)}$.

- (A) $\frac{1}{7}$
- (B) $\frac{1}{5}$
- (C) $\frac{1}{12}$
- (D) $\frac{5}{7}$
- (E) $\frac{1}{35}$

Questions 4–6

4. Let $X_1, \dots, X_{14}$ be independent uniform random variables on $[0, 1]$. What is $E[(\max_i X_i)^2]$?

- (A) $\frac{7}{16}$
- (B) $\frac{7}{24}$
- (C) $\frac{15}{16}$
- (D) $\frac{1}{8}$
- (E) $\frac{7}{8}$

5. What is the maximum of x^5y^7 over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\frac{1}{2^{12}}$
- (B) $\frac{35}{144}$
- (C) $\left(\frac{5}{12}\right)^7 \left(\frac{7}{12}\right)^5$
- (D) $\left(\frac{5}{12}\right)^5 \left(\frac{7}{12}\right)^7$
- (E) $12 \left(\left(\frac{5}{12}\right)^5 \left(\frac{7}{12}\right)^7 \right)$

6. For real t , let $M(t) = \begin{pmatrix} t-28 & 5 & 0 \\ 0 & t-28 & 1 \\ 0 & 0 & t-28 \end{pmatrix}$. At $t = 28$, what is $\dim \ker(M(t)^2) - \dim \ker M(t)$?

- (A) 0
- (B) 2
- (C) 4
- (D) 3
- (E) 1

Questions 7–9

7. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{7t} \cos(5t)$, and $y = e^{7t} \sin(5t)$, find d^2f/dt^2 at $t = 1$.

- (A) 0
- (B) 196
- (C) 148
- (D) 10
- (E) 14

8. Independent trials have success probability $1/14$. Let T count trials up to and including the first success. Find $E[T^2]$.

- (A) 504
- (B) 126
- (C) 378
- (D) 252
- (E) 630

9. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 7z^2 = x + 5y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) -70
- (B) 70
- (C) -14
- (D) -5
- (E) -35

Questions 10–12

10. Let $u = (6, 5)^T$ and $v = (1, 0)^T$. Find the squared distance from $(6, 0)^T$ to $\text{span}(u)$.
- (A) $\frac{1861}{122}$
(B) $\frac{839}{61}$
(C) $\frac{900}{61}$
(D) $\frac{961}{61}$
(E) $\frac{1739}{122}$
11. Find the upward flux of $\mathbf{F} = (x, y, z)$ through $z = 7x^2 + 5y^2$, $0 \leq x, y \leq 1$.
- (A) -8
(B) -4
(C) -3
(D) -2
(E) 4
12. A uniformly random subset of 14 objects is selected from a collection with 15 red and 16 blue objects. If X is the number of red objects selected, find $E[X]$.
- (A) $\frac{272}{31}$
(B) $\frac{241}{31}$
(C) $\frac{420}{31}$
(D) $\frac{105}{31}$
(E) $\frac{210}{31}$

Questions 13–15

13. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-7)^j}{j^{\frac{4}{3}} (\log j)^m}$ converge absolutely at $x = 6$? Give the least such m , or 0 if none exists.

- (A) 4
- (B) 1
- (C) 3
- (D) 0
- (E) 2

14. An idempotent 34×34 real matrix P has rank 3. What is $\det(I + P)$?

- (A) 10
- (B) 8
- (C) 9
- (D) 6
- (E) 7

15. Evaluate $\int_0^{\log 2} e^x (e^x + 7)^5 dx$.

- (A) $\frac{9^6 - 8^6}{6}$
- (B) $\frac{9^6}{6}$
- (C) $6 \left(\frac{9^6 - 8^6}{6} \right)$
- (D) $-\frac{9^6 - 8^6}{6}$
- (E) $9^5 - 8^5$

Questions 16–18

16. X, Y are independent Poisson variables with means 14 and 15. Given $X + Y = 16$, find $\text{Var}(X | X + Y)$.

- (A) $\frac{5042}{841}$
- (B) $\frac{1680}{841}$
- (C) $\frac{3360}{841}$
- (D) $\frac{6720}{841}$
- (E) $\frac{4201}{841}$

17. Solve $y' + 7y = x^5 e^{-7x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{1}{5e^7}$
- (B) e^{-7}
- (C) $\frac{e^7}{6}$
- (D) $\frac{1}{6e^7}$
- (E) $\frac{7}{6e^7}$

18. For $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 28 \\ 5 \\ 33 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find the minimum squared residual.

