

GRE Mathematics Practice Test 41

Advanced Challenge

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

The most demanding form in this five-test cycle. Emphasis on connected ideas, non-routine reasoning, and careful conceptual distinctions. This is cycle 8 of 19, position 1 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. For the curve $x = t^6$, $y = t^7 + 9t$, find d^2y/dx^2 at $t = 1$.

- (A) $-\frac{3}{2}$
- (B) $\frac{8}{3}$
- (C) $\frac{13}{9}$
- (D) $-\frac{19}{18}$
- (E) $-\frac{19}{3}$

2. Linear maps $T : \mathbb{R}^{44} \rightarrow \mathbb{R}^{44}$ and $U : \mathbb{R}^{44} \rightarrow \mathbb{R}^{44}$ have ranks 4 and 4. What is the largest possible rank of UT ?

- (A) 2
- (B) 6
- (C) 5
- (D) 4
- (E) 3

3. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(9+6j)(15+6j)}$.

- (A) $\frac{1}{54}$
- (B) $\frac{1}{6}$
- (C) $\frac{2}{3}$
- (D) $\frac{1}{15}$
- (E) $\frac{1}{9}$

Questions 4–6

4. A random variable has mean 0 and variance 13. What upper bound does the Chebyshev inequality give for $P(|X| \geq 26)$?

- (A) $\frac{1}{52}$
- (B) $\frac{51}{52}$
- (C) $\frac{1}{156}$
- (D) $\frac{1}{104}$
- (E) $\frac{53}{104}$

5. Under $x = u + 9v$, $y = u - 6v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) 54
- (B) 225
- (C) 3
- (D) 15
- (E) $\frac{1}{15}$

6. Fix a nonzero vector $u \in \mathbb{R}^{43}$. What is the dimension of the space of skew-symmetric 43×43 matrices A satisfying $Au = 0$?

- (A) 861
- (B) 859
- (C) 863
- (D) 860
- (E) 862

Questions 7–9

7. Find the coefficient of x^4 in $e^{5x} \log(1 + 9x)$.

- (A) -735
- (B) $-\frac{6561}{4}$
- (C) -744
- (D) -17856
- (E) 744

8. A box is chosen uniformly from two boxes. Box A contains 18 red and 20 blue balls; box B contains 20 red and 18 blue balls. Two balls are drawn without replacement and both are red. What is the conditional probability that A was chosen?

- (A) $\frac{153}{343}$
- (B) $\frac{248}{343}$
- (C) $\frac{153}{686}$
- (D) $\frac{190}{343}$
- (E) $\frac{51}{343}$

9. Evaluate $\int_0^\infty x^6 e^{-9x} dx$.

- (A) $\frac{1}{6} \left(\frac{6!}{9^7} \right)$
- (B) $9 \left(\frac{6!}{9^7} \right)$
- (C) $\frac{6!}{9^7}$
- (D) $\frac{1}{9} \left(\frac{6!}{9^7} \right)$
- (E) 720

Questions 10–12

10. Let $A = \begin{bmatrix} 1 & 37 & 0 \\ 0 & 1 & 8 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 3)$ entry of A^5 ?

- (A) 2959
- (B) 2958
- (C) 2962
- (D) 2961
- (E) 2960

11. Evaluate $\iiint_B 9z^2(x^2 + y^2 + z^2)^6 dV$, where B is the unit ball.

- (A) $\frac{6\pi}{17}$
- (B) $1 + \frac{12\pi}{17}$
- (C) $-\frac{12\pi}{17}$
- (D) $\frac{24\pi}{17}$
- (E) $\frac{12\pi}{17}$

12. Let $X_1, \dots, X_{18}$ be independent uniform random variables on $[0, 1]$. What is $E[(\max_i X_i)^2]$?

- (A) $\frac{3}{10}$
- (B) $\frac{1}{10}$
- (C) $\frac{9}{10}$
- (D) $\frac{9}{20}$
- (E) $\frac{19}{20}$

Questions 13–15

13. Find the directional derivative of $f(x, y, z) = x^2y + 6xz^2 + 9y^2z$ at $(1, 1, 1)$ in the direction $(1, 2, 0)$.

- (A) $-\frac{46\sqrt{5}}{5}$
- (B) $1 + \frac{46\sqrt{5}}{5}$
- (C) $\frac{46\sqrt{5}}{5}$
- (D) $\frac{23\sqrt{5}}{5}$
- (E) $\frac{92\sqrt{5}}{5}$

