

GRE Mathematics Practice Test 37

Advanced Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A demanding practice form with frequent multi-step arguments and a mix of computational and conceptual questions. This is cycle 7 of 19, position 2 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Let $f(x, y) = e^{8x+5y}$. Find $\partial^5 f / \partial x^4 \partial y$ at $(0, 0)$.

- (A) $\frac{625}{4096} (5 \cdot 8^4)$
- (B) $5 \cdot 8^4$
- (C) $\frac{125}{512} (5 \cdot 8^4)$
- (D) $\frac{8}{5} (5 \cdot 8^4)$
- (E) $\frac{1}{5} (5 \cdot 8^4)$

2. Let $P = \text{diag}(\underbrace{1, \dots, 1}_4, \underbrace{0, \dots, 0}_{36})$. What is the dimension of the space of 40×40 real matrices X satisfying $PX = X$ and $XP = 0$?

- (A) 143
- (B) 145
- (C) 142
- (D) 144
- (E) 146

3. Let $F(x) = \int_0^x (x-t)^4 e^{8t} dt$. Find $F^{(5)}(0)$.

- (A) 192
- (B) 24
- (C) 0
- (D) 120
- (E) $\frac{1}{24}$

Questions 4–6

4. X has density cx^{14} on $0 < x < 1$ and zero elsewhere. Find $E[X^2]$.

- (A) $\frac{15}{34}$
- (B) $\frac{5}{17}$
- (C) $\frac{15}{17}$
- (D) $\frac{16}{17}$
- (E) $\frac{2}{17}$

5. For $x > 0$, $x^2y'' + (1 - 13)xy' + 40y = 0$, $y(1) = 0$, $y'(1) = 8$. Find $y(e)$.

- (A) $-e^5 + e^8$
- (B) $-\frac{8(1 - e^3)e^5}{3}$
- (C) $\frac{e^8}{13}$
- (D) $e^5 + e^8$
- (E) $\frac{8(1 - e^3)e^5}{3}$

6. A 3×3 matrix A has characteristic polynomial $(t - 33)(t + 1)(t - 2)$. What is $\text{tr}(A^3 - 2A + I)$?

- (A) 35880
- (B) 35881
- (C) 35879
- (D) 35878
- (E) 35877

Questions 7–9

7. Solve $y' + 8y = x^5e^{-8x}$, $y(0) = 0$, and find $y(1)$.

- (A) e^{-8}
- (B) $\frac{1}{5e^8}$
- (C) $\frac{e^8}{6}$
- (D) $\frac{1}{6e^8}$
- (E) $\frac{4}{3e^8}$

8. Independent exponential waiting times X, Y have rates 14 and 16. Find $P(X < Y)$.

- (A) $\frac{11}{15}$
- (B) $\frac{7}{15}$
- (C) $\frac{7}{45}$
- (D) $\frac{30}{8}$
- (E) $\frac{8}{15}$

9. If $y' + 8y = 5x^5e^{-8x}$ and $y(0) = 3$, find $y(1)$.

- (A) $\frac{23}{3e^8}$
- (B) $\frac{23}{6e^8}$
- (C) $\frac{23}{6e^8} + 1$
- (D) $\frac{23}{12e^8}$
- (E) $-\frac{23}{6e^8}$

Questions 10–12

10. Let N be the 39×39 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\dim(\ker N^{10} / \ker N^5)$.

- (A) 5
- (B) 4
- (C) 7
- (D) 3
- (E) 6

11. Let $\mathbf{F} = (8y^5, z^5, x^5)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (1, 1, 3)/\sqrt{11}$.

- (A) $-\frac{260\sqrt{11}}{11}$
- (B) $1 - \frac{130\sqrt{11}}{11}$
- (C) $-\frac{65\sqrt{11}}{11}$
- (D) $\frac{130\sqrt{11}}{11}$
- (E) $-\frac{130\sqrt{11}}{11}$

12. $X_1, \dots, X_{14}$ are independent with common mean and variance 16. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 70
- (B) 277
- (C) 208
- (D) 139
- (E) 346

Questions 13–15

13. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 8x)^{1/(4x)} - e^{8/4}}{x}$.

