

GRE Mathematics Practice Test 33

Upper-Intermediate Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

Intermediate reinforcement, combining standard applications with selected more demanding arguments. This is cycle 6 of 19, position 3 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Let $f(x) = \log(1 + 7x)$. Find $f^{(4)}(0)$.

- (A) $\frac{1}{7} \left((-1)^3 3! 7^4 \right)$
- (B) $(-1)^3 3! 7^4$
- (C) $-(-1)^3 3! 7^4$
- (D) $\frac{1}{4} \left((-1)^3 3! 7^4 \right)$
- (E) $4 \left((-1)^3 3! 7^4 \right)$

2. What is the coefficient of x^7 in $(x + 6)^6(1 - x)^3$?

- (A) -435
- (B) -437
- (C) -433
- (D) -436
- (E) -434

3. If $y'' - 14y' + 49y = 4e^{7x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $2e^7$
- (B) $-2e^7$
- (C) e^7
- (D) $1 + 2e^7$
- (E) $4e^7$

Questions 4–6

4. Let $X_1, \dots, X_{15}$ be independent uniform random variables on $[0, 1]$. What is $E[(\max_i X_i)^2]$?
- (A) $\frac{5}{17}$
(B) $\frac{15}{17}$
(C) $\frac{15}{34}$
(D) $\frac{2}{17}$
(E) $\frac{16}{17}$
5. Evaluate $\iiint_B 7z^2(x^2 + y^2 + z^2)^4 dV$, where B is the unit ball.
- (A) $1 + \frac{28\pi}{39}$
(B) $-\frac{28\pi}{39}$
(C) $\frac{28\pi}{39}$
(D) $\frac{14\pi}{39}$
(E) $\frac{56\pi}{39}$
6. A matrix A satisfies $(A - 30I)(A - 32I) = 0$, and its minimal polynomial has degree two. If $A^4 = uA + vI$, find u .
- (A) 119287
(B) 119286
(C) 119290
(D) 119288
(E) 119289

Questions 7–9

7. Let C be $x^2/16 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C \left(-\frac{7y^3}{3}\right) dx + \left(\frac{x^3}{3}\right) dy$.
- (A) -23π
(B) 23π
(C) $1 + 23\pi$
(D) 46π
(E) $\frac{23\pi}{2}$
8. Independent trials have success probability $1/15$. Let T count trials up to and including the first success. Find $E[T]$.
- (A) 10
(B) 5
(C) 15
(D) 20
(E) 25
9. Evaluate $\int_C (4x^3 + 7y) dx + (7x + 4y^3) dy$, where C is $r(t) = (t, 4t^2)$, $0 \leq t \leq 1$.
- (A) $\frac{285}{2}$
(B) 285
(C) 570
(D) 286
(E) -285

Questions 10–12

10. Let $V = \begin{bmatrix} 1 & 29 & 841 \\ 1 & 31 & 961 \\ 1 & 34 & 1156 \end{bmatrix}$. Find $\det(V^T V)$.

- (A) 902
- (B) 899
- (C) 900
- (D) 898
- (E) 901

11. Evaluate $\int_0^{\log 2} e^x (e^x + 7)^4 dx$.

- (A) $\frac{9^5 - 8^5}{5}$
- (B) $-\frac{9^5 - 8^5}{5}$
- (C) $5 \left(\frac{9^5 - 8^5}{5} \right)$
- (D) $\frac{9^5}{5}$
- (E) $9^4 - 8^4$

12. A uniformly random subset of 15 objects is selected from a collection with 16 red and 17 blue objects. If X is the number of red objects selected, find $E[X]$.

- (A) $\frac{102}{11}$
- (B) $\frac{91}{11}$
- (C) $\frac{160}{11}$
- (D) $\frac{80}{11}$
- (E) $\frac{40}{11}$