14. An invertible 43×43 matrix A has determinant 4. Find $\det(\text{adj } A)$.

- (A) 2^{84}
- (B) $2^{84} \cdot 3$
- (C) 2^{85}
- (D) $2^{84} \cdot 5$
- (E) 2^{83}

15. Evaluate $\int_C (6x^5 + 9y) dx + (9x + 6y^5) dy$, where C is $r(t) = (t, 2t^2)$, $0 \leq t \leq 1$.

- (A) $\frac{83}{2}$
- (B) 83
- (C) 84
- (D) 166
- (E) -83

Questions 16–18

16. Independent trials have success probability $1/18$. Let T count trials up to and including the first success. Find $E[T^2]$.

- (A) 630
- (B) 210
- (C) 840
- (D) 1050
- (E) 420

17. For $\mathbf{F} = (9x^6y, y^6z, xz^6)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) -306
- (B) 306
- (C) 307
- (D) 612
- (E) 153

18. On P_{44} , the real polynomials of degree at most 44, define $T(p) = x^2p'' - 12xp' + 42p$. Find its rank.

- (A) 44
- (B) 43
- (C) 42
- (D) 45
- (E) 41

Questions 19–21

19. The solution of $y' = 9x^5y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{3^{\frac{2}{3}}}{3}$
- (B) $\frac{3^{\frac{5}{6}}}{3}$
- (C) $\frac{2}{3}$
- (D) $\frac{\sqrt[6]{2} \cdot 3^{\frac{5}{6}}}{3}$
- (E) $\frac{2^{\frac{5}{6}} \sqrt[6]{3}}{2}$

20. A uniformly random subset of 18 objects is selected from a collection with 19 red and 20 blue objects. If X is the number of red objects selected, find $\text{Var}(X)$.

- (A) $\frac{589}{169}$
- (B) $\frac{840}{169}$
- (C) $\frac{169}{758}$
- (D) $\frac{169}{210}$
- (E) $\frac{169}{420}$

21. If $y'' - 18y' + 81y = 6e^{9x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $\frac{3e^9}{2}$
- (B) $3e^9$
- (C) $6e^9$
- (D) $-3e^9$
- (E) $1 + 3e^9$

Questions 22–24

22. Subspaces U, V of $\mathbb{R}^{45}$ have dimensions 5 and 6. What is the smallest possible value of $\dim(U \cap V)$?

- (A) 4
- (B) 1
- (C) 0
- (D) 3
- (E) 2

23. Let $f(x, y) = e^{9x+6y}$. Find $\partial^6 f / \partial x^5 \partial y$ at $(0, 0)$.

- (A) $6 \cdot 9^5$
- (B) $\frac{3}{2} (6 \cdot 9^5)$
- (C) $\frac{32}{243} (6 \cdot 9^5)$
- (D) $\frac{1}{6} (6 \cdot 9^5)$
- (E) $\frac{16}{81} (6 \cdot 9^5)$

24. Which subset of $\mathbb{R}$ is compact, with 13 fixed as displayed?

- (A) $\{0\} \cup \{13/k : k \geq 1\}$
- (B) $\{13/k : k \geq 1\}$
- (C) $\mathbb{Q} \cap [0, 13]$
- (D) $(0, 13)$
- (E) $13\mathbb{Z}$

Questions 25–27

25. For $f(x) = x^6 e^{9x}$, find $f^{(8)}(0)$.

- (A) $2 \left(\frac{8! 9^2}{2} \right)$
- (B) $\frac{1}{20160} \left(\frac{8! 9^2}{2} \right)$
- (C) $\frac{2}{9} \left(\frac{8! 9^2}{2} \right)$
- (D) $\frac{1}{56} \left(\frac{8! 9^2}{2} \right)$
- (E) $\frac{8! 9^2}{2}$

26. Let V consist of real polynomials of degree at most 10 that have both value and first derivative zero at 37, 38, and 40. Find $\dim V$.