- (A) $-e^2$
- (B) e^2
- (C) $-16e^2$
- (D) $-8e^2$
- (E) $8e^2$

14. Let A be the 39×39 matrix whose diagonal entries are 36 and whose off-diagonal entries are 1. What is its largest eigenvalue?

- (A) 75
- (B) 74
- (C) 72
- (D) 73
- (E) 76

15. Find the total outward flux of $\mathbf{F} = (8x^5y, y^5z, xz^5)$ through the boundary of $[0, 1]^3$.

- (A) 10
- (B) -5
- (C) 5
- (D) 6
- (E) $\frac{5}{2}$

Questions 16–18

16. A permutation of 16 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z^2]$.

- (A) 4
- (B) 3
- (C) 0
- (D) 1
- (E) 2

17. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(8+5j)(13+5j)}$.

- (A) $\frac{1}{40}$
- (B) $\frac{1}{8}$
- (C) $\frac{5}{8}$
- (D) $\frac{1}{5}$
- (E) $\frac{1}{13}$

18. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 198$ and $x + y = 39$. Find $x^2 + y^2$.

- (A) 1127
- (B) 1126
- (C) 1124
- (D) 1123
- (E) 1125

Questions 19–21

19. Find the arc length of $r(t) = (8(t - t^3/3), 8t^2)$ for $0 \leq t \leq 1/5$.

- (A) $\frac{304}{375}$
- (B) $\frac{608}{375}$
- (C) $\frac{983}{375}$
- (D) $\frac{1216}{375}$
- (E) $-\frac{608}{375}$

20. A coin success probability p has uniform prior on $[0, 1]$. After observing 14 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?

- (A) $\frac{1}{6}$
- (B) $\frac{5}{12}$
- (C) $\frac{11}{12}$
- (D) $\frac{5}{18}$
- (E) $\frac{5}{6}$

21. Find the coefficient of x^5 in the Taylor series of $(1 - 8x)^{-2}$ at zero.

- (A) $\frac{5}{6} (6 \cdot 8^5)$
- (B) $\frac{1}{8} (6 \cdot 8^5)$
- (C) $-6 \cdot 8^5$
- (D) $6 \cdot 8^5$
- (E) $\frac{1}{6} (6 \cdot 8^5)$

Questions 22–24

22. A sequence satisfies $u_0 = 33$ and $u_{k+1} = 2u_k + 6$ for all $k \geq 0$. Find u_6 .

- (A) 2492
- (B) 2489
- (C) 2490
- (D) 2491
- (E) 2488

23. Let $f(x, y) = x^2 + 16xy + (69)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 10
- (B) 69
- (C) 5
- (D) 276
- (E) 20

24. For which real a does $f_m(x) = m^a x^{14}(1-x)^m$ converge uniformly to zero on $[0, 1]$?

- (A) $a \leq 14$
- (B) $a > 14$
- (C) $a < 14$
- (D) All real a
- (E) $a < 0$

Questions 25–27

25. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^8 y^4 dV$.

- (A) $\frac{1}{45}$
- (B) $\frac{1}{6} \left(\frac{8! 4!}{15!} \right)$
- (C) $\frac{8! 4!}{15!}$
- (D) $6 \left(\frac{8! 4!}{15!} \right)$
- (E) $15 \left(\frac{8! 4!}{15!} \right)$

26. Linear maps $T : \mathbb{R}^{40} \rightarrow \mathbb{R}^{40}$ and $U : \mathbb{R}^{40} \rightarrow \mathbb{R}^{40}$ have ranks 3 and 3. What is the largest possible rank of UT ?

- (A) 1
- (B) 5
- (C) 2
- (D) 3
- (E) 4

27. For $f(x) = x^5 e^{8x}$, find $f^{(7)}(0)$.

- (A) $\frac{1}{2520} \left(\frac{7! 8^2}{2} \right)$
- (B) $\frac{7! 8^2}{2}$
- (C) $2 \left(\frac{7! 8^2}{2} \right)$
- (D) $\frac{1}{42} \left(\frac{7! 8^2}{2} \right)$
- (E) $\frac{1}{4} \left(\frac{7! 8^2}{2} \right)$

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{14p}]$ converge?

