

# GRE Mathematics Practice Test 39

Bridge to Standard

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

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## Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural,  $\mathbb{N} = \{1, 2, 3, \dots\}$ , and distances in  $\mathbb{R}^n$  are Euclidean.  $I_n$  denotes the  $n \times n$  identity matrix. Graphs are finite and simple unless stated otherwise.

## Difficulty target

A bridge toward standard practice, with routine multi-step questions and a smaller number of challenging concepts. This is cycle 7 of 19, position 4 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

## Coverage

|                 |                                                                                                                           |
|-----------------|---------------------------------------------------------------------------------------------------------------------------|
| Calculus / 33   | Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations. |
| Algebra / 17    | Elementary and linear algebra, groups, rings, fields, and number theory.                                                  |
| Additional / 16 | Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.              |

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

**Questions 1–3**

1. Find the total outward flux of  $\mathbf{F} = (8x^3, y^3, z^3)$  through the boundary of  $[0, 1]^3$ .
- (A) 10  
(B) 5  
(C)  $-10$   
(D) 20  
(E) 11
2. On the space  $P_{42}$  of real polynomials of degree at most 42, define  $T(p) = p'$ . What is the rank of  $T$ ?
- (A) 42  
(B) 40  
(C) 43  
(D) 44  
(E) 41
3. For  $D = \{(x, y) : x^2 + y^2 \leq 8\}$ , evaluate  $\iint_D (x^2 + y^2)^2 dA$ .
- (A)  $\frac{512\pi}{5}$   
(B)  $\frac{1024\pi}{3}$   
(C)  $512\pi$   
(D)  $\frac{512\pi}{3}$   
(E)  $\frac{64\pi}{3}$

**Questions 4–6**

4.  $X$  has density  $cx^{16}$  on  $0 < x < 1$  and zero elsewhere. Find the normalizing constant  $c$ .

- (A) 7
- (B) 17
- (C) 22
- (D) 27
- (E) 12

5. Evaluate  $\lim_{x \rightarrow 0} (1 + 8x)^{1/(2x)}$ .

- (A)  $e^{\frac{1}{4}}$
- (B)  $e^4$
- (C)  $e^{64}$
- (D)  $e^9$
- (E) 8

6. Subspaces  $U, V$  of  $\mathbb{R}^{43}$  have dimensions 3 and 4. What is the largest possible value of  $\dim(U \cap V)$ ?

- (A) 1
- (B) 2
- (C) 3
- (D) 4
- (E) 5

**Questions 7–9**

7. For  $y' = 8y(1 - y/3)$ , what is the derivative of the right-hand side with respect to  $y$  at the positive equilibrium?

- (A)  $-7$
- (B)  $8$
- (C)  $-8$
- (D)  $-4$
- (E)  $-16$

8. Independent exponential waiting times  $X, Y$  have rates 16 and 18. Find  $P(X < Y)$ .

- (A)  $\frac{8}{17}$
- (B)  $\frac{25}{34}$
- (C)  $\frac{8}{51}$
- (D)  $\frac{4}{17}$
- (E)  $\frac{9}{17}$

9. Evaluate  $\int_0^1 x^8(1 - x)^2 dx$ .

- (A)  $\frac{55}{3} \left( \frac{8! 2!}{11!} \right)$
- (B)  $11 \left( \frac{8! 2!}{11!} \right)$
- (C)  $9 \left( \frac{8! 2!}{11!} \right)$
- (D)  $\frac{8! 2!}{11!}$
- (E)  $\frac{1}{3} \left( \frac{8! 2!}{11!} \right)$

**Questions 10–12**

**10.** Let  $V$  consist of real polynomials of degree at most 6 that vanish at 35, 36, and 38. Find  $\dim V$ .

- (A) 6
- (B) 5
- (C) 3
- (D) 2
- (E) 4

**11.** A lamina occupies  $0 \leq x \leq 1$ ,  $0 \leq y \leq 8x$  and has density  $\rho(x, y) = x$ . Find its centroid coordinate  $\bar{x}$ .