Questions 13–15

13. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(7x) - \sum_{j=0}^2 (-1)^j (7x)^{2j+1} / (2j+1)!}{x^7}$.
- (A) $-\frac{(-1)^3 7^7}{7!}$
(B) $7 \left(\frac{(-1)^3 7^7}{7!} \right)$
(C) $\frac{1}{7} \left(\frac{(-1)^3 7^7}{7!} \right)$
(D) $\frac{(-1)^3 7^7}{7!}$
(E) $2 \left(\frac{(-1)^3 7^7}{7!} \right)$
14. Let $u = (1, 2, \dots, 35)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $\det B$.
- (A) 631
(B) 632
(C) 630
(D) 633
(E) 629
15. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(7j)!}{(j!)^7} x^{4j}$?
- (A) $7^{-\frac{4}{7}}$
(B) $\frac{1}{7}$
(C) $7^{\frac{7}{4}}$
(D) 7^{-7}
(E) $7^{-\frac{7}{4}}$

Questions 16–18

16. X, Y are independent Poisson variables with means 15 and 16. Given $X + Y = 17$, find $\text{Var}(X | X + Y)$.

- (A) $\frac{4080}{961}$
- (B) $\frac{5041}{961}$
- (C) $\frac{6002}{961}$
- (D) $\frac{8160}{961}$
- (E) $\frac{2040}{961}$

17. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-7)^j}{j^1 (\log j)^m}$ converge absolutely at $x = 6$? Give the least such m , or 0 if none exists.

- (A) 2
- (B) 0
- (C) 1
- (D) 3
- (E) 4

18. Let $D = \text{diag}(\underbrace{1, \dots, 1}_5, \underbrace{2, \dots, 2}_{30})$. Find the dimension of the space of real matrices X with $XD = DX$.

- (A) 927
- (B) 926
- (C) 925
- (D) 923
- (E) 924

Questions 19–21

19. Evaluate $\int_0^1 \int_0^x x^7 y^4 dy dx$.

- (A) $\frac{1}{60}$
- (B) $\frac{1}{78}$
- (C) $\frac{1}{13}$
- (D) $\frac{1}{65}$
- (E) $\frac{1}{40}$

20. X, Y are independent, with variances 15 and 16. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 9
- (B) 13
- (C) 17
- (D) 21
- (E) 5

21. Evaluate $\int_0^\infty x^4 e^{-7x} dx$.

- (A) $16807 \left(\frac{4!}{7^5} \right)$
- (B) $\frac{1}{4} \left(\frac{4!}{7^5} \right)$
- (C) $\frac{4!}{7^5}$
- (D) $\frac{1}{7} \left(\frac{4!}{7^5} \right)$
- (E) $7 \left(\frac{4!}{7^5} \right)$

Questions 22–24

22. When $x^3 + 1$ is divided by $(x - 30)(x - 31)$, the remainder is $ux + v$. Find u .

- (A) 2792
- (B) 2793
- (C) 2789
- (D) 2791
- (E) 2790

23. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(7 + 4j)(11 + 4j)}$.

- (A) $\frac{4}{7}$
- (B) $\frac{1}{7}$
- (C) $\frac{1}{4}$
- (D) $\frac{1}{11}$
- (E) $\frac{1}{28}$

24. A continuous $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(q) = q^{10}$ for every rational q . Find $f(2)$.

- (A) 2^9
- (B) 2^{10}
- (C) 2^{11}
- (D) 2^{13}
- (E) 2^{12}

Questions 25–27

25. Find the directional derivative of $f(x, y, z) = x^2y + 4xz^2 + 7y^2z$ at $(1, 1, 1)$ in the direction $(1, 4, 0)$.

- (A) $\frac{66\sqrt{17}}{17}$
- (B) $\frac{132\sqrt{17}}{17}$
- (C) $1 + \frac{66\sqrt{17}}{17}$
- (D) $-\frac{66\sqrt{17}}{17}$
- (E) $\frac{33\sqrt{17}}{17}$

26. Let $P = \text{diag}(\underbrace{1, \dots, 1}_3, \underbrace{0, \dots, 0}_{33})$. What is the dimension of the space of 36×36 real matrices X satisfying $PX = X$ and $XP = 0$?