- (A) $p < 1/14$
- (B) $p \geq 1/14$
- (C) $p > 0$
- (D) All real p
- (E) $p > 1/14$

29. Evaluate $\int_C (5x^4 + 8y) dx + (8x + 5y^4) dy$, where C is $r(t) = (t, 3t^2)$, $0 \leq t \leq 1$.

- (A) 536
- (B) -268
- (C) 268
- (D) 269
- (E) 134

30. Fix a nonzero vector $u \in \mathbb{R}^{39}$. What is the dimension of the space of skew-symmetric 39×39 matrices A satisfying $Au = 0$?

- (A) 701
- (B) 705
- (C) 704
- (D) 703
- (E) 702

Questions 31–33

31. Find the curvature of the helix $r(t) = (8 \cos t, 8 \sin t, 5t)$.

- (A) $-\frac{8}{89}$
- (B) $\frac{8}{89}$
- (C) $\frac{4}{89}$
- (D) $\frac{97}{89}$
- (E) $\frac{16}{89}$

32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{14 + 1/k : k \geq 1\}$ have?

- (A) 2
- (B) 1
- (C) 4
- (D) 0
- (E) 3

33. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{8t} \cos(5t)$, and $y = e^{8t} \sin(5t)$, find d^2f/dt^2 at $t = 1$.

- (A) 0
- (B) 256
- (C) 16
- (D) 178
- (E) 10

Questions 34–36

34. Let $A = \begin{bmatrix} 1 & 33 & 0 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 3)$ entry of A^5 ?

- (A) 1980
- (B) 1978
- (C) 1981
- (D) 1982
- (E) 1979

35. If $y' = y(1 - y/5)$ and $y(0) = 5/2$, find $y(-\log 8)$.

- (A) $\frac{14}{9}$
- (B) $-\frac{5}{9}$
- (C) $\frac{5}{18}$
- (D) $\frac{5}{9}$
- (E) $\frac{10}{9}$

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 14\}$ have?

- (A) 20
- (B) 15
- (C) 10
- (D) 5
- (E) 25

Questions 37–39

37. Find the upward flux of $\mathbf{F} = (x, y, z)$ through $z = 8x^2 + 5y^2$, $0 \leq x, y \leq 1$.

- (A) $-\frac{10}{3}$
- (B) $\frac{13}{3}$
- (C) $-\frac{13}{6}$
- (D) $-\frac{26}{3}$
- (E) $-\frac{13}{3}$

38. An invertible 39×39 matrix A has determinant 3. Find $\det(\text{adj } A)$.

- (A) $2 \cdot 3^{38}$
- (B) 3^{39}
- (C) 3^{38}
- (D) $3^{38} \cdot 5$
- (E) 3^{37}

39. Find the coefficient of x^6 in $\cos(8x)/(1 - 4x^2)$.

- (A) -93184
- (B) $\frac{5824}{45}$
- (C) $-\frac{16384}{45}$
- (D) $-\frac{5869}{45}$
- (E) $-\frac{5824}{45}$

Questions 40–42

40. Let $\zeta = e^{2\pi i/11}$. Find $\sum_{k=0}^{10} \zeta^k$.

- (A) 0
- (B) 3
- (C) 4
- (D) 2
- (E) 1

41. Evaluate $\lim_{x \rightarrow 0} \frac{e^{8x} - \sum_{j=0}^4 (8x)^j / j!}{x^5}$.

- (A) $\frac{8^5}{5!}$
- (B) $\frac{1}{8} \left(\frac{8^5}{5!} \right)$
- (C) $-\frac{8^5}{5!}$
- (D) $5 \left(\frac{8^5}{5!} \right)$
- (E) $120 \left(\frac{8^5}{5!} \right)$

42. The field K has 7^8 elements. Find the largest possible multiplicative order of a nonzero element of K .

- (A) $7^7 - 1$
- (B) 7^8
- (C) $7^8 + 1$
- (D) $7^8 - 1$
- (E) $7^9 - 1$

Questions 43–45

43. Evaluate $\int_0^{\log 2} e^x(e^x + 8)^5 dx$.

- (A) $\frac{10^6 - 9^6}{6}$
- (B) $10^5 - 9^5$
- (C) $\frac{10^6}{6}$
- (D) $6 \left(\frac{10^6 - 9^6}{6} \right)$
- (E) $-\frac{10^6 - 9^6}{6}$

