

GRE Mathematics Practice Test 38

Upper-Intermediate Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

Intermediate reinforcement, combining standard applications with selected more demanding arguments. This is cycle 7 of 19, position 3 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Find the directional derivative of $f(x, y, z) = x^2y + 4xz^2 + 8y^2z$ at $(1, 1, 1)$ in the direction $(1, 4, 0)$.

- (A) $-\frac{74\sqrt{17}}{17}$
- (B) $\frac{37\sqrt{17}}{17}$
- (C) $\frac{148\sqrt{17}}{17}$
- (D) $1 + \frac{74\sqrt{17}}{17}$
- (E) $\frac{74\sqrt{17}}{17}$

2. A sequence satisfies $u_0 = 34$ and $u_{k+1} = 2u_k + 9$ for all $k \geq 0$. Find u_5 .

- (A) 1367
- (B) 1368
- (C) 1365
- (D) 1369
- (E) 1366

3. If $y' + 8y = 4x^4e^{-8x}$ and $y(0) = 4$, find $y(1)$.

- (A) $-\frac{24}{5e^8}$
- (B) $\frac{24}{5e^8}$
- (C) $\frac{24}{5e^8} + 1$
- (D) $\frac{48}{5e^8}$
- (E) $\frac{12}{5e^8}$

Questions 4–6

4. X has density cx^{15} on $0 < x < 1$ and zero elsewhere. Find the normalizing constant c .

- (A) 21
- (B) 26
- (C) 16
- (D) 11
- (E) 6

5. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(8j)!}{(j!)^8} x^{4j}$?

- (A) 8^2
- (B) 8^{-2}
- (C) 8^{-8}
- (D) $8(8^{-2})$
- (E) $8^{-\frac{1}{2}}$

6. Linear maps $T : \mathbb{R}^{41} \rightarrow \mathbb{R}^{41}$ and $U : \mathbb{R}^{41} \rightarrow \mathbb{R}^{41}$ have ranks 2 and 2. What is the largest possible rank of UT ?

- (A) 3
- (B) 1
- (C) 0
- (D) 4
- (E) 2

Questions 7–9

7. Let $f(x) = \log(1 + 8x)$. Find $f^{(4)}(0)$.

- (A) $-(-1)^3 3! 8^4$
- (B) $\frac{1}{8} \left((-1)^3 3! 8^4 \right)$
- (C) $\frac{1}{4} \left((-1)^3 3! 8^4 \right)$
- (D) $4 \left((-1)^3 3! 8^4 \right)$
- (E) $(-1)^3 3! 8^4$

8. Independent exponential waiting times X, Y have rates 15 and 17. Find $P(X < Y)$.

- (A) $\frac{17}{32}$
- (B) $\frac{15}{64}$
- (C) $\frac{15}{32}$
- (D) $\frac{5}{32}$
- (E) $\frac{47}{64}$

9. Evaluate $\lim_{x \rightarrow 0} \frac{e^{8x} - \sum_{j=0}^3 (8x)^j / j!}{x^4}$.

- (A) $\frac{8^4}{4!}$
- (B) $\frac{1}{8} \left(\frac{8^4}{4!} \right)$
- (C) $4 \left(\frac{8^4}{4!} \right)$
- (D) $24 \left(\frac{8^4}{4!} \right)$
- (E) $-\frac{8^4}{4!}$

Questions 10–12

10. Fix a nonzero vector $u \in \mathbb{R}^{40}$. What is the dimension of the space of skew-symmetric 40×40 matrices A satisfying $Au = 0$?

- (A) 743
- (B) 740
- (C) 742
- (D) 741
- (E) 739

11. Under $x = u + 8v$, $y = u - 4v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) $\frac{1}{12}$
- (B) 4
- (C) 12
- (D) 144
- (E) 32

12. $X_1, \dots, X_{15}$ are independent with common mean and variance 17. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 317
- (B) 396
- (C) 238
- (D) 159
- (E) 80

Questions 13–15

13. Evaluate $\int_0^1 \int_0^x x^8 y^4 dy dx$.

- (A) $\frac{1}{70}$
- (B) $\frac{1}{14}$
- (C) $\frac{1}{65}$
- (D) $\frac{1}{84}$
- (E) $\frac{1}{45}$

14. Let $A = \begin{bmatrix} 1 & 34 & 0 \\ 0 & 1 & 9 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 3)$ entry of A^6 ?