- (A) $\frac{867}{2}$
- (B) 433
- (C) $\frac{869}{2}$
- (D) 434
- (E) $\frac{865}{2}$

Questions 19–21

19. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^7 y^4 dV$.

- (A) $\frac{1}{40}$
- (B) $6 \left(\frac{7! 4!}{14!} \right)$
- (C) $\frac{1}{6} \left(\frac{7! 4!}{14!} \right)$
- (D) $\frac{7! 4!}{14!}$
- (E) $14 \left(\frac{7! 4!}{14!} \right)$

20. X, Y are independent, with variances 14 and 15. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 8
- (B) 4
- (C) 12
- (D) 16
- (E) 20

21. If $y' = 14xy$ and $y(0) = 1$, find the coefficient of x^{10} in the Maclaurin series of y .

- (A) $\frac{16807}{120}$
- (B) $-\frac{16807}{120}$
- (C) $\frac{16807}{60}$
- (D) $\frac{16807}{240}$
- (E) $\frac{16927}{120}$

Questions 22–24

22. What is the coefficient of x^7 in $(x + 6)^7(1 - x)^3$?

- (A) -5416
- (B) -5417
- (C) -5418
- (D) -5415
- (E) -5419

23. Two solutions of $y'' + 7x^5y' + 5y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $\frac{1}{2e^{\frac{7}{6}}}$
- (B) $e^{-\frac{7}{6}} + 1$
- (C) $\frac{2}{e^{\frac{7}{6}}}$
- (D) $-\frac{1}{e^{\frac{7}{6}}}$
- (E) $e^{-\frac{7}{6}}$

24. A continuous $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(q) = q^9$ for every rational q . Find $f(2)$.

- (A) 2^9
- (B) 2^{12}
- (C) 2^{11}
- (D) 2^{10}
- (E) 2^8

Questions 25–27

25. The solution of $y' = 7x^5y^3$ has $y(0) = 1/8$. Find $y(1)$.

- (A) $-\frac{\sqrt{555}}{185}$
- (B) $\frac{\sqrt{555}}{370}$
- (C) $\frac{2\sqrt{555}}{185}$
- (D) $\frac{\sqrt{555}}{185}$
- (E) $\frac{\sqrt{555}}{185} + 1$

26. A matrix A satisfies $(A - 29I)(A - 31I) = 0$, and its minimal polynomial has degree two. If $A^5 = uA + vI$, find u .

- (A) 66541
- (B) $61 \cdot 66541$
- (C) $5 \cdot 61 \cdot 66541$
- (D) $3 \cdot 61 \cdot 66541$
- (E) $2 \cdot 61 \cdot 66541$

27. Evaluate $\int_0^{\pi/2} \sin^{11} t \cos^{15} t \, dt$.

- (A) $2 \left(\frac{5! 7!}{2 \cdot 13!} \right)$
- (B) $13 \left(\frac{5! 7!}{2 \cdot 13!} \right)$
- (C) $\frac{1}{2} \left(\frac{5! 7!}{2 \cdot 13!} \right)$
- (D) $\frac{5! 7!}{2 \cdot 13!}$
- (E) $792 \left(\frac{5! 7!}{2 \cdot 13!} \right)$

Questions 28–30

28. Find the radius of convergence of $\sum_{k \geq 1} k^2 z^{2k} / 9^{2k} + \sum_{k \geq 0} z^{2k+1} / (10)^{2k+1}$.

- (A) 15
- (B) 12
- (C) 3
- (D) 6
- (E) 9

29. Evaluate $\int_0^1 x^6 (\log x)^4 dx$.

- (A) $\frac{2401}{24} \left(\frac{(-1)^4 4!}{7^5} \right)$
- (B) $-\frac{(-1)^4 4!}{7^5}$
- (C) $7 \left(\frac{(-1)^4 4!}{7^5} \right)$
- (D) $\frac{1}{5} \left(\frac{(-1)^4 4!}{7^5} \right)$
- (E) $\frac{(-1)^4 4!}{7^5}$