44. An analytic function has real part $u(x, y) = x^2 - y^2 + 9x$ and satisfies $f(0) = 0$. Find $\text{Im } f(1 + i)$.

- (A) 14
- (B) 5
- (C) 17
- (D) 8
- (E) 11

45. Evaluate $\int_0^1 x^8(1 - x)^4 dx$.

- (A) $143 \left(\frac{8! 4!}{13!} \right)$
- (B) $\frac{8! 4!}{13!}$
- (C) $\frac{1}{5} \left(\frac{8! 4!}{13!} \right)$
- (D) $13 \left(\frac{8! 4!}{13!} \right)$
- (E) $9 \left(\frac{8! 4!}{13!} \right)$

Questions 46–48

46. Let $G = \mathbb{Z}/228\mathbb{Z} \times \mathbb{Z}/156\mathbb{Z}$. Find the order of G/H , where $H = \{g \in G : 12g = 0\}$.

- (A) 249
- (B) 245
- (C) 246
- (D) 247
- (E) 248

47. Evaluate $\int_0^1 \int_0^x x^8 y^5 dy dx$.

- (A) $\frac{1}{90}$
- (B) $\frac{1}{84}$
- (C) $\frac{1}{54}$
- (D) $\frac{1}{105}$
- (E) $\frac{1}{15}$

48. For Newton iteration on $f(x) = x^2 - 196$ near the root $r = 14$, find $\lim_{k \rightarrow \infty} (x_{k+1} - r)/(x_k - r)^2$ for nonterminating convergent iterates.

- (A) $\frac{29}{56}$
- (B) $\frac{27}{28}$
- (C) $\frac{1}{28}$
- (D) $\frac{1}{84}$
- (E) $\frac{1}{56}$

Questions 49–51

49. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 8z^2 = x + 5y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) -40
- (B) -16
- (C) -80
- (D) 80
- (E) -5

50. How many group homomorphisms $\mathbb{Z}/228\mathbb{Z} \rightarrow \mathbb{Z}/156\mathbb{Z}$ have image of order 12?

- (A) 3
- (B) 5
- (C) 2
- (D) 6
- (E) 4

51. Let $f(x) = \log(1 + 8x)$. Find $f^{(5)}(0)$.

- (A) $\frac{1}{8} \left((-1)^4 4! 8^5 \right)$
- (B) $5 \left((-1)^4 4! 8^5 \right)$
- (C) $\frac{1}{5} \left((-1)^4 4! 8^5 \right)$
- (D) $-(-1)^4 4! 8^5$
- (E) $(-1)^4 4! 8^5$

Questions 52–54

52. Bisection starts with an interval of length 2^{14} . After 16 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{1}{8}$
- (B) $\frac{7}{8}$
- (C) $\frac{1}{24}$
- (D) $\frac{1}{16}$
- (E) $\frac{9}{16}$

53. The solution of $y' = 8x^4y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{2^{\frac{3}{5}} \cdot 5^{\frac{4}{5}}}{5}$
- (B) $\frac{2^{\frac{2}{5}}}{2}$
- (C) $\frac{\sqrt[5]{10}}{2}$
- (D) $\frac{5}{8}$
- (E) $\frac{2^{\frac{2}{5}} \sqrt[5]{5}}{2}$

54. How many 2-dimensional subspaces does $\mathbb{F}_{17}^6$ have?

- (A) $\frac{(17^8 - 1)(17^8 - 17)}{(17^2 - 1)(17^2 - 17)}$
- (B) $\frac{(17^5 - 1)(17^5 - 17)}{(17^2 - 1)(17^2 - 17)}$
- (C) $\frac{(17^9 - 1)(17^9 - 17)}{(17^2 - 1)(17^2 - 17)}$
- (D) $\frac{(17^6 - 1)(17^6 - 17)}{(17^2 - 1)(17^2 - 17)}$
- (E) $\frac{(17^7 - 1)(17^7 - 17)}{(17^2 - 1)(17^2 - 17)}$