- (A)  $\frac{3}{2}$
- (B)  $-\frac{3}{4}$
- (C)  $\frac{7}{4}$
- (D)  $\frac{3}{4}$
- (E)  $\frac{3}{8}$

**12.**  $X_1, \dots, X_{16}$  are independent with common mean and variance 18. With  $\bar{X}$  their sample mean, find  $E[\sum_i (X_i - \bar{X})^2]$ .

- (A) 450
- (B) 90
- (C) 180
- (D) 270
- (E) 360

## Questions 13–15

13. For  $f(x) = x^3e^{8x}$ , find  $f^{(5)}(0)$ .

- (A)  $\frac{1}{60} \left( \frac{5! 8^2}{2} \right)$
- (B)  $\frac{5! 8^2}{2}$
- (C)  $\frac{1}{20} \left( \frac{5! 8^2}{2} \right)$
- (D)  $\frac{1}{4} \left( \frac{5! 8^2}{2} \right)$
- (E)  $2 \left( \frac{5! 8^2}{2} \right)$

14. The roots  $r, s$  of  $x^2 - 42x + 245 = 0$  are counted with multiplicity. Find  $r^2 + s^2$ .

- (A) 1272
- (B) 1276
- (C) 1275
- (D) 1273
- (E) 1274

15. Find the arc length of  $r(t) = (8t, 3t)$  for  $0 \leq t \leq 1$ .

- (A)  $2\sqrt{73}$
- (B)  $\sqrt{73}$
- (C)  $-\sqrt{73}$
- (D)  $\frac{\sqrt{73}}{2}$
- (E)  $1 + \sqrt{73}$

## Questions 16–18

16. A permutation of 18 objects is chosen uniformly. Let  $Z$  be the number of fixed points. Find  $E[Z]$ .

- (A) 4
- (B) 0
- (C) 1
- (D) 3
- (E) 2

17. Let  $f(x, y) = e^{8x+3y}$ . Find  $\partial^3 f / \partial x^2 \partial y$  at  $(0, 0)$ .

- (A)  $\frac{3}{8} (3 \cdot 8^2)$
- (B)  $\frac{8}{3} (3 \cdot 8^2)$
- (C)  $3 \cdot 8^2$
- (D)  $\frac{9}{64} (3 \cdot 8^2)$
- (E)  $\frac{1}{3} (3 \cdot 8^2)$

18. For real  $t$ , let  $M(t) = \begin{pmatrix} t-35 & 7 & 0 \\ 0 & t-35 & 1 \\ 0 & 0 & t-35 \end{pmatrix}$ . At  $t = 35$ , what is  $\text{rank } M(t)$ ?

- (A) 2
- (B) 4
- (C) 3
- (D) 0
- (E) 1

## Questions 19–21

19. Evaluate  $\lim_{x \rightarrow 0} \frac{e^{8x} - \sum_{j=0}^2 (8x)^j / j!}{x^3}$ .

- (A)  $\frac{8^3}{3!}$
- (B)  $6 \left( \frac{8^3}{3!} \right)$
- (C)  $-\frac{8^3}{3!}$
- (D)  $\frac{1}{8} \left( \frac{8^3}{3!} \right)$
- (E)  $3 \left( \frac{8^3}{3!} \right)$

20. A coin success probability  $p$  has uniform prior on  $[0, 1]$ . After observing 16 successes and 2 failures in independent tosses conditional on  $p$ , what is the predictive probability of success on the next toss?

- (A)  $\frac{17}{20}$
- (B)  $\frac{17}{60}$
- (C)  $\frac{17}{40}$
- (D)  $\frac{3}{20}$
- (E)  $\frac{37}{40}$

21. If  $f(x, y) = \log(x^2 + y^2)$ ,  $x = e^{8t} \cos(3t)$ , and  $y = e^{8t} \sin(3t)$ , find  $d^2f/dt^2$  at  $t = 1$ .

- (A) 146
- (B) 6
- (C) 256
- (D) 16
- (E) 0

## Questions 22–24

**22.** Let  $u = (7, 3)^T$  and  $v = (1, 0)^T$ . Find the squared norm of the orthogonal projection of  $v$  onto  $\text{span}(u)$ .

