

GRE Mathematics Practice Test 40

Standard-Level Target

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A mixed standard-level target, intended to approximate public GRE Mathematics practice in scope and overall demand. This is cycle 7 of 19, position 5 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Let C be the unit circle $x^2 + y^2 = 1$, $z = 2$, counterclockwise viewed from above. Evaluate $\int_C -8y(x^2 + y^2) dx + (8x(x^2 + y^2)) dy$.

- (A) -16π
- (B) $1 + 16\pi$
- (C) 8π
- (D) 32π
- (E) 16π

2. Let $u = (7, 2)^T$ and $v = (1, 0)^T$. Find the squared norm of the orthogonal projection of v onto $\text{span}(u)$.

- (A) $\frac{257}{106}$
- (B) $\frac{45}{106}$
- (C) $\frac{49}{53}$
- (D) $\frac{102}{53}$
- (E) $\frac{151}{106}$

3. Evaluate $\iiint_B 8(x^2 + y^2 + z^2)^1 dV$, where B is the unit ball.

- (A) $\frac{32\pi}{5}$
- (B) $-\frac{32\pi}{5}$
- (C) $\frac{64\pi}{5}$
- (D) $\frac{16\pi}{5}$
- (E) $1 + \frac{32\pi}{5}$

Questions 4–6

4. X has density cx^{17} on $0 < x < 1$ and zero elsewhere. Find the normalizing constant c .

- (A) 6
- (B) 18
- (C) 30
- (D) 12
- (E) 24

5. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(8j)!}{(j!)^8} x^{2j}$?

- (A) $512(8^{-4})$
- (B) 8^{-8}
- (C) $8^{-\frac{1}{4}}$
- (D) 8^{-4}
- (E) 8^4

6. An idempotent 42×42 real matrix P has rank 2. What is $\text{tr } P$?

- (A) 1
- (B) 0
- (C) 3
- (D) 4
- (E) 2

Questions 7–9

7. Find the directional derivative of $f(x, y, z) = x^2 + 8y^2 + 2z^2$ at $(1, 1, 1)$ in the direction $(1, 6, 0)$.

- (A) $\frac{98\sqrt{37}}{37}$
- (B) $\frac{49\sqrt{37}}{37}$
- (C) $\frac{196\sqrt{37}}{37}$
- (D) $1 + \frac{98\sqrt{37}}{37}$
- (E) $-\frac{98\sqrt{37}}{37}$

8. Independent exponential waiting times X, Y have rates 17 and 19. Find $P(X < Y)$.

- (A) $\frac{19}{36}$
- (B) $\frac{17}{72}$
- (C) $\frac{17}{36}$
- (D) $\frac{53}{72}$
- (E) $\frac{17}{108}$

9. Two solutions of $y'' + 8x^1y' + 2y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $-\frac{1}{e^4}$
- (B) e^{-4}
- (C) $e^{-4} + 1$
- (D) $\frac{1}{2e^4}$
- (E) $\frac{2}{e^4}$

Questions 10–12

10. For $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $b = \begin{bmatrix} 36 \\ 5 \\ 41 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find x_2 .

- (A) 5
- (B) 4
- (C) 3
- (D) 6
- (E) 7

11. Let $F(x) = \int_0^{x^2} e^{8t} dt$. Find $F'(1)$.

- (A) e^8
- (B) e^{16}
- (C) $16e^8$
- (D) $2e^8$
- (E) $\frac{e^8}{2}$

12. $X_1, \dots, X_{17}$ are independent with common mean and variance 19. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 506
- (B) 102
- (C) 203
- (D) 405
- (E) 304

Questions 13–15

13. Evaluate $\int_0^1 \int_0^x x^8 y^2 dy dx$.

- (A) $\frac{1}{33}$
- (B) $\frac{1}{27}$
- (C) $\frac{1}{48}$
- (D) $\frac{1}{36}$
- (E) $\frac{1}{12}$

14. What is the coefficient of x^2 in $(x + 7)^4$?

- (A) 294
- (B) 292
- (C) 296
- (D) 295
- (E) 293

15. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{8t} \cos(2t)$, and $y = e^{8t} \sin(2t)$, find $d^2 f / dt^2$ at $t = 1$.

- (A) 16
- (B) 256
- (C) 136
- (D) 4
- (E) 0

Questions 16–18

16. A permutation of 19 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z]$.

- (A) 1
- (B) 0
- (C) 3
- (D) 2
- (E) 4

17. Evaluate $\int_C (2x + 8y) dx + (8x + 2y) dy$, where C is the line segment from $(0, 0)$ to $(6, 1)$.

- (A) 86
- (B) 85
- (C) 170
- (D) $\frac{85}{2}$
- (E) -85

18. A matrix A satisfies $(A - 37I)(A - 39I) = 0$, and its minimal polynomial has degree two. If $A^2 = uA + vI$, find u .