30. Let $V = \begin{bmatrix} 1 & 28 & 784 \\ 1 & 30 & 900 \\ 1 & 33 & 1089 \end{bmatrix}$. Find $\det(V^T V)$.

- (A) 902
- (B) 898
- (C) 900
- (D) 899
- (E) 901

Questions 31–33

31. Find the coefficient of x^6 in $\cos(7x)/(1 - 4x^2)$.

- (A) $\frac{65689}{720}$
- (B) $-\frac{66409}{720}$
- (C) $-\frac{65689}{720}$
- (D) $-\frac{65689}{720}$
- (E) $-\frac{117649}{720}$

32. On $[0, 1]$, f is 9 on rationals and $1/9$ on irrationals. Find its upper Darboux integral minus its lower Darboux integral.

- (A) $\frac{89}{9}$
- (B) $\frac{80}{9}$
- (C) $\frac{160}{9}$
- (D) $\frac{40}{9}$
- (E) $\frac{98}{9}$

33. Let C be the unit circle $x^2 + y^2 = 1$, $z = 5$, counterclockwise viewed from above. Evaluate $\int_C -7y(x^2 + y^2)^4 dx + (7x(x^2 + y^2)^4) dy + 3z dz$.

- (A) $1 + 14\pi$
- (B) 7π
- (C) -14π
- (D) 14π
- (E) 28π

Questions 34–36

34. Let $u = (1, 2, \dots, 34)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $v^T B^{-1}u$.

- (A) $\frac{595}{596}$
- (B) $\frac{596}{297}$
- (C) $\frac{596}{1191}$
- (D) $\frac{596}{1489}$
- (E) $\frac{893}{596}$

35. Let $f(x) = \log(1 + 7x)$. Find $f^{(5)}(0)$.

- (A) $5 \left((-1)^4 4! 7^5 \right)$
- (B) $\frac{1}{5} \left((-1)^4 4! 7^5 \right)$
- (C) $-(-1)^4 4! 7^5$
- (D) $(-1)^4 4! 7^5$
- (E) $\frac{1}{7} \left((-1)^4 4! 7^5 \right)$

36. A set with 9 elements has the discrete topology. How many of its subsets are simultaneously open and closed?

- (A) 2^{11}
- (B) 2^9
- (C) 2^8
- (D) 2^{12}
- (E) 2^{10}

Questions 37–39

37. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(7x) - \sum_{j=0}^3 (-1)^j (7x)^{2j+1} / (2j+1)!}{x^9}$.

- (A) $\frac{1}{7} \left(\frac{(-1)^4 7^9}{9!} \right)$
(B) $7 \left(\frac{(-1)^4 7^9}{9!} \right)$
(C) $9 \left(\frac{(-1)^4 7^9}{9!} \right)$
(D) $-\frac{(-1)^4 7^9}{9!}$
(E) $\frac{(-1)^4 7^9}{9!}$

38. Let $D = \text{diag}(\underbrace{1, \dots, 1}_5, \underbrace{2, \dots, 2}_{29})$. Find the dimension of the space of real matrices X with $XD = DX$.

- (A) 866
(B) 867
(C) 865
(D) 864
(E) 868

39. If $y' + 7y = 5x^5 e^{-7x}$ and $y(0) = 3$, find $y(1)$.

- (A) $\frac{23}{12e^7}$
(B) $\frac{23}{6e^7}$
(C) $\frac{23}{3e^7}$
(D) $-\frac{23}{6e^7}$
(E) $\frac{23}{6e^7} + 1$

Questions 40–42

40. How many zeros, counted with multiplicity, does $z^9 + 12z + 1$ have in $|z| < 1$?

- (A) 0
- (B) 2
- (C) 4
- (D) 1
- (E) 3

41. Evaluate $\iiint_B 7z^2(x^2 + y^2 + z^2)^5 dV$, where B is the unit ball.

- (A) $1 + \frac{28\pi}{45}$
- (B) $\frac{14\pi}{45}$
- (C) $\frac{28\pi}{45}$
- (D) $\frac{56\pi}{45}$
- (E) $-\frac{28\pi}{45}$