- (A) 101
- (B) 98
- (C) 99
- (D) 97
- (E) 100

27. What is the maximum of x^4y^7 over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $11 \left(\left(\frac{4}{11} \right)^4 \left(\frac{7}{11} \right)^7 \right)$
- (B) $\left(\frac{4}{11} \right)^7 \left(\frac{7}{11} \right)^4$
- (C) $\frac{1}{2^{11}}$
- (D) $\frac{28}{121}$
- (E) $\left(\frac{4}{11} \right)^4 \left(\frac{7}{11} \right)^7$

Questions 28–30

28. Find the radius of convergence of $\sum_{k=1}^{\infty} k^{10} x^k / 10^k$.

- (A) 10
- (B) 7
- (C) 16
- (D) 13
- (E) 4

29. Find the coefficient of x^6 in $\cos(7x)/(1 - 3x^2)$.

- (A) -40879
- (B) $\frac{40879}{720}$
- (C) $-\frac{41599}{720}$
- (D) $-\frac{117649}{720}$
- (E) $-\frac{40879}{720}$

30. A 3×3 matrix A has characteristic polynomial $(t - 29)(t + 1)(t - 2)$. What is $\text{tr}(A^3)$?

- (A) 24394
- (B) 24395
- (C) 24398
- (D) 24396
- (E) 24397

Questions 31–33

31. For $f(x) = x^4 e^{7x}$, find $f^{(6)}(0)$.

- (A) $2 \left(\frac{6! 7^2}{2} \right)$
- (B) $\frac{1}{30} \left(\frac{6! 7^2}{2} \right)$
- (C) $\frac{6! 7^2}{2}$
- (D) $\frac{1}{360} \left(\frac{6! 7^2}{2} \right)$
- (E) $\frac{2}{7} \left(\frac{6! 7^2}{2} \right)$

32. On $[0, 1]$, f is 10 on rationals and $1/10$ on irrationals. Find its upper Darboux integral minus its lower Darboux integral.

- (A) $\frac{99}{10}$
- (B) $\frac{99}{20}$
- (C) $\frac{109}{10}$
- (D) $\frac{99}{5}$
- (E) $\frac{119}{10}$

33. Evaluate $\lim_{x \rightarrow 0} \frac{e^{7x} - \sum_{j=0}^3 (7x)^j / j!}{x^4}$.

- (A) $-\frac{7^4}{4!}$
- (B) $\frac{1}{7} \left(\frac{7^4}{4!} \right)$
- (C) $\frac{7^4}{4!}$
- (D) $24 \left(\frac{7^4}{4!} \right)$
- (E) $4 \left(\frac{7^4}{4!} \right)$

Questions 34–36

34. Let N be the 35×35 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\dim(\ker N^8 / \ker N^4)$.

- (A) 2
- (B) 3
- (C) 6
- (D) 5
- (E) 4

35. For $x > 0$, $x^2 y'' + (-17)xy' + 77y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $\frac{(1 - e^4) e^7}{4}$
- (B) $-\frac{(1 - e^4) e^7}{4}$
- (C) $-\frac{e^7}{4} + 1 + \frac{e^{11}}{4}$
- (D) $-\frac{(1 - e^4) e^7}{2}$
- (E) $-\frac{(1 - e^4) e^7}{8}$

36. A set with 10 elements has the discrete topology. How many of its subsets are simultaneously open and closed?

- (A) 2^{11}
- (B) 2^9
- (C) 2^{10}
- (D) 2^{12}
- (E) 2^{13}

Questions 37–39

37. Let $F(x) = \int_0^{x^4} e^{7t} dt$. Find $F'(1)$.

- (A) $28e^7$
- (B) e^7
- (C) e^{28}
- (D) $\frac{e^7}{4}$
- (E) $4e^7$

38. Let A be the 35×35 matrix whose diagonal entries are 32 and whose off-diagonal entries are 1. What is its largest eigenvalue?

- (A) 68
- (B) 64
- (C) 65
- (D) 67
- (E) 66

39. If $y' = y(1 - y/4)$ and $y(0) = 2$, find $y(-\log 7)$.

- (A) $\frac{3}{2}$
- (B) $\frac{1}{2}$
- (C) 1
- (D) $\frac{1}{4}$
- (E) $-\frac{1}{2}$

Questions 40–42

40. How many zeros, counted with multiplicity, does $z^{10} + 13z + 1$ have in $|z| < 1$?

- (A) 3
- (B) 2
- (C) 1
- (D) 0
- (E) 4

41. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 7x)^{1/(3x)} - e^{7/3}}{x}$.