- (A) 76
- (B) 77
- (C) 78
- (D) 74
- (E) 75

Questions 19–21

19. Evaluate $\int_0^1 x^8(1-x)^1 dx$.

- (A) $5 \left(\frac{8! 1!}{10!} \right)$
- (B) $\frac{1}{2} \left(\frac{8! 1!}{10!} \right)$
- (C) $10 \left(\frac{8! 1!}{10!} \right)$
- (D) $9 \left(\frac{8! 1!}{10!} \right)$
- (E) $\frac{8! 1!}{10!}$

20. A coin success probability p has uniform prior on $[0, 1]$. After observing 17 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?

- (A) $\frac{2}{7}$
- (B) $\frac{3}{7}$
- (C) $\frac{1}{7}$
- (D) $\frac{6}{7}$
- (E) $\frac{13}{14}$

21. Find the arc length of $r(t) = (8t, 2t)$ for $0 \leq t \leq 1$.

- (A) $-2\sqrt{17}$
- (B) $1 + 2\sqrt{17}$
- (C) $2\sqrt{17}$
- (D) $\sqrt{17}$
- (E) $4\sqrt{17}$

Questions 22–24

22. Let $V = \begin{bmatrix} 1 & 36 & 1296 \\ 1 & 38 & 1444 \\ 1 & 41 & 1681 \end{bmatrix}$. Find $\det V$.

- (A) 29
- (B) 32
- (C) 31
- (D) 28
- (E) 30

23. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 8z^2 = x + 2y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) -2
- (B) -64
- (C) -32
- (D) 32
- (E) -16

24. For which real a does $f_m(x) = x^{17m}/m^a$ converge uniformly to zero on $[0, 1]$?

- (A) $a > 1$
- (B) $a > 0$
- (C) $a = 0$
- (D) $a < 0$
- (E) All real a

Questions 25–27

25. For the curve $x = t^2$, $y = t^3 + 8t$, find d^2y/dx^2 at $t = 1$.

- (A) $-\frac{5}{2}$
- (B) $\frac{11}{4}$
- (C) $-\frac{5}{4}$
- (D) -4
- (E) $\frac{11}{2}$

26. Let $u = (1, 2, \dots, 42)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $\det B$.

- (A) 905
- (B) 902
- (C) 906
- (D) 904
- (E) 903

27. If $y'' + 4y = 0$, $y(0) = 8$, and $y'(0) = 6$, find $y(\pi/4)$.

- (A) -3
- (B) 4
- (C) 3
- (D) $\frac{3}{2}$
- (E) 6

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{17p}]$ converge?

- (A) $p > 1/17$
- (B) $p > 0$
- (C) All real p
- (D) $p < 1/17$
- (E) $p \geq 1/17$

29. The solution of $y' = 8x^1y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{\sqrt{2}}{4}$
- (B) 1
- (C) $\frac{1}{4}$
- (D) 2
- (E) $\frac{1}{2}$

30. Let D be a diagonal 42×42 real matrix with distinct diagonal entries. What is the dimension of the space of matrices X satisfying $XD = DX$?

- (A) 41
- (B) 40
- (C) 44
- (D) 42
- (E) 43

Questions 31–33

31. Let $F(x) = \int_0^x (x-t)^1 e^{8t} dt$. Find $F^{(2)}(0)$.

- (A) $\frac{1}{2}$
- (B) 2
- (C) 1
- (D) 8
- (E) 0

32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{17 + 1/k : k \geq 1\}$ have?

- (A) 4
- (B) 2
- (C) 3
- (D) 0
- (E) 1

33. For $f(x, y) = x^2 + 8xy + 2y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) 52
- (B) 10
- (C) -6
- (D) 4
- (E) $2\sqrt{13}$

Questions 34–36

34. What is the remainder when $x^2 + 1$ is divided by $x - 37$?

- (A) 1368
- (B) 1370
- (C) 1372
- (D) 1371
- (E) 1369

35. Evaluate $\int_0^{\pi/2} \sin^5 t \cos^{17} t \, dt$.

- (A) $45 \left(\frac{2! 8!}{2 \cdot 11!} \right)$
- (B) $11 \left(\frac{2! 8!}{2 \cdot 11!} \right)$
- (C) $2 \left(\frac{2! 8!}{2 \cdot 11!} \right)$
- (D) $\frac{1}{2} \left(\frac{2! 8!}{2 \cdot 11!} \right)$
- (E) $\frac{2! 8!}{2 \cdot 11!}$

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 17\}$ have?