- (A) 5
- (B) 6
- (C) 3
- (D) 7
- (E) 4

27. Evaluate $\int_0^1 \int_x^1 9y^6 \, dy \, dx$.

- (A) $\frac{9}{7}$
- (B) $\frac{9}{2}$
- (C) $\frac{9}{56}$
- (D) $\frac{3}{2}$
- (E) $\frac{9}{8}$

Questions 28–30

28. For $a_k = (-1)^k + 13/k$, find $\limsup_{k \rightarrow \infty} a_k - \liminf_{k \rightarrow \infty} a_k$.

- (A) 3
- (B) 0
- (C) 2
- (D) 4
- (E) 1

29. Find the coefficient of x^6 in the Taylor series of $(1 - 9x)^{-2}$ at zero.

- (A) $2(7 \cdot 9^6)$
- (B) $\frac{6}{7}(7 \cdot 9^6)$
- (C) $\frac{1}{7}(7 \cdot 9^6)$
- (D) $7 \cdot 9^6$
- (E) $\frac{1}{9}(7 \cdot 9^6)$

30. The roots r, s of $x^2 - 45x + 296 = 0$ are counted with multiplicity. Find $r^4 + s^4$.

- (A) $45^4 - 4(296)(45^2)$
- (B) $45^4 - 2(296)(45^2) + 2(296^2)$
- (C) $45^4 + 2(296^2)$
- (D) $45^4 - 4(296)(45^2) + 296^2$
- (E) $45^4 - 4(296)(45^2) + 2(296^2)$

Questions 31–33

31. Evaluate $\int_0^1 x^8(\log x)^5 dx$.

- (A) $9 \left(\frac{(-1)^5 5!}{9^6} \right)$
- (B) $-\frac{(-1)^5 5!}{9^6}$
- (C) $\frac{(-1)^5 5!}{9^6}$
- (D) $\frac{1}{6} \left(\frac{(-1)^5 5!}{9^6} \right)$
- (E) $\frac{19683}{40} \left(\frac{(-1)^5 5!}{9^6} \right)$

32. For which real a is $f(x) = x^a$ in $L^{13}(0, 1)$?

- (A) $a \geq -1/13$
- (B) $a < 1/13$
- (C) All real a
- (D) $a > 0$
- (E) $a > -1/13$

33. Find the curvature of the helix $r(t) = (9 \cos t, 9 \sin t, 6t)$.

- (A) $-\frac{1}{13}$
- (B) $\frac{2}{13}$
- (C) $\frac{1}{13}$
- (D) $\frac{1}{26}$
- (E) $\frac{14}{13}$

Questions 34–36

34. For real t , let $M(t) = \begin{pmatrix} t-37 & 8 & 0 \\ 0 & t-37 & 1 \\ 0 & 0 & t-37 \end{pmatrix}$. At $t = 37$, what is $\dim \ker(M(t)^2) - \dim \ker M(t)$?

- (A) 4
- (B) 3
- (C) 2
- (D) 1
- (E) 0

35. The region $0 \leq x \leq 1$, $0 \leq y \leq 9x^6$ is revolved about the line $x = -2$. Find its volume.

- (A) $1 + \frac{207\pi}{28}$
- (B) $\frac{207\pi}{28}$
- (C) $\frac{207\pi}{56}$
- (D) $\frac{207\pi}{14}$
- (E) $-\frac{207\pi}{28}$

36. $T : \mathbb{R} \rightarrow \mathbb{R}$ has Lipschitz constant at most $1/13$. If $x_1 = T(x_0)$ and $|x_1 - x_0| \leq 1$, what is the sharp upper bound on $|T^3(x_0) - x_*|$, where x_* is its fixed point?

- (A) $\frac{1}{6084}$
- (B) $\frac{1}{4056}$
- (C) $\frac{2028}{2027}$
- (D) $\frac{2028}{2029}$
- (E) $\frac{1}{2028}$

Questions 37–39

37. If $y' + 9y = 6x^6e^{-9x}$ and $y(0) = 2$, find $y(1)$.

- (A) $\frac{40}{7e^9}$
- (B) $-\frac{20}{7e^9}$
- (C) $\frac{10}{7e^9}$
- (D) $\frac{20}{7e^9} + 1$
- (E) $\frac{20}{7e^9}$

38. Let $u = (8, 6)^T$ and $v = (1, 0)^T$. Find the squared distance from $(8, 0)^T$ to $\text{span}(u)$.

- (A) $\frac{1177}{50}$
- (B) $\frac{576}{25}$
- (C) $\frac{1127}{50}$
- (D) $\frac{551}{25}$
- (E) $\frac{601}{25}$