42. How many permutations in S_{34} consist of one 6-cycle and 28 fixed points?

- (A) $2^6 \cdot 3 \cdot 5^2 \cdot 11 \cdot 17 \cdot 29 \cdot 31$
- (B) $2^6 \cdot 3^2 \cdot 5 \cdot 11 \cdot 17 \cdot 29 \cdot 31$
- (C) $2^7 \cdot 3 \cdot 5 \cdot 11 \cdot 17 \cdot 29 \cdot 31$
- (D) $2^6 \cdot 3 \cdot 5 \cdot 11 \cdot 17 \cdot 29 \cdot 31$
- (E) $2^5 \cdot 3 \cdot 5 \cdot 11 \cdot 17 \cdot 29 \cdot 31$

Questions 43–45

43. Let C be $x^2/25 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C \left(-\frac{7y^3}{3}\right) dx + \left(\frac{x^3}{3}\right) dy$.

- (A) 80π
- (B) $1 + 40\pi$
- (C) 40π
- (D) 20π
- (E) -40π

44. A meromorphic function has 11 zeros and 2 poles inside a positively oriented simple closed contour, counted with multiplicity, and no zeros or poles on it. Find $\frac{1}{2\pi i} \int f'(z)/f(z) dz$.

- (A) 9
- (B) 6
- (C) 12
- (D) 15
- (E) 3

45. For $f(x, y) = x^2 + 7xy + 5y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) 34
- (B) $\sqrt{34}$
- (C) -3
- (D) -5
- (E) 8

Questions 46–48

46. How many maximal proper subgroups does the cyclic group of order 384 have?

- (A) 2
- (B) 4
- (C) 0
- (D) 1
- (E) 3

47. For the curve $x = t^5$, $y = t^6 + 7t$, find d^2y/dx^2 at $t = 1$.

- (A) $-\frac{22}{5}$
- (B) $\frac{13}{5}$
- (C) $-\frac{22}{25}$
- (D) $-\frac{7}{5}$
- (E) $\frac{34}{25}$

48. How many nonnegative integer triples sum to 28, if each coordinate is at most 14?

- (A) 160
- (B) 120
- (C) 200
- (D) 40
- (E) 80

Questions 49–51

49. Find the total outward flux of $\mathbf{F} = (7x^5y, y^5z, xz^5)$ through the boundary of $[0, 1]^3$.

- (A) $\frac{9}{2}$
- (B) $-\frac{9}{2}$
- (C) $\frac{9}{4}$
- (D) 9
- (E) $\frac{11}{2}$

50. How many residues x modulo 42 satisfy $7x \equiv 21 \pmod{42}$ and $2x \equiv 0 \pmod{42}$?

- (A) 2
- (B) 4
- (C) 3
- (D) 1
- (E) 0

51. The local inverse $y(x)$ of $x = 7y + y^3$ satisfies $y(0) = 0$. Find $y^{(5)}(0)$.

- (A) $\frac{1}{7} \left(\frac{360}{7^7} \right)$
- (B) $\frac{1}{120} \left(\frac{360}{7^7} \right)$
- (C) $7 \left(\frac{360}{7^7} \right)$
- (D) $-\frac{360}{7^7}$
- (E) $\frac{360}{7^7}$

Questions 52–54

52. A tree has 14 vertices of degree 3; every other vertex has degree 1. How many leaves does it have?

- (A) 6
- (B) 11
- (C) 26
- (D) 21
- (E) 16

53. For $x > 0$, $x^2 y'' + (1 - 12)xy' + 35y = 0$, $y(1) = 0$, $y'(1) = 7$. Find $y(e)$.

- (A) $\frac{e^7}{12}$
- (B) $-\frac{7(1 - e^2)e^5}{2}$
- (C) $e^5 + e^7$
- (D) $\frac{7(1 - e^2)e^5}{2}$
- (E) $-e^5 + e^7$

54. The field K has 7^7 elements. Find the largest possible multiplicative order of a nonzero element of K .

- (A) $7^7 + 1$
- (B) $7^7 - 1$
- (C) $7^8 - 1$
- (D) 7^7
- (E) $7^6 - 1$

Questions 55–57

55. Find the curvature of the helix $r(t) = (7 \cos t, 7 \sin t, 5t)$.

- (A) $\frac{81}{74}$
- (B) $\frac{7}{148}$
- (C) $\frac{7}{37}$
- (D) $\frac{7}{74}$
- (E) $-\frac{7}{74}$