- (A) 30
- (B) 24
- (C) 18
- (D) 6
- (E) 12

Questions 37–39

37. For $x > 0$, $x^2 y'' + (1 - 10)xy' + 16y = 0$, $y(1) = 0$, $y'(1) = 8$. Find $y(e)$.

- (A) $\frac{4(1 - e^6)e^2}{3}$
- (B) $e^2 + e^8$
- (C) $\frac{e^8}{10}$
- (D) $-e^2 + e^8$
- (E) $-\frac{4(1 - e^6)e^2}{3}$

38. Let $P = \text{diag}(\underbrace{1, \dots, 1}_3, \underbrace{0, \dots, 0}_{40})$. What is the dimension of the space of 43×43 real matrices X satisfying $PX = X$?

- (A) 129
- (B) 128
- (C) 131
- (D) 130
- (E) 127

39. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(8x) - \sum_{j=0}^0 (-1)^j (8x)^{2j+1} / (2j+1)!}{x^3}$.

- (A) $\frac{(-1)^1 8^3}{3!}$
- (B) $3 \left(\frac{(-1)^1 8^3}{3!} \right)$
- (C) $-\frac{(-1)^1 8^3}{3!}$
- (D) $\frac{1}{8} \left(\frac{(-1)^1 8^3}{3!} \right)$
- (E) $8 \left(\frac{(-1)^1 8^3}{3!} \right)$

Questions 40–42

40. Let $\zeta = e^{2\pi i/14}$. Find $\sum_{k=0}^{13} \zeta^k$.
- (A) 0
(B) 1
(C) 2
(D) 3
(E) 4
41. Let C be $x^2/4 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-y) dx + (8x) dy$.
- (A) $1 + 18\pi$
(B) 18π
(C) 9π
(D) 36π
(E) -18π
42. A 3×3 matrix A has characteristic polynomial $(t - 36)(t + 1)(t - 2)$. What is $\text{tr}(A^2)$?
- (A) 1300
(B) 1299
(C) 1303
(D) 1301
(E) 1302

Questions 43–45

43. Find the curvature at $t = 0$ of $r(t) = (2t, 8t^2)$.

- (A) 4
- (B) -4
- (C) 5
- (D) 8
- (E) 2

44. An analytic function has real part $u(x, y) = x^2 - y^2 + 12x$ and satisfies $f(0) = 0$. Find $\text{Im } f(1 + i)$.

- (A) 6
- (B) 10
- (C) 18
- (D) 14
- (E) 22

45. Find the upward flux of $\mathbf{F} = (0, 0, z)$ through $z = 8x^2 + 2y^2$, $0 \leq x, y \leq 1$.

- (A) $-\frac{10}{3}$
- (B) $\frac{20}{3}$
- (C) $\frac{5}{3}$
- (D) $\frac{13}{3}$
- (E) $\frac{10}{3}$

Questions 46–48

46. Let N be the 42×42 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\text{rank } N^5$.

- (A) 38
- (B) 36
- (C) 37
- (D) 39
- (E) 35

47. For $f(x) = x^2 e^{8x}$, find $f^{(4)}(0)$.

- (A) $\frac{4! 8^2}{2}$
- (B) $2 \left(\frac{4! 8^2}{2} \right)$
- (C) $\frac{1}{4} \left(\frac{4! 8^2}{2} \right)$
- (D) $\frac{1}{12} \left(\frac{4! 8^2}{2} \right)$
- (E) $\frac{1}{2} \left(\frac{4! 8^2}{2} \right)$

48. Apply one Newton iteration to solve $x^2 - 17 = 0$, starting at $x_0 = 1$. What is x_1 ?

- (A) 3
- (B) 12
- (C) 6
- (D) 9
- (E) 15

Questions 49–51

49. If $y' = 8y$, $y(0) = 1$, what is the coefficient of x^2 in the Maclaurin series of y ?
- (A) 33
(B) 16
(C) 32
(D) 64
(E) -32
50. Let $G = \mathbb{Z}/246\mathbb{Z} \times \mathbb{Z}/168\mathbb{Z}$. Find the order of $(1, 1)$.
- (A) 6886
(B) 6888
(C) 6887
(D) 6889
(E) 6890
51. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 8x$ and has density $\rho(x, y) = 1$. Find its centroid coordinate $\bar{x}$.
- (A) $\frac{4}{3}$
(B) $\frac{1}{3}$
(C) $\frac{2}{3}$
(D) $-\frac{2}{3}$
(E) $\frac{5}{3}$