- (A) $-\frac{49e^{\frac{7}{3}}}{3}$
- (B) $-\frac{7e^{\frac{7}{3}}}{6}$
- (C) $e^{\frac{7}{3}}$
- (D) $\frac{49e^{\frac{7}{3}}}{6}$
- (E) $-\frac{49e^{\frac{7}{3}}}{6}$

42. How many monic irreducible polynomials of degree 3 are there over $\mathbb{F}_{13}$?

- (A) 727
- (B) 729
- (C) 728
- (D) 726
- (E) 730

Questions 43–45

43. Find the arc length of $r(t) = (7(t - t^3/3), 7t^2)$ for $0 \leq t \leq 1/4$.

- (A) $\frac{343}{96}$
- (B) $\frac{343}{384}$
- (C) $-\frac{343}{192}$
- (D) $\frac{343}{192}$
- (E) $\frac{535}{192}$

44. A meromorphic function has 12 zeros and 2 poles inside a positively oriented simple closed contour, counted with multiplicity, and no zeros or poles on it. Find $\frac{1}{2\pi i} \int f'(z)/f(z) dz$.

- (A) 4
- (B) 7
- (C) 16
- (D) 13
- (E) 10

45. Let $f(x, y) = e^{7x+4y}$. Find $\partial^4 f / \partial x^3 \partial y$ at $(0, 0)$.

- (A) $\frac{1}{4} (4 \cdot 7^3)$
- (B) $\frac{7}{4} (4 \cdot 7^3)$
- (C) $\frac{64}{343} (4 \cdot 7^3)$
- (D) $4 \cdot 7^3$
- (E) $\frac{16}{49} (4 \cdot 7^3)$

Questions 46–48

46. How many elements z of $\mathbb{F}_{57}$ satisfy $z^{25} = z$?

- (A) 4
- (B) 3
- (C) 6
- (D) 7
- (E) 5

47. Find the coefficient of x^4 in the Taylor series of $(1 - 7x)^{-2}$ at zero.

- (A) $\frac{1}{5} (5 \cdot 7^4)$
- (B) $2 (5 \cdot 7^4)$
- (C) $\frac{1}{7} (5 \cdot 7^4)$
- (D) $5 \cdot 7^4$
- (E) $\frac{4}{5} (5 \cdot 7^4)$

48. How many nonnegative integer triples sum to 30, if each coordinate is at most 15?

- (A) 46
- (B) 226
- (C) 91
- (D) 181
- (E) 136

Questions 49–51

49. Two solutions of $y'' + 7x^4y' + 4y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $\frac{1}{2e^{\frac{7}{5}}}$
- (B) $\frac{2}{e^{\frac{7}{5}}}$
- (C) $e^{-\frac{7}{5}}$
- (D) $e^{-\frac{7}{5}} + 1$
- (E) $-\frac{1}{e^{\frac{7}{5}}}$

50. How many generators does the cyclic group of order 28561 have?

- (A) $13^5(13 - 1)$
- (B) $13^6(13 - 1)$
- (C) $13^2(13 - 1)$
- (D) $13^3(13 - 1)$
- (E) $13^4(13 - 1)$

51. Under $x = u + 7v$, $y = u - 4v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) $\frac{1}{11}$
- (B) 121
- (C) 28
- (D) 3
- (E) 11

Questions 52–54

52. A tree has 19 vertices. What is the sum of its vertex degrees?

- (A) 24
- (B) 36
- (C) 60
- (D) 12
- (E) 48

53. Let C be the unit circle $x^2 + y^2 = 1$, $z = 4$, counterclockwise viewed from above. Evaluate $\int_C -7y(x^2 + y^2)^3 dx + (7x(x^2 + y^2)^3) dy + 4z dz$.

- (A) -14π
- (B) 14π
- (C) 7π
- (D) $1 + 14\pi$
- (E) 28π

54. Let $R = (\mathbb{Z}/50\mathbb{Z})/([10])$, where $([10])$ is the ideal generated by $[10]$. How many units does R have?