- (A)  $\frac{39}{29}$
- (B)  $\frac{68}{29}$
- (C)  $\frac{49}{58}$
- (D)  $\frac{107}{58}$
- (E)  $\frac{10}{29}$

**23.** Let  $F(x) = \int_0^x (x - t)^2 e^{8t} dt$ . Find  $F^{(3)}(0)$ .

- (A) 2
- (B) 0
- (C) 6
- (D) 16
- (E)  $\frac{1}{2}$

**24.** For which real  $a$  does  $f_m(x) = x^{16m}/m^a$  converge uniformly to zero on  $[0, 1]$ ?

- (A)  $a > 0$
- (B) All real  $a$
- (C)  $a < 0$
- (D)  $a = 0$
- (E)  $a > 1$

## Questions 25–27

25. For the curve  $x = t^3$ ,  $y = t^4 + 8t$ , find  $d^2y/dx^2$  at  $t = 1$ .

- (A)  $-\frac{4}{3}$
- (B)  $\frac{20}{9}$
- (C) 4
- (D)  $-4$
- (E)  $-\frac{8}{3}$

26. An idempotent  $41 \times 41$  real matrix  $P$  has rank 1. What is  $\text{tr } P$ ?

- (A) 1
- (B) 0
- (C) 4
- (D) 3
- (E) 2

27. For  $x > 0$ ,  $x^2y'' + (1 - 11)xy' + 24y = 0$ ,  $y(1) = 0$ ,  $y'(1) = 8$ . Find  $y(e)$ .

- (A)  $e^3 + e^8$
- (B)  $\frac{e^8}{11}$
- (C)  $\frac{8(1 - e^5)e^3}{5}$
- (D)  $-\frac{8(1 - e^5)e^3}{5}$
- (E)  $-e^3 + e^8$

## Questions 28–30

28. For which real  $p$  does  $\sum_{k=2}^{\infty} 1/[k(\log k)^{16p}]$  converge?

- (A)  $p > 0$
- (B)  $p \geq 1/16$
- (C)  $p < 1/16$
- (D)  $p > 1/16$
- (E) All real  $p$

29. Evaluate  $\lim_{x \rightarrow 0} \frac{\sin(8x) - \sum_{j=0}^1 (-1)^j (8x)^{2j+1} / (2j+1)!}{x^5}$ .

- (A)  $8 \left( \frac{(-1)^2 8^5}{5!} \right)$
- (B)  $5 \left( \frac{(-1)^2 8^5}{5!} \right)$
- (C)  $\frac{1}{8} \left( \frac{(-1)^2 8^5}{5!} \right)$
- (D)  $-\frac{(-1)^2 8^5}{5!}$
- (E)  $\frac{(-1)^2 8^5}{5!}$

30. For  $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$  and  $b = \begin{bmatrix} 35 \\ 7 \\ 42 \end{bmatrix}$ , let  $x$  minimize  $\|Ax - b\|^2$ . Find  $x_2$ .

- (A) 5
- (B) 9
- (C) 6
- (D) 8
- (E) 7

## Questions 31–33

31. Evaluate  $\int_C (3x^2 + 8y) dx + (8x + 3y^2) dy$ , where  $C$  is the line segment from  $(0, 0)$  to  $(5, 1)$ .

- (A)  $-166$
- (B)  $167$
- (C)  $332$
- (D)  $166$
- (E)  $83$

32. How many accumulation points in  $\mathbb{R}$  does  $S = \{1/k : k \geq 1\} \cup \{16 + 1/k : k \geq 1\}$  have?

- (A) 1
- (B) 3
- (C) 4
- (D) 0
- (E) 2

33. Find the coefficient of  $x^3$  in  $e^{2x} \log(1 + 8x)$ .

- (A)  $\frac{392}{3}$
- (B)  $\frac{368}{3}$
- (C)  $736$
- (D)  $\frac{512}{3}$
- (E)  $-\frac{368}{3}$

## Questions 34–36

34. What is the coefficient of  $x^3$  in  $(x + 7)^5$ ?

- (A) 488
- (B) 492
- (C) 490
- (D) 491
- (E) 489

35. Let  $C$  be the unit circle  $x^2 + y^2 = 1$ ,  $z = 3$ , counterclockwise viewed from above. Evaluate  $\int_C -8y(x^2 + y^2)^2 dx + (8x(x^2 + y^2)^2) dy$ .