Questions 52–54

52. Bisection starts with an interval of length 2^{17} . After 19 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{1}{24}$
- (B) $\frac{1}{8}$
- (C) $\frac{7}{8}$
- (D) $\frac{1}{16}$
- (E) $\frac{9}{16}$

53. The region $0 \leq x \leq 1$, $0 \leq y \leq 8x^2$ is revolved about the x -axis. Find its volume.

- (A) $\frac{32\pi}{5}$
- (B) $\frac{128\pi}{5}$
- (C) $1 + \frac{64\pi}{5}$
- (D) $\frac{64\pi}{5}$
- (E) $-\frac{64\pi}{5}$

54. How many group homomorphisms $\mathbb{Z}/246\mathbb{Z} \rightarrow \mathbb{Z}/168\mathbb{Z}$ are there?

- (A) 7
- (B) 4
- (C) 5
- (D) 8
- (E) 6

Questions 55–57

55. For $y' = 8y(1 - y/2)$, what is the derivative of the right-hand side with respect to y at the positive equilibrium?

- (A) -16
- (B) -7
- (C) 8
- (D) -8
- (E) -4

56. What is the least positive multiplicative inverse of 2 modulo 35?

- (A) 24
- (B) 18
- (C) 30
- (D) 12
- (E) 6

57. Find the coefficient of x^2 in $e^{1x} \log(1 + 8x)$.

- (A) -48
- (B) -16
- (C) 24
- (D) -24
- (E) -32

Questions 58–60

58. How many 1-dimensional subspaces does $\mathbb{F}_{17}^3$ have?

- (A) $\frac{17^4 - 1}{17 - 1}$
- (B) $\frac{17^3 - 1}{17 - 1}$
- (C) $\frac{17^6 - 1}{17 - 1}$
- (D) $\frac{17^5 - 1}{17 - 1}$
- (E) $\frac{17^2 - 1}{17 - 1}$

59. Evaluate $\int_0^1 \int_x^1 8y^2 \, dy \, dx$.

- (A) 4
- (B) $\frac{2}{3}$
- (C) 1
- (D) $\frac{8}{3}$
- (E) 2

60. How many truth assignments to $p, q, r_1, \dots, r_{15}$ make $p \Rightarrow q$ false?

- (A) 2^{16}
- (B) 2^{14}
- (C) 2^{15}
- (D) 2^{17}
- (E) 2^{18}

Questions 61–63

61. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^8 y^1 dV$.

- (A) $12 \left(\frac{8! 1!}{12!} \right)$
- (B) $\frac{8! 1!}{12!}$
- (C) $6 \left(\frac{8! 1!}{12!} \right)$
- (D) $\frac{1}{18}$
- (E) $\frac{1}{6} \left(\frac{8! 1!}{12!} \right)$

62. How many monic irreducible polynomials of degree 2 are there over $\mathbb{F}_{17}$?

- (A) 138
- (B) 134
- (C) 137
- (D) 136
- (E) 135

63. Find the coefficient of x^4 in $\cos(8x)/(1 - 1x^2)$.

- (A) $\frac{416}{3}$
- (B) $-\frac{419}{3}$
- (C) $\frac{419}{3}$
- (D) 3352
- (E) $\frac{512}{3}$

Questions 64–66

64. What is the coefficient of x^2 in $(1 + 2x)^{17}$?

- (A) 544
- (B) 725
- (C) 906
- (D) 182
- (E) 363

65. Solve $y'' - 18y' + 80y = 0$ with $y(0) = 1$, $y'(0) = 0$. Find $y(1)$.

- (A) $(10 - 8e^2)e^8$
- (B) $(5 - 4e^2)e^8$
- (C) $(5 - 4e^2)e^8 + 1$
- (D) $\frac{(5 - 4e^2)e^8}{2}$
- (E) $(-5 + 4e^2)e^8$

66. How many elements z of $\mathbb{F}_{2^8}$ satisfy $z^2 = z$?

- (A) 3
- (B) 2
- (C) 4
- (D) 1
- (E) 0

Answer Key

Practice Test 40 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	E	23	C	45	E
2	C	24	B	46	C
3	A	25	C	47	A
4	B	26	D	48	D
5	D	27	C	49	C
6	E	28	A	50	B
7	A	29	E	51	C
8	C	30	D	52	B
9	B	31	C	53	D
10	A	32	B	54	E
11	D	33	E	55	D
12	E	34	B	56	B
13	D	35	E	57	D
14	A	36	C	58	B
15	E	37	E	59	E
16	A	38	A	60	C
17	B	39	A	61	B
18	A	40	A	62	D
19	E	41	B	63	C
20	D	42	D	64	A
21	C	43	A	65	B
22	E	44	D	66	B

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