- (A) 3
- (B) 2
- (C) 5
- (D) 6
- (E) 4

Questions 55–57

55. The solution of $y' = 7x^3y^2$, $y(0) = 1$, has its first positive singularity at which x ?
- (A) $\frac{\sqrt{2}\sqrt[4]{7}}{2}$
(B) $\frac{4}{7}$
(C) $\frac{\sqrt{2} \cdot 7^{\frac{3}{4}}}{7}$
(D) $\frac{7^{\frac{3}{4}}}{7}$
(E) $\frac{\sqrt[4]{2} \cdot 7^{\frac{3}{4}}}{7}$
56. A connected planar graph has 18 vertices and 31 edges. How many faces, including the unbounded face, are in a planar embedding?
- (A) 25
(B) 10
(C) 15
(D) 20
(E) 5
57. The region $0 \leq x \leq 1$, $0 \leq y \leq 7x^4$ is revolved about the line $x = -4$. Find its volume.
- (A) $-\frac{203\pi}{15}$
(B) $\frac{203\pi}{15}$
(C) $1 + \frac{203\pi}{15}$
(D) $\frac{406\pi}{15}$
(E) $\frac{203\pi}{30}$

Questions 58–60

58. What is the largest integer k such that 3^k divides $38!$?

- (A) 17
- (B) 18
- (C) 15
- (D) 19
- (E) 16

59. Let $\mathbf{F} = (7y^4, z^4, x^4)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (1, 1, 4)/\sqrt{18}$.

- (A) $-10\sqrt{2}$
- (B) $-20\sqrt{2}$
- (C) $20\sqrt{2}$
- (D) $1 - 20\sqrt{2}$
- (E) $-40\sqrt{2}$

60. How many integers in $\{1, \dots, 90\}$ are divisible by 2 or 3?

- (A) 60
- (B) 40
- (C) 100
- (D) 20
- (E) 80

Questions 61–63

61. Evaluate $\iiint_{x^2+y^2+z^2 \leq 7} (x^2 + y^2 + z^2)^3 dV$.

- (A) $\frac{4\pi 7^{\frac{9}{2}}}{9}$
- (B) $\frac{1}{2} \left(\frac{4\pi 7^{\frac{9}{2}}}{9} \right)$
- (C) $9 \left(\frac{4\pi 7^{\frac{9}{2}}}{9} \right)$
- (D) $\frac{9}{49} \left(\frac{4\pi 7^{\frac{9}{2}}}{9} \right)$
- (E) $\frac{1}{3} \left(\frac{4\pi 7^{\frac{9}{2}}}{9} \right)$

62. In the ring $\mathbb{Z}/198\mathbb{Z}$, how many elements satisfy $x^2 = 1$?

- (A) 4
- (B) 6
- (C) 3
- (D) 2
- (E) 5

63. If $y' + 7y = 4x^4 e^{-7x}$ and $y(0) = 4$, find $y(1)$.

- (A) $\frac{24}{5e^7} + 1$
- (B) $\frac{12}{5e^7}$
- (C) $\frac{48}{5e^7}$
- (D) $\frac{24}{5e^7}$
- (E) $-\frac{24}{5e^7}$

Questions 64–66

64. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 15 have no adjacent 1s?

- (A) F_{18}
- (B) F_{17}
- (C) F_{15}
- (D) F_{19}
- (E) F_{16}

65. Evaluate $\int_0^1 x^7(1-x)^3 dx$.

- (A) $8 \left(\frac{7! 3!}{11!} \right)$
- (B) $\frac{165}{4} \left(\frac{7! 3!}{11!} \right)$
- (C) $\frac{1}{4} \left(\frac{7! 3!}{11!} \right)$
- (D) $\frac{7! 3!}{11!}$
- (E) $11 \left(\frac{7! 3!}{11!} \right)$

66. A permutation consists of disjoint cycles of lengths 5 and 10, together with a disjoint transposition when the sum of these two lengths is odd. It fixes all other points. What is its order?

- (A) 11
- (B) 8
- (C) 12
- (D) 10
- (E) 9

Answer Key

Practice Test 33 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	B	23	E	45	D
2	A	24	B	46	E
3	A	25	A	47	D
4	B	26	C	48	E
5	C	27	E	49	C
6	D	28	A	50	D
7	B	29	E	51	E
8	C	30	D	52	B
9	B	31	C	53	B
10	C	32	A	54	E
11	A	33	C	55	C
12	D	34	E	56	C
13	D	35	B	57	B
14	A	36	C	58	A
15	E	37	E	59	B
16	A	38	E	60	A
17	A	39	B	61	A
18	C	40	C	62	A
19	D	41	E	63	D
20	B	42	C	64	B
21	C	43	D	65	D
22	D	44	E	66	D

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