

GRE Mathematics Practice Test 36

Advanced Challenge

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

The most demanding form in this five-test cycle. Emphasis on connected ideas, non-routine reasoning, and careful conceptual distinctions. This is cycle 7 of 19, position 1 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Evaluate $\lim_{x \rightarrow 0} \frac{e^{8x} - \sum_{j=0}^5 (8x)^j / j!}{x^6}$.

- (A) $\frac{1}{8} \left(\frac{8^6}{6!} \right)$
- (B) $6 \left(\frac{8^6}{6!} \right)$
- (C) $-\frac{8^6}{6!}$
- (D) $720 \left(\frac{8^6}{6!} \right)$
- (E) $\frac{8^6}{6!}$

2. A matrix A satisfies $(A - 33I)(A - 35I) = 0$, and its minimal polynomial has degree two. If $A^5 = uA + vI$, find u .

- (A) $2 \cdot 31 \cdot 71 \cdot 3041$
- (B) $3 \cdot 31 \cdot 71 \cdot 3041$
- (C) $5 \cdot 31 \cdot 71 \cdot 3041$
- (D) $31 \cdot 71 \cdot 3041$
- (E) 215911

3. For $x > 0$, $x^2 y'' + (1 - 14x)y' + 48y = 0$, $y(1) = 0$, $y'(1) = 8$. Find $y(e)$.

- (A) $-4(1 - e^2)e^6$
- (B) $\frac{e^8}{14}$
- (C) $-e^6 + e^8$
- (D) $4(1 - e^2)e^6$
- (E) $e^6 + e^8$

Questions 4–6

4. X has density cx^{13} on $0 < x < 1$ and zero elsewhere. Find $E[X^2]$.

- (A) $\frac{7}{8}$
- (B) $\frac{1}{8}$
- (C) $\frac{7}{16}$
- (D) $\frac{15}{16}$
- (E) $\frac{7}{24}$

5. For $f(x) = x^6 e^{8x}$, find $f^{(8)}(0)$.

- (A) $\frac{1}{20160} \left(\frac{8! 8^2}{2} \right)$
- (B) $\frac{1}{4} \left(\frac{8! 8^2}{2} \right)$
- (C) $\frac{1}{56} \left(\frac{8! 8^2}{2} \right)$
- (D) $\frac{8! 8^2}{2}$
- (E) $2 \left(\frac{8! 8^2}{2} \right)$

6. Let $V = \begin{bmatrix} 1 & 32 & 1024 \\ 1 & 34 & 1156 \\ 1 & 37 & 1369 \end{bmatrix}$. Find $\det(V^T V)$.

- (A) 901
- (B) 900
- (C) 898
- (D) 899
- (E) 902

Questions 7–9

7. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 8x)^{1/(5x)} - e^{8/5}}{x}$.

(A) $e^{\frac{8}{5}}$

(B) $-\frac{4e^{\frac{8}{5}}}{5}$

(C) $\frac{32e^{\frac{8}{5}}}{5}$

(D) $-\frac{32e^{\frac{8}{5}}}{5}$

(E) $-\frac{64e^{\frac{8}{5}}}{5}$

8. Independent exponential waiting times X, Y have rates 13 and 15. Find $E[\min(X, Y)]$.

(A) $\frac{1}{56}$

(B) $\frac{56}{29}$

(C) $\frac{1}{84}$

(D) $\frac{1}{28}$

(E) $\frac{27}{28}$

9. For $x > 0$, $x^2 y'' + (-21)xy' + 112y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

(A) $-\frac{e^8}{6} + 1 + \frac{e^{14}}{6}$

(B) $-\frac{(1 - e^6)e^8}{6}$

(C) $-\frac{(1 - e^6)e^8}{12}$

(D) $-\frac{(1 - e^6)e^8}{3}$

(E) $\frac{(1 - e^6)e^8}{6}$

Questions 10–12

10. Let $u = (1, 2, \dots, 38)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $v^T B^{-1}u$.

- (A) $\frac{185}{371}$
- (B) $\frac{1483}{742}$
- (C) $\frac{741}{742}$
- (D) $\frac{927}{371}$
- (E) $\frac{556}{371}$

11. If $y'' + 36y = 8$ and $y(0) = y'(0) = 0$, find $y(\pi/6)$.

- (A) $\frac{4}{9}$
- (B) $\frac{2}{9}$
- (C) $\frac{13}{9}$
- (D) $-\frac{4}{9}$
- (E) $\frac{8}{9}$