Questions 55–57

55. Evaluate $\int_0^{\pi/2} \sin^{11} t \cos^{17} t \, dt$.

- (A) $2 \left(\frac{5! 8!}{2 \cdot 14!} \right)$
- (B) $\frac{5! 8!}{2 \cdot 14!}$
- (C) $14 \left(\frac{5! 8!}{2 \cdot 14!} \right)$
- (D) $\frac{1}{2} \left(\frac{5! 8!}{2 \cdot 14!} \right)$
- (E) $1287 \left(\frac{5! 8!}{2 \cdot 14!} \right)$

56. What is the least positive multiplicative inverse of 2 modulo 29?

- (A) 15
- (B) 25
- (C) 5
- (D) 20
- (E) 10

57. For the curve $x = t^5$, $y = t^6 + 8t$, find d^2y/dx^2 at $t = 1$.

- (A) $-\frac{8}{5}$
- (B) $-\frac{26}{5}$
- (C) $\frac{14}{5}$
- (D) $\frac{38}{25}$
- (E) $-\frac{26}{25}$

Questions 58–60

58. How many monic irreducible polynomials of degree 4 are there over $\mathbb{F}_{17}$?

- (A) $2(17^4 - 17^2)$
- (B) $\frac{3(17^4 - 17^2)}{4}$
- (C) $\frac{2(17^4 - 17^2)}{4}$
- (D) $\frac{17^4 - 17^2}{4}$
- (E) $17^4 - 17^2$

59. Evaluate $\int_0^\infty x^5 e^{-8x} dx$.

- (A) 120
- (B) $\frac{1}{8} \left(\frac{5!}{8^6} \right)$
- (C) $\frac{1}{5} \left(\frac{5!}{8^6} \right)$
- (D) $\frac{5!}{8^6}$
- (E) $8 \left(\frac{5!}{8^6} \right)$

60. How many truth assignments to $p, q, r_1, \dots, r_{12}$ make $p \Rightarrow q$ false?

- (A) 2^{14}
- (B) 2^{12}
- (C) 2^{15}
- (D) 2^{11}
- (E) 2^{13}

Questions 61–63

61. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 8x$ and has density $\rho(x, y) = x^3y$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{3}{7}$
- (B) $-\frac{6}{7}$
- (C) $\frac{6}{7}$
- (D) $\frac{12}{7}$
- (E) $\frac{13}{7}$

62. How many elements z of $\mathbb{F}_{7^8}$ satisfy $z^{49} = z$?

- (A) 51
- (B) 47
- (C) 50
- (D) 49
- (E) 48

63. For $\mathbf{F} = (8x^5y, y^5z, xz^5)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) 165
- (B) 166
- (C) 330
- (D) -165
- (E) $\frac{165}{2}$

Questions 64–66

64. Find the coefficient of x^3 in $(1+x)^{14}(1-x)^{14}(1+2x)^2$.

- (A) -20
- (B) -92
- (C) -74
- (D) -56
- (E) -38

65. For $x > 0$, $x^2y'' + (-20)xy' + 104y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $-\frac{(1-e^5)e^8}{5}$
- (B) $-\frac{2(1-e^5)e^8}{5}$
- (C) $\frac{(1-e^5)e^8}{5}$
- (D) $-\frac{e^8}{5} + 1 + \frac{e^{13}}{5}$
- (E) $-\frac{(1-e^5)e^8}{10}$

66. How many generators does the cyclic group of order 1419857 have?

- (A) $17^7(17-1)$
- (B) $17^5(17-1)$
- (C) $17^4(17-1)$
- (D) $17^3(17-1)$
- (E) $17^6(17-1)$

Answer Key

Practice Test 37 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	B	23	E	45	B
2	D	24	C	46	D
3	B	25	C	47	A
4	C	26	D	48	C
5	B	27	B	49	C
6	C	28	E	50	E
7	D	29	C	51	E
8	B	30	D	52	A
9	B	31	B	53	E
10	A	32	A	54	D
11	E	33	A	55	B
12	C	34	A	56	A
13	D	35	D	57	E
14	B	36	B	58	D
15	C	37	E	59	D
16	E	38	C	60	B
17	A	39	E	61	C
18	E	40	A	62	D
19	B	41	A	63	A
20	E	42	D	64	D
21	D	43	A	65	A
22	C	44	E	66	C

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