- (A)  $32\pi$
- (B)  $8\pi$
- (C)  $16\pi$
- (D)  $1 + 16\pi$
- (E)  $-16\pi$

36. How many connected components does  $\mathbb{R} \setminus \{1, 2, \dots, 16\}$  have?

- (A) 22
- (B) 7
- (C) 27
- (D) 12
- (E) 17

## Questions 37–39

37. Two solutions of  $y'' + 8x^1y' + 3y = 0$  have Wronskian  $W(0) = 1$ . Find  $W(1)$ .

- (A)  $e^{-4}$
- (B)  $\frac{2}{e^4}$
- (C)  $-\frac{1}{e^4}$
- (D)  $e^{-4} + 1$
- (E)  $\frac{1}{2e^4}$

38. A matrix  $A$  satisfies  $(A - 36I)(A - 38I) = 0$ , and its minimal polynomial has degree two. If  $A^2 = uA + vI$ , find  $u$ .

- (A) 74
- (B) 72
- (C) 75
- (D) 76
- (E) 73

39. Find the directional derivative of  $f(x, y, z) = x^2 + 8y^2 + 3z^2$  at  $(1, 1, 1)$  in the direction  $(1, 5, 0)$ .

- (A)  $\frac{82\sqrt{26}}{13}$
- (B)  $\frac{41\sqrt{26}}{26}$
- (C)  $-\frac{41\sqrt{26}}{13}$
- (D)  $1 + \frac{41\sqrt{26}}{13}$
- (E)  $\frac{41\sqrt{26}}{13}$

**Questions 40–42**

**40.** Let  $\zeta = e^{2\pi i/13}$ . Find  $\sum_{k=0}^{12} \zeta^k$ .

- (A) 2
- (B) 0
- (C) 1
- (D) 4
- (E) 3

**41.** For  $x > 0$ ,  $xy' = 8y$  and  $y(1) = 1$ . Find  $y(e^3)$ .

- (A)  $1 + e^{24}$
- (B)  $-e^{24}$
- (C)  $2e^{24}$
- (D)  $e^{24}$
- (E)  $\frac{e^{24}}{2}$

**42.** Let  $V = \begin{bmatrix} 1 & 35 & 1225 \\ 1 & 37 & 1369 \\ 1 & 40 & 1600 \end{bmatrix}$ . Find  $\det V$ .

- (A) 28
- (B) 32
- (C) 31
- (D) 29
- (E) 30

## Questions 43–45

43. For  $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$ , evaluate  $\iiint_T x^8 y^2 dV$ .

- (A)  $6 \left( \frac{8! 2!}{13!} \right)$
- (B)  $13 \left( \frac{8! 2!}{13!} \right)$
- (C)  $\frac{1}{6} \left( \frac{8! 2!}{13!} \right)$
- (D)  $\frac{8! 2!}{13!}$
- (E)  $\frac{1}{27}$

44. An analytic function has real part  $u(x, y) = x^2 - y^2 + 11x$  and satisfies  $f(0) = 0$ . Find  $\text{Im } f(1 + i)$ .