- (A) 4592
- (B) 4590
- (C) 4591
- (D) 4589
- (E) 4588

15. Evaluate $\int_0^1 \int_x^1 8y^4 dy dx$.

- (A) $\frac{4}{15}$
- (B) 4
- (C) 2
- (D) $\frac{4}{3}$
- (E) $\frac{8}{5}$

Questions 16–18

16. A permutation of 17 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z^2]$.

- (A) 4
- (B) 3
- (C) 0
- (D) 1
- (E) 2

17. The solution of $y' = 8x^3y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{1}{2}$
- (B) $\frac{2^{\frac{3}{4}}}{2}$
- (C) $\sqrt[4]{2}$
- (D) $\frac{\sqrt[4]{2}}{2}$
- (E) $\frac{\sqrt{2}}{2}$

18. An invertible 40×40 matrix A has determinant 4. Find $\det(\text{adj } A)$.

- (A) 2^{78}
- (B) 2^{79}
- (C) 2^{77}
- (D) $2^{78} \cdot 3$
- (E) $2^{78} \cdot 5$

Questions 19–21

19. Find the total outward flux of $\mathbf{F} = (8x^4y, y^4z, xz^4)$ through the boundary of $[0, 1]^3$.
- (A) 6
(B) 5
(C) $\frac{5}{2}$
(D) 10
(E) -5
20. A coin success probability p has uniform prior on $[0, 1]$. After observing 15 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?
- (A) $\frac{16}{19}$
(B) $\frac{3}{19}$
(C) $\frac{8}{19}$
(D) $\frac{35}{38}$
(E) $\frac{16}{57}$
21. Evaluate $\int_0^1 \frac{x^3}{1 + 8x^4} dx$.
- (A) $\frac{\log(3)}{2}$
(B) $-\frac{\log(3)}{16}$
(C) $\frac{\log(3)}{16}$
(D) $\log(9)$
(E) $\frac{\log(3)}{4}$

Questions 22–24

22. On P_{41} , the real polynomials of degree at most 41, define $T(p) = x^2 p'' - 20p$. Find its rank.

- (A) 39
- (B) 40
- (C) 42
- (D) 41
- (E) 43

23. Evaluate $\int_0^1 x^7 (\log x)^3 dx$.

- (A) $8 \left(\frac{(-1)^3 3!}{8^4} \right)$
- (B) $-\frac{(-1)^3 3!}{8^4}$
- (C) $\frac{1}{4} \left(\frac{(-1)^3 3!}{8^4} \right)$
- (D) $\frac{(-1)^3 3!}{8^4}$
- (E) $\frac{256}{3} \left(\frac{(-1)^3 3!}{8^4} \right)$

24. For which real a does $f_m(x) = m^a x^{15} (1-x)^m$ converge uniformly to zero on $[0, 1]$?

- (A) $a < 0$
- (B) $a < 15$
- (C) $a \leq 15$
- (D) All real a
- (E) $a > 15$

Questions 25–27

- 25.** Let $f(x, y) = x^2 + 16xy + (68)y^2$. What is the product of the eigenvalues of the Hessian of f ?
- (A) 16
(B) 272
(C) 8
(D) 4
(E) 68
- 26.** Subspaces U, V of $\mathbb{R}^{42}$ have dimensions 3 and 4. What is the smallest possible value of $\dim(U \cap V)$?
- (A) 0
(B) 4
(C) 2
(D) 3
(E) 1
- 27.** If $y' = y(1 - y/4)$ and $y(0) = 2$, find $y(-\log 8)$.
- (A) $-\frac{4}{9}$
(B) $\frac{4}{9}$
(C) $\frac{8}{9}$
(D) $\frac{13}{9}$
(E) $\frac{2}{9}$

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{15p}]$ converge?

- (A) $p > 1/15$
- (B) $p > 0$
- (C) $p < 1/15$
- (D) All real p
- (E) $p \geq 1/15$

29. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 8x)^{1/(3x)} - e^{8/3}}{x}$.

- (A) $-\frac{64e^{\frac{8}{3}}}{3}$
- (B) $e^{\frac{8}{3}}$
- (C) $\frac{32e^{\frac{8}{3}}}{3}$
- (D) $-\frac{32e^{\frac{8}{3}}}{3}$
- (E) $-\frac{4e^{\frac{8}{3}}}{3}$

30. Let V consist of real polynomials of degree at most 7 that vanish at 34, 35, and 37. Find $\dim V$.

- (A) 4
- (B) 6
- (C) 3
- (D) 7
- (E) 5

Questions 31–33

31. What is the maximum of x^4y^8 over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\frac{59049}{128} \left(\left(\frac{4}{12} \right)^4 \left(\frac{8}{12} \right)^8 \right)$
- (B) $12 \left(\left(\frac{4}{12} \right)^4 \left(\frac{8}{12} \right)^8 \right)$
- (C) $\left(\frac{4}{12} \right)^8 \left(\frac{8}{12} \right)^4$
- (D) $\frac{1}{2^{12}}$
- (E) $\left(\frac{4}{12} \right)^4 \left(\frac{8}{12} \right)^8$

32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{15 + 1/k : k \geq 1\}$ have?