56. A connected planar graph has 17 vertices and 29 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 14
- (B) 18
- (C) 6
- (D) 10
- (E) 22

57. Evaluate $\lim_{x \rightarrow 0} \frac{e^{7x} - \sum_{j=0}^4 (7x)^j / j!}{x^5}$.

- (A) $120 \left(\frac{7^5}{5!} \right)$
- (B) $5 \left(\frac{7^5}{5!} \right)$
- (C) $-\frac{7^5}{5!}$
- (D) $\frac{7^5}{5!}$
- (E) $\frac{1}{7} \left(\frac{7^5}{5!} \right)$

Questions 58–60

58. Let $G = \mathbb{Z}/198\mathbb{Z} \times \mathbb{Z}/136\mathbb{Z}$. Find the order of G/H , where $H = \{g \in G : 2g = 0\}$.

- (A) 6733
- (B) 6734
- (C) 6731
- (D) 6730
- (E) 6732

59. For $D = \{(x, y) : x^2 + y^2 \leq 7\}$, evaluate $\iint_D (x^2 + y^2)^4 dA$.

- (A) $\frac{33614\pi}{5}$
- (B) 16807π
- (C) $\frac{16807\pi}{5}$
- (D) $\frac{2401\pi}{5}$
- (E) $\frac{16807\pi}{9}$

60. How many integers in $\{1, \dots, 84\}$ are divisible by 2 or 3?

- (A) 38
- (B) 92
- (C) 74
- (D) 20
- (E) 56

Questions 61–63

61. Under $x = u + 7v$, $y = u - 5v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) 35
- (B) 144
- (C) 12
- (D) $\frac{1}{12}$
- (E) 2

62. How many group homomorphisms $\mathbb{Z}/198\mathbb{Z} \rightarrow \mathbb{Z}/136\mathbb{Z}$ have image of order 2?

- (A) 1
- (B) 0
- (C) 4
- (D) 3
- (E) 2

63. Find the coefficient of x^5 in the Taylor series of $(1 - 7x)^{-2}$ at zero.

- (A) $\frac{1}{6} (6 \cdot 7^5)$
- (B) $-6 \cdot 7^5$
- (C) $\frac{1}{7} (6 \cdot 7^5)$
- (D) $\frac{5}{6} (6 \cdot 7^5)$
- (E) $6 \cdot 7^5$

Questions 64–66

64. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 14 have no adjacent 1s?

- (A) F_{14}
- (B) F_{15}
- (C) F_{18}
- (D) F_{17}
- (E) F_{16}

65. If $y' = y(1 - y/5)$ and $y(0) = 5/2$, find $y(-\log 7)$.

- (A) $\frac{5}{4}$
- (B) $\frac{5}{8}$
- (C) $\frac{13}{8}$
- (D) $-\frac{5}{8}$
- (E) $\frac{5}{16}$

66. How many 2-dimensional subspaces does $\mathbb{F}_{13}^6$ have?

- (A) $\frac{(13^9 - 1)(13^9 - 13)}{(13^2 - 1)(13^2 - 13)}$
- (B) $\frac{(13^8 - 1)(13^8 - 13)}{(13^2 - 1)(13^2 - 13)}$
- (C) $\frac{(13^6 - 1)(13^6 - 13)}{(13^2 - 1)(13^2 - 13)}$
- (D) $\frac{(13^5 - 1)(13^5 - 13)}{(13^2 - 1)(13^2 - 13)}$
- (E) $\frac{(13^7 - 1)(13^7 - 13)}{(13^2 - 1)(13^2 - 13)}$

Answer Key

Practice Test 32 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	C	23	E	45	B
2	C	24	A	46	A
3	E	25	D	47	C
4	E	26	B	48	B
5	D	27	D	49	A
6	E	28	E	50	D
7	A	29	E	51	E
8	C	30	C	52	E
9	A	31	D	53	B
10	C	32	B	54	B
11	B	33	D	55	D
12	E	34	A	56	A
13	B	35	D	57	D
14	B	36	B	58	E
15	A	37	E	59	C
16	C	38	A	60	E
17	D	39	B	61	C
18	A	40	D	62	A
19	D	41	C	63	E
20	C	42	D	64	E
21	A	43	C	65	B
22	B	44	A	66	C

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