- (A) 21
- (B) 9
- (C) 13
- (D) 5
- (E) 17

45. Let  $f(x, y) = x^2 + 16xy + (67)y^2$ . What is the product of the eigenvalues of the Hessian of  $f$ ?

- (A) 3
- (B) 67
- (C) 268
- (D) 6
- (E) 12

**Questions 46–48**

**46.** In  $\mathbb{Z}/1038\mathbb{Z}$ , how many residues are idempotent ( $x^2 = x$ )?

- (A) 10
- (B) 7
- (C) 6
- (D) 9
- (E) 8

**47.** Find the coefficient of  $x^6$  in  $\cos(8x)/(1 - 2x^2)$ .

- (A)  $-\frac{6469}{45}$
- (B)  $\frac{6424}{45}$
- (C)  $-\frac{6424}{45}$
- (D)  $-\frac{16384}{45}$
- (E)  $-102784$

**48.** Apply one Newton iteration to solve  $x^2 - 16 = 0$ , starting at  $x_0 = 1$ . What is  $x_1$ ?

- (A)  $\frac{21}{2}$
- (B)  $\frac{17}{4}$
- (C)  $\frac{17}{2}$
- (D) 17
- (E)  $\frac{19}{2}$

## Questions 49–51

49. Evaluate  $\int_0^{\pi/2} \sin^7 t \cos^{17} t \, dt$ .

- (A)  $165 \left( \frac{3! 8!}{2 \cdot 12!} \right)$
- (B)  $\frac{3! 8!}{2 \cdot 12!}$
- (C)  $\frac{1}{2} \left( \frac{3! 8!}{2 \cdot 12!} \right)$
- (D)  $2 \left( \frac{3! 8!}{2 \cdot 12!} \right)$
- (E)  $12 \left( \frac{3! 8!}{2 \cdot 12!} \right)$

50. In the additive group  $\mathbb{Z}/66\mathbb{Z}$ , find the order of  $[2]$ .

- (A) 31
- (B) 33
- (C) 32
- (D) 35
- (E) 34

51. Solve  $y'' - 19y' + 88y = 0$  with  $y(0) = 1$ ,  $y'(0) = 0$ . Find  $y(1)$ .

- (A)  $\frac{(-11 + 8e^3) e^8}{3}$
- (B)  $\frac{(11 - 8e^3) e^8}{3} + 1$
- (C)  $\frac{(11 - 8e^3) e^8}{6}$
- (D)  $\frac{(11 - 8e^3) e^8}{3}$
- (E)  $\frac{2(11 - 8e^3) e^8}{3}$

## Questions 52–54

**52.** Bisection starts with an interval of length  $2^{16}$ . After 18 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A)  $\frac{9}{16}$
- (B)  $\frac{1}{24}$
- (C)  $\frac{1}{16}$
- (D)  $\frac{7}{8}$
- (E)  $\frac{1}{8}$

**53.** Near  $(0, 0)$ ,  $z = z(x, y)$  satisfies  $z + 8z^2 = x + 3y$  and  $z(0, 0) = 0$ . Find  $z_{xy}(0, 0)$ .

- (A)  $-24$
- (B)  $48$
- (C)  $-16$
- (D)  $-48$
- (E)  $-3$

**54.** How many invertible  $2 \times 2$  matrices over  $\mathbb{F}_{17}$  are there with determinant 1?

- (A)  $\frac{17^2 17^2}{16}$
- (B)  $\frac{(17^2 - 17^0)(17^2 - 17^0)}{16}$
- (C)  $\frac{(17^2 - 17^1)(17^2 - 17^2)}{16}$
- (D)  $\frac{(17^2 - 17^0)(17^2 - 17^1)}{16}$
- (E)  $\frac{(17^2 - 17^0)}{16}$

**Questions 55–57**

**55.** For  $f(x, y) = x^2 + 8xy + 3y^2$ , what is its maximum directional derivative at  $(1, -1)$ ?

- (A)  $-6$
- (B)  $40$
- (C)  $2\sqrt{10}$
- (D)  $8$
- (E)  $2$

**56.** What is the least positive multiplicative inverse of 2 modulo 33?

- (A) 7
- (B) 17
- (C) 22
- (D) 27
- (E) 12

**57.** The function  $y(x)$  satisfies  $y^3 + 8y = 2x$  and  $y(0) = 0$ . Find  $y'(0)$ .

- (A)  $-\frac{1}{4}$
- (B) 0
- (C)  $\frac{1}{4}$
- (D) 8
- (E)  $\frac{2}{11}$

**Questions 58–60**

**58.** How many permutations in  $S_{41}$  consist of one 3-cycle and 38 fixed points?

- (A) 21319
- (B) 21320
- (C) 21322
- (D) 21318
- (E) 21321

**59.** If  $y' + 8y = 3e^{-8x}$  and  $y(0) = 5$ , find  $y(1)$ .

- (A)  $\frac{4}{e^8}$
- (B)  $\frac{8}{e^8}$
- (C)  $-\frac{8}{e^8}$
- (D)  $\frac{8}{e^8} + 1$
- (E)  $\frac{16}{e^8}$