39. Find the coefficient of x^6 in $\cos(9x)/(1 - 5x^2)$.

- (A) $\frac{20699}{80}$
- (B) -186291
- (C) $-\frac{20699}{80}$
- (D) $-\frac{59049}{80}$
- (E) $-\frac{20779}{80}$

Questions 40–42

40. Find the coefficient of z^{-1} in $e^{8z}/[z^2(1-z)]$ for $0 < |z| < 1$.

- (A) 3
- (B) 6
- (C) 9
- (D) 12
- (E) 15

41. Evaluate $\int_0^{\log 2} e^x(e^x + 9)^6 dx$.

- (A) $-\frac{11^7 - 10^7}{7}$
- (B) $\frac{11^7 - 10^7}{7}$
- (C) $11^6 - 10^6$
- (D) $7\left(\frac{11^7 - 10^7}{7}\right)$
- (E) $\frac{11^7}{7}$

42. How many invertible 4×4 matrices over $\mathbb{F}_{19}$ are there with determinant 1?

- (A) $\frac{19^4 19^4 19^4 19^4}{18}$
- (B) $\frac{(19^4 - 19^0)(19^4 - 19^1)(19^4 - 19^2)}{18}$
- (C) $\frac{(19^4 - 19^0)(19^4 - 19^1)(19^4 - 19^2)(19^4 - 19^3)}{18}$
- (D) $\frac{(19^4 - 19^1)(19^4 - 19^2)(19^4 - 19^3)(19^4 - 19^4)}{18}$
- (E) $\frac{(19^4 - 19^0)(19^4 - 19^0)(19^4 - 19^0)(19^4 - 19^0)}{18}$

Questions 43–45

43. For $x > 0$, $x^2 y'' + (-23)xy' + 135y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $-\frac{(1 - e^6)e^9}{3}$
- (B) $-\frac{e^9}{6} + 1 + \frac{e^{15}}{6}$
- (C) $\frac{(1 - e^6)e^9}{6}$
- (D) $-\frac{(1 - e^6)e^9}{6}$
- (E) $-\frac{(1 - e^6)e^9}{12}$

44. An entire function satisfies $|f(z)| \leq C(1 + |z|^8)$ for some $C > 0$. What is the largest possible degree of f ?

- (A) 6
- (B) 8
- (C) 4
- (D) 12
- (E) 10

45. Let $f(x, y) = x^2 + 18xy + (87)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 24
- (B) 348
- (C) 6
- (D) 12
- (E) 87

Questions 46–48

46. How many permutations in S_{43} consist of one 5-cycle and 38 fixed points?
- (A) $2^4 \cdot 3^2 \cdot 7 \cdot 13 \cdot 41 \cdot 43$
(B) $2^3 \cdot 3^2 \cdot 7 \cdot 13 \cdot 41 \cdot 43$
(C) $2^4 \cdot 3^2 \cdot 5 \cdot 7 \cdot 13 \cdot 41 \cdot 43$
(D) $2^5 \cdot 3^2 \cdot 7 \cdot 13 \cdot 41 \cdot 43$
(E) $2^4 \cdot 3^3 \cdot 7 \cdot 13 \cdot 41 \cdot 43$
47. For $f(x, y) = x^2 + 9xy + 6y^2$, what is its maximum directional derivative at $(1, -1)$?
- (A) 10
(B) $\sqrt{58}$
(C) 58
(D) -3
(E) -7
48. How many distinct unoriented cycles of length 4 are in $K_{13,15}$?
- (A) 10920
(B) 5460
(C) 8190
(D) 13650
(E) 2730

Questions 49–51

49. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{9t} \cos(6t)$, and $y = e^{9t} \sin(6t)$, find d^2f/dt^2 at $t = 1$.

- (A) 324
- (B) 12
- (C) 234
- (D) 0
- (E) 18

50. How many maximal proper subgroups does the cyclic group of order 492 have?

- (A) 3
- (B) 4
- (C) 5
- (D) 1
- (E) 2

51. For $D = \{(x, y) : x^2 + y^2 \leq 9\}$, evaluate $\iint_D (x^2 + y^2)^5 dA$.

- (A) 177147π
- (B) 531441π
- (C) $\frac{531441\pi}{11}$
- (D) $\frac{19