- (A) 2
- (B) 1
- (C) 4
- (D) 3
- (E) 0

33. For $D = \{(x, y) : x^2 + y^2 \leq 8\}$, evaluate $\iint_D (x^2 + y^2)^3 dA$.

- (A) 128π
- (B) 1024π
- (C) 2048π
- (D) $\frac{4096\pi}{7}$
- (E) 4096π

Questions 34–36

34. The roots r, s of $x^2 - 43x + 306 = 0$ are counted with multiplicity. Find $r^3 + s^3$.

- (A) 40035
- (B) 40031
- (C) 40033
- (D) 40032
- (E) 40034

35. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 8z^2 = x + 4y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) -32
- (B) -64
- (C) -4
- (D) -16
- (E) 64

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 15\}$ have?

- (A) 26
- (B) 11
- (C) 16
- (D) 6
- (E) 21

Questions 37–39

37. For $f(x, y) = x^2 + 8xy + 4y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) 0
- (B) 36
- (C) -6
- (D) 12
- (E) 6

38. For real t , let $M(t) = \begin{pmatrix} t-34 & 9 & 0 \\ 0 & t-34 & 1 \\ 0 & 0 & t-34 \end{pmatrix}$. At $t = 34$, what is $\dim \ker(M(t)^2) - \dim \ker M(t)$?

- (A) 3
- (B) 2
- (C) 0
- (D) 1
- (E) 4

39. Let $F(x) = \int_0^{x^4} e^{8t} dt$. Find $F'(1)$.

- (A) $4e^8$
- (B) $32e^8$
- (C) e^8
- (D) $\frac{e^8}{4}$
- (E) e^{32}

Questions 40–42

40. Let $\zeta = e^{2\pi i/12}$. Find $\sum_{k=0}^{11} \zeta^k$.

- (A) 1
- (B) 4
- (C) 3
- (D) 0
- (E) 2

41. For $\mathbf{F} = (8x^4y, y^4z, xz^4)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) 100
- (B) -100
- (C) 50
- (D) 200
- (E) 101

42. Let $R = (\mathbb{Z}/44\mathbb{Z})/([11])$, where $([11])$ is the ideal generated by $[11]$. How many units does R have?

- (A) 10
- (B) 11
- (C) 9
- (D) 12
- (E) 8

Questions 43–45

43. If $y'' - 16y' + 64y = 4e^{8x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) e^8
- (B) $1 + 2e^8$
- (C) $-2e^8$
- (D) $4e^8$
- (E) $2e^8$

44. An analytic function has real part $u(x, y) = x^2 - y^2 + 10x$ and satisfies $f(0) = 0$. Find $\text{Im } f(1 + i)$.

- (A) 16
- (B) 4
- (C) 20
- (D) 12
- (E) 8

45. Evaluate $\int_0^{\log 2} e^x (e^x + 8)^4 dx$.

- (A) $-\frac{10^5 - 9^5}{5}$
- (B) $\frac{10^5}{5}$
- (C) $5 \left(\frac{10^5 - 9^5}{5} \right)$
- (D) $10^4 - 9^4$
- (E) $\frac{10^5 - 9^5}{5}$

Questions 46–48

46. What is the largest integer k such that 3^k divides $43!$?

- (A) 18
- (B) 17
- (C) 21
- (D) 19
- (E) 20

47. Find the coefficient of x^4 in $e^{3x} \log(1 + 8x)$.

- (A) -1024
- (B) -14880
- (C) 620
- (D) -620
- (E) -612

48. Apply one Newton iteration to solve $x^2 - 15 = 0$, starting at $x_0 = 1$. What is x_1 ?

- (A) 4
- (B) 6
- (C) 10
- (D) 8
- (E) 12

Questions 49–51

49. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{8t} \cos(4t)$, and $y = e^{8t} \sin(4t)$, find d^2f/dt^2 at $t = 1$.

- (A) 160
- (B) 8
- (C) 16
- (D) 256
- (E) 0

50. In the ring $\mathbb{Z}/228\mathbb{Z}$, how many elements satisfy $x^2 = 1$?

- (A) 8
- (B) 7
- (C) 9
- (D) 10
- (E) 6

51. Evaluate $\int_0^1 x^8(1-x)^3 dx$.

- (A) $55 \left(\frac{8! 3!}{12!} \right)$
- (B) $9 \left(\frac{8! 3!}{12!} \right)$
- (C) $\frac{1}{4} \left(\frac{8! 3!}{12!} \right)$
- (D) $\frac{8! 3!}{12!}$
- (E) $12 \left(\frac{8! 3!}{12!} \right)$

Questions 52–54

52. Bisection starts with an interval of length 2^{15} . After 17 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{9}{16}$
- (B) $\frac{1}{8}$
- (C) $\frac{1}{24}$
- (D) $\frac{1}{16}$
- (E) $\frac{7}{8}$

53. Let C be the unit circle $x^2 + y^2 = 1$, $z = 4$, counterclockwise viewed from above. Evaluate $\int_C -8y(x^2 + y^2)^3 dx + (8x(x^2 + y^2)^3) dy + 4z dz$.