**60.** How many truth assignments to  $p, q, r_1, \dots, r_{14}$  make  $p \Rightarrow q$  false?

- (A)  $2^{17}$
- (B)  $2^{15}$
- (C)  $2^{13}$
- (D)  $2^{16}$
- (E)  $2^{14}$

## Questions 61–63

61. What is the radius of convergence about zero of  $\sum_{j=1}^{\infty} \frac{(8j)!}{(j!)^8} x^{3j}$ ?

- (A)  $8^{-\frac{3}{8}}$
- (B)  $8^{-8}$
- (C)  $32 \left( 8^{-\frac{8}{3}} \right)$
- (D)  $8^{-\frac{8}{3}}$
- (E)  $8^{\frac{8}{3}}$

62. How many subgroups does the cyclic group of order 468 have?

- (A) 20
- (B) 19
- (C) 17
- (D) 16
- (E) 18

63. Let  $C$  be  $x^2/9 + y^2 = 1$ , oriented counterclockwise. Evaluate  $\int_C (-y) dx + (8x) dy$ .

- (A)  $54\pi$
- (B)  $27\pi$
- (C)  $-27\pi$
- (D)  $1 + 27\pi$
- (E)  $\frac{27\pi}{2}$

**Questions 64–66**

**64.** What is the coefficient of  $x^2$  in  $(1 + 2x)^{16}$ ?

- (A) 480
- (B) 640
- (C) 800
- (D) 160
- (E) 320

**65.** Evaluate  $\iiint_B 8(x^2 + y^2 + z^2)^{3/2} dV$ , where  $B$  is the unit ball.

- (A)  $\frac{32\pi}{3}$
- (B)  $\frac{16\pi}{3}$
- (C)  $1 + \frac{16\pi}{3}$
- (D)  $-\frac{16\pi}{3}$
- (E)  $\frac{8\pi}{3}$

**66.** How many residues  $x$  modulo 66 satisfy  $7x \equiv 28 \pmod{66}$ ?

- (A) 4
- (B) 0
- (C) 1
- (D) 2
- (E) 3

# Answer Key

Practice Test 39 / Questions 1–66

| Question | Answer | Question | Answer | Question | Answer |
|----------|--------|----------|--------|----------|--------|
| 1        | A      | 23       | A      | 45       | E      |
| 2        | A      | 24       | A      | 46       | E      |
| 3        | D      | 25       | A      | 47       | C      |
| 4        | B      | 26       | A      | 48       | C      |
| 5        | B      | 27       | D      | 49       | B      |
| 6        | C      | 28       | D      | 50       | B      |
| 7        | C      | 29       | E      | 51       | D      |
| 8        | A      | 30       | E      | 52       | E      |
| 9        | D      | 31       | D      | 53       | D      |
| 10       | E      | 32       | E      | 54       | D      |
| 11       | D      | 33       | B      | 55       | C      |
| 12       | D      | 34       | C      | 56       | B      |
| 13       | B      | 35       | C      | 57       | C      |
| 14       | E      | 36       | E      | 58       | B      |
| 15       | B      | 37       | A      | 59       | B      |
| 16       | C      | 38       | A      | 60       | E      |
| 17       | C      | 39       | E      | 61       | D      |
| 18       | A      | 40       | B      | 62       | E      |
| 19       | A      | 41       | D      | 63       | B      |
| 20       | A      | 42       | E      | 64       | A      |
| 21       | E      | 43       | D      | 65       | B      |
| 22       | C      | 44       | C      | 66       | C      |

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