12. $X_1, \dots, X_{13}$ are independent with common mean and variance 15. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 60
- (B) 300
- (C) 180
- (D) 120
- (E) 240

Questions 13–15

13. The region $0 \leq x \leq 1$, $0 \leq y \leq 8x^6$ is revolved about the line $x = -2$. Find its volume.

- (A) $-\frac{46\pi}{7}$
- (B) $\frac{46\pi}{7}$
- (C) $\frac{92\pi}{7}$
- (D) $\frac{23\pi}{7}$
- (E) $1 + \frac{46\pi}{7}$

14. Let $D = \text{diag}(\underbrace{1, \dots, 1}_6, \underbrace{2, \dots, 2}_{32})$. Find the dimension of the space of real matrices X with $XD = DX$ and $\text{tr } X = 0$.

- (A) 1058
- (B) 1061
- (C) 1059
- (D) 1060
- (E) 1057

15. For $\mathbf{F} = (8x^6y, y^6z, xz^6)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) 147
- (B) 295
- (C) 294
- (D) -294
- (E) 588

Questions 16–18

16. A permutation of 15 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z^2]$.

- (A) 2
- (B) 1
- (C) 3
- (D) 4
- (E) 0

17. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{8t} \cos(6t)$, and $y = e^{8t} \sin(6t)$, find d^2f/dt^2 at $t = 1$.

- (A) 0
- (B) 16
- (C) 256
- (D) 12
- (E) 200

18. When $x^3 + 1$ is divided by $(x - 33)(x - 34)$, the remainder is $ux + v$. Find u .

- (A) 3369
- (B) 3366
- (C) 3367
- (D) 3368
- (E) 3365

Questions 19–21

19. Under $x = u + 8v$, $y = u - 6v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) $\frac{1}{14}$
- (B) 196
- (C) 48
- (D) 14
- (E) 2

20. A coin success probability p has uniform prior on $[0, 1]$. After observing 13 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?

- (A) $\frac{31}{34}$
- (B) $\frac{3}{17}$
- (C) $\frac{17}{14}$
- (D) $\frac{17}{14}$
- (E) $\frac{14}{51}$

21. If $y' + 8y = 6x^6e^{-8x}$ and $y(0) = 2$, find $y(1)$.

- (A) $\frac{10}{7e^8}$
- (B) $-\frac{20}{7e^8}$
- (C) $\frac{40}{7e^8}$
- (D) $\frac{20}{7e^8} + 1$
- (E) $\frac{20}{7e^8}$

Questions 22–24

22. Let $P = \text{diag}(\underbrace{1, \dots, 1}_3, \underbrace{0, \dots, 0}_{36})$. What is the dimension of the space of 39×39 real matrices X satisfying $PX = X$ and $XP = 0$?

- (A) 107
- (B) 110
- (C) 106
- (D) 109
- (E) 108

23. For $f(x, y) = x^2 + 8xy + 6y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) 10
- (B) $2\sqrt{13}$
- (C) -4
- (D) 52
- (E) -6

24. For which real a does $f_m(x) = m^a x^{13}(1-x)^m$ converge uniformly to zero on $[0, 1]$?

- (A) $a < 0$
- (B) $a > 13$
- (C) $a < 13$
- (D) All real a
- (E) $a \leq 13$

Questions 25–27

25. Find the coefficient of x^6 in the Taylor series of $(1 - 8x)^{-2}$ at zero.

- (A) $2(7 \cdot 8^6)$
- (B) $\frac{6}{7}(7 \cdot 8^6)$
- (C) $\frac{1}{8}(7 \cdot 8^6)$
- (D) $7 \cdot 8^6$
- (E) $\frac{1}{7}(7 \cdot 8^6)$

26. A 3×3 matrix A has characteristic polynomial $(t - 32)(t + 1)(t - 2)$. What is $\text{tr}(A^3 - 2A + I)$?

- (A) 32713
- (B) 32714
- (C) 32710
- (D) 32711
- (E) 32712

27. Let $\mathbf{F} = (8y^6, z^6, x^6)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (1, 1, 2)/\sqrt{6}$.

- (A) $18\sqrt{6}$
- (B) $-36\sqrt{6}$
- (C) $1 - 18\sqrt{6}$
- (D) $-18\sqrt{6}$
- (E) $-9\sqrt{6}$

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{13p}]$ converge?