- (A) 8π
- (B) $1 + 16\pi$
- (C) 16π
- (D) -16π
- (E) 32π

54. A permutation consists of disjoint cycles of lengths 4 and 8, together with a disjoint transposition when the sum of these two lengths is odd. It fixes all other points. What is its order?

- (A) 9
- (B) 8
- (C) 7
- (D) 6
- (E) 10

Questions 55–57

55. For the curve $x = t^4$, $y = t^5 + 8t$, find d^2y/dx^2 at $t = 1$.

- (A) $-\frac{19}{4}$
- (B) $\frac{13}{4}$
- (C) $-\frac{19}{16}$
- (D) $\frac{29}{16}$
- (E) -2

56. What is the least positive multiplicative inverse of 2 modulo 31?

- (A) 6
- (B) 21
- (C) 11
- (D) 26
- (E) 16

57. Find the upward flux of $\mathbf{F} = (x, y, z)$ through $z = 8x^2 + 4y^2$, $0 \leq x, y \leq 1$.

- (A) 4
- (B) -2
- (C) -4
- (D) -3
- (E) -8

Questions 58–60

58. In $R = \mathbb{F}_5[x]/(x^8)$, how many elements are units?

- (A) $5^{10} - 5^9$
- (B) $5^9 - 5^7$
- (C) $5^8 - 5^0$
- (D) $5^9 - 5^8$
- (E) $5^8 - 5^7$

59. The region $0 \leq x \leq 1$, $0 \leq y \leq 8x^4$ is revolved about the line $x = -4$. Find its volume.

- (A) $\frac{464\pi}{15}$
- (B) $\frac{116\pi}{15}$
- (C) $-\frac{232\pi}{15}$
- (D) $1 + \frac{232\pi}{15}$
- (E) $\frac{232\pi}{15}$

60. How many truth assignments to $p, q, r_1, \dots, r_{13}$ make $p \Rightarrow q$ false?

- (A) 2^{14}
- (B) 2^{15}
- (C) 2^{12}
- (D) 2^{13}
- (E) 2^{16}

Questions 61–63

61. Find the coefficient of x^6 in $\cos(8x)/(1 - 3x^2)$.

- (A) $\frac{5089}{45}$
- (B) $-\frac{5089}{45}$
- (C) $-\frac{5134}{45}$
- (D) -81424
- (E) $-\frac{16384}{45}$

62. Integers x satisfy $x \equiv 0 \pmod{11}$ and $x \equiv 12 \pmod{16}$. Find the least nonnegative solution.

- (A) 46
- (B) 45
- (C) 44
- (D) 42
- (E) 43

63. If $y' = 16xy$ and $y(0) = 1$, find the coefficient of x^8 in the Maclaurin series of y .

- (A) $\frac{1024}{3}$
- (B) $\frac{512}{3}$
- (C) $-\frac{512}{3}$
- (D) $\frac{256}{3}$
- (E) $\frac{515}{3}$

Questions 64–66

64. What is the coefficient of x^2 in $(1 + 2x)^{15}$?

- (A) 420
- (B) 700
- (C) 140
- (D) 560
- (E) 280

65. The local inverse $y(x)$ of $x = 8y + y^3$ satisfies $y(0) = 0$. Find $y^{(3)}(0)$.

- (A) $\frac{6}{8^4}$
- (B) $8\left(-\frac{6}{8^4}\right)$
- (C) $-\frac{6}{8^4}$
- (D) $\frac{1}{8}\left(-\frac{6}{8^4}\right)$
- (E) $\frac{1}{6}\left(-\frac{6}{8^4}\right)$

66. The symmetric group S_{40} acts on the two-element subsets of $\{1, \dots, 40\}$. What is the index of the setwise stabilizer of $\{1, 2\}$?

- (A) 778
- (B) 781
- (C) 780
- (D) 782
- (E) 779

Answer Key

Practice Test 38 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	E	23	D	45	E
2	A	24	B	46	D
3	B	25	A	47	D
4	C	26	A	48	D
5	B	27	B	49	E
6	E	28	A	50	A
7	E	29	D	51	D
8	C	30	E	52	B
9	A	31	E	53	C
10	D	32	A	54	B
11	C	33	B	55	C
12	C	34	C	56	E
13	A	35	B	57	C
14	B	36	C	58	E
15	D	37	E	59	E
16	E	38	D	60	D
17	B	39	A	61	B
18	A	40	D	62	C
19	B	41	A	63	B
20	A	42	A	64	A
21	C	43	E	65	C
22	D	44	D	66	C

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