- (A) $p > 1/13$
- (B) $p < 1/13$
- (C) All real p
- (D) $p \geq 1/13$
- (E) $p > 0$

29. Evaluate $\int_0^1 \int_0^x x^8 y^6 dy dx$.

- (A) $\frac{1}{105}$
- (B) $\frac{1}{112}$
- (C) $\frac{1}{63}$
- (D) $\frac{1}{128}$
- (E) $\frac{1}{16}$

30. Let N be the 38×38 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\dim(\ker N^{10} / \ker N^5)$.

- (A) 4
- (B) 6
- (C) 5
- (D) 7
- (E) 3

Questions 31–33

31. Let C be the unit circle $x^2 + y^2 = 1$, $z = 6$, counterclockwise viewed from above. Evaluate $\int_C -8y(x^2 + y^2)^5 dx + (8x(x^2 + y^2)^5) dy + 2z dz$.

- (A) $1 + 16\pi$
- (B) 16π
- (C) -16π
- (D) 8π
- (E) 32π

32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{13 + 1/k : k \geq 1\}$ have?

- (A) 3
- (B) 2
- (C) 4
- (D) 1
- (E) 0

33. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j(x-8)^j}{j^{\frac{5}{3}}(\log j)^m}$ converge absolutely at $x = 7$? Give the least such m , or 0 if none exists.

- (A) 1
- (B) 2
- (C) 4
- (D) 0
- (E) 3

Questions 34–36

34. Let A be the 38×38 matrix whose diagonal entries are 35 and whose off-diagonal entries are 1. What is its largest eigenvalue?

- (A) 72
- (B) 73
- (C) 74
- (D) 70
- (E) 71

35. Two solutions of $y'' + 8x^6y' + 6y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $\frac{2}{e^{\frac{8}{7}}}$
- (B) $\frac{1}{2e^{\frac{8}{7}}}$
- (C) $e^{-\frac{8}{7}} + 1$
- (D) $-\frac{1}{e^{\frac{8}{7}}}$
- (E) $e^{-\frac{8}{7}}$

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 13\}$ have?

- (A) 6
- (B) 10
- (C) 14
- (D) 18
- (E) 22

Questions 37–39

37. Evaluate $\int_0^1 x^8(1-x)^5 dx$.

- (A) $\frac{1001}{3} \left(\frac{8!5!}{14!} \right)$
- (B) $\frac{1}{6} \left(\frac{8!5!}{14!} \right)$
- (C) $9 \left(\frac{8!5!}{14!} \right)$
- (D) $\frac{8!5!}{14!}$
- (E) $14 \left(\frac{8!5!}{14!} \right)$

38. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 256$ and $x + y = 40$. Find $x^2 + y^2$.

- (A) 1087
- (B) 1090
- (C) 1086
- (D) 1088
- (E) 1089

39. Find the coefficient of x^6 in $\cos(8x)/(1-5x^2)$.

- (A) $\frac{8359}{45}$
- (B) $-\frac{8359}{45}$
- (C) $-\frac{16384}{45}$
- (D) -133744
- (E) $-\frac{8404}{45}$

Questions 40–42

40. Let $\zeta = e^{2\pi i/10}$. Find $\sum_{k=0}^9 \zeta^k$.

- (A) 3
- (B) 0
- (C) 2
- (D) 1
- (E) 4

41. The solution of $y' = 8x^5y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{\sqrt[3]{2} \cdot 3^{\frac{5}{6}}}{3}$
- (B) $\frac{3}{4}$
- (C) $\frac{2^{\frac{2}{3}} \sqrt[6]{3}}{2}$
- (D) $\frac{\sqrt{2}}{2}$
- (E) $\frac{\sqrt{2} \sqrt[6]{3}}{2}$

42. The symmetric group S_{38} acts on the two-element subsets of $\{1, \dots, 38\}$. What is the index of the setwise stabilizer of $\{1, 2\}$?

- (A) 703
- (B) 704
- (C) 702
- (D) 701
- (E) 705

Questions 43–45

43. If $y' = 16xy$ and $y(0) = 1$, find the coefficient of x^{12} in the Maclaurin series of y .
- (A) $\frac{8192}{45}$
(B) $\frac{16429}{45}$
(C) $-\frac{16384}{45}$
(D) $\frac{32768}{45}$
(E) $\frac{16384}{45}$
44. An analytic function has real part $u(x, y) = x^2 - y^2 + 8x$ and satisfies $f(0) = 0$. Find $\text{Im } f(1 + i)$.
- (A) 16
(B) 4
(C) 7
(D) 13
(E) 10
45. Evaluate $\int_0^1 \frac{x^5}{1 + 8x^6} dx$.
- (A) $\frac{\log(3)}{4}$
(B) $\frac{\log(3)}{3}$
(C) $\frac{\log(3)}{24}$
(D) $-\frac{\log(3)}{24}$
(E) $\log(9)$

Questions 46–48

46. In $\mathbb{Z}/942\mathbb{Z}$, how many residues are not idempotent ($x^2 \neq x$)?

- (A) 935
- (B) 932
- (C) 936
- (D) 933
- (E) 934

47. Evaluate $\int_0^\infty x^6 e^{-8x} dx$.

- (A) $\frac{1}{6} \left(\frac{6!}{8^7} \right)$
- (B) $8 \left(\frac{6!}{8^7} \right)$
- (C) $\frac{6!}{8^7}$
- (D) 720
- (E) $\frac{1}{8} \left(\frac{6!}{8^7} \right)$

48. For Newton iteration on $f(x) = x^2 - 169$ near the root $r = 13$, find $\lim_{k \rightarrow \infty} (x_{k+1} - r)/(x_k - r)^2$ for nonterminating convergent iterates.

- (A) $\frac{27}{52}$
- (B) $\frac{25}{26}$
- (C) $\frac{1}{52}$
- (D) $\frac{1}{26}$
- (E) $\frac{1}{78}$

Questions 49–51

49. For the curve $x = t^6$, $y = t^7 + 8t$, find d^2y/dx^2 at $t = 1$.

- (A) $-\frac{11}{2}$
- (B) $\frac{5}{2}$
- (C) $\frac{47}{36}$
- (D) $-\frac{4}{3}$
- (E) $-\frac{11}{12}$

50. In the additive group $\mathbb{Z}/48\mathbb{Z}$, find the index of the subgroup generated by $[2]$.

- (A) 2
- (B) 0
- (C) 3
- (D) 4
- (E) 1

51. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(8x) - \sum_{j=0}^4 (-1)^j (8x)^{2j+1} / (2j+1)!}{x^{11}}$.

- (A) $11 \left(\frac{(-1)^5 8^{11}}{11!} \right)$
- (B) $\frac{(-1)^5 8^{11}}{11!}$
- (C) $8 \left(\frac{(-1)^5 8^{11}}{11!} \right)$
- (D) $-\frac{(-1)^5 8^{11}}{11!}$
- (E) $\frac{1}{8} \left(\frac{(-1)^5 8^{11}}{11!} \right)$

Questions 52–54

52. Bisection starts with an interval of length 2^{13} . After 15 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{1}{8}$
- (B) $\frac{1}{16}$
- (C) $\frac{7}{8}$
- (D) $\frac{1}{24}$
- (E) $\frac{9}{16}$

53. Evaluate $\int_0^1 x^7(\log x)^5 dx$.

- (A) $8 \left(\frac{(-1)^5 5!}{8^6} \right)$
- (B) $\frac{4096}{15} \left(\frac{(-1)^5 5!}{8^6} \right)$
- (C) $\frac{(-1)^5 5!}{8^6}$
- (D) $-\frac{(-1)^5 5!}{8^6}$
- (E) $\frac{1}{6} \left(\frac{(-1)^5 5!}{8^6} \right)$

54. How many invertible 4×4 matrices over $\mathbb{F}_{17}$ are there whose first column is $(1, 0, 0, 0)^T$?

- (A) $\frac{(17^4 - 17^0)(17^4 - 17^0)(17^4 - 17^0)(17^4 - 17^0)}{17^4 - 1}$
- (B) $\frac{17^4 17^4 17^4 17^4}{17^4 - 1}$
- (C) $\frac{(17^4 - 17^0)(17^4 - 17^1)(17^4 - 17^2)(17^4 - 17^3)}{17^4 - 1}$
- (D) $\frac{(17^4 - 17^1)(17^4 - 17^2)(17^4 - 17^3)(17^4 - 17^4)}{17^4 - 1}$
- (E) $\frac{(17^4 - 17^0)(17^4 - 17^1)(17^4 - 17^2)}{17^4 - 1}$

Questions 55–57

55. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 8x$ and has density $\rho(x, y) = x^4y$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{7}{8}$
- (B) $\frac{7}{16}$
- (C) $\frac{7}{4}$
- (D) $-\frac{7}{8}$
- (E) $\frac{15}{8}$

56. What is the least positive multiplicative inverse of 2 modulo 27?

- (A) 22
- (B) 18
- (C) 10
- (D) 6
- (E) 14

57. Evaluate $\int_0^{\log 2} e^x(e^x + 8)^6 dx$.

- (A) $\frac{10^7 - 9^7}{7}$
- (B) $10^6 - 9^6$
- (C) $\frac{10^7}{7}$
- (D) $-\frac{10^7 - 9^7}{7}$
- (E) $7 \left(\frac{10^7 - 9^7}{7} \right)$

Questions 58–60

58. How many permutations in S_{38} consist of one 4-cycle and 34 fixed points?
- (A) 442892
(B) 442891
(C) 442889
(D) 442888
(E) 442890
59. Find the directional derivative of $f(x, y, z) = x^2y + 6xz^2 + 8y^2z$ at $(1, 1, 1)$ in the direction $(1, 2, 0)$.
- (A) $\frac{84\sqrt{5}}{5}$
(B) $\frac{42\sqrt{5}}{5}$
(C) $\frac{21\sqrt{5}}{5}$
(D) $-\frac{42\sqrt{5}}{5}$
(E) $1 + \frac{42\sqrt{5}}{5}$
60. How many truth assignments to $p, q, r_1, \dots, r_{11}$ make $p \Rightarrow q$ false?
- (A) 2^{10}
(B) 2^{11}
(C) 2^{12}
(D) 2^{14}
(E) 2^{13}

Questions 61–63

61. For $D = \{(x, y) : x^2 + y^2 \leq 8\}$, evaluate $\iint_D (x^2 + y^2)^5 dA$.
- (A) $\frac{262144\pi}{3}$
(B) 262144π
(C) $\frac{262144\pi}{11}$
(D) $\frac{16384\pi}{3}$
(E) $\frac{131072\pi}{3}$
62. How many maximal proper subgroups does the cyclic group of order 432 have?
- (A) 1
(B) 2
(C) 3
(D) 0
(E) 4
63. Evaluate $\iiint_B 8z^2(x^2 + y^2 + z^2)^6 dV$, where B is the unit ball.
- (A) $\frac{16\pi}{51}$
(B) $-\frac{32\pi}{51}$
(C) $\frac{64\pi}{51}$
(D) $\frac{32\pi}{51}$
(E) $1 + \frac{32\pi}{51}$

Questions 64–66

64. Find the coefficient of x^3 in $(1+x)^{13}(1-x)^{13}(1+2x)^2$.

- (A) -52
- (B) -18
- (C) -86
- (D) -69
- (E) -35

65. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(8j)!}{(j!)^8} x^{6j}$?

- (A) 8^{-8}
- (B) $2\left(8^{-\frac{4}{3}}\right)$
- (C) $8^{-\frac{3}{4}}$
- (D) $8^{\frac{4}{3}}$
- (E) $8^{-\frac{4}{3}}$

66. How many residues x modulo 48 satisfy $4x \equiv 4 \pmod{48}$ and $2x \equiv 0 \pmod{48}$?

- (A) 3
- (B) 2
- (C) 4
- (D) 0
- (E) 1

Answer Key

Practice Test 36 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	E	23	B	45	C
2	D	24	C	46	E
3	A	25	D	47	C
4	A	26	E	48	D
5	D	27	D	49	E
6	B	28	A	50	A
7	D	29	B	51	B
8	D	30	C	52	A
9	B	31	B	53	C
10	C	32	B	54	C
11	A	33	A	55	A
12	C	34	A	56	E
13	B	35	E	57	A
14	C	36	C	58	E
15	C	37	D	59	B
16	A	38	D	60	B
17	A	39	B	61	E
18	C	40	B	62	B
19	D	41	C	63	D
20	D	42	A	64	A
21	E	43	E	65	E
22	E	44	E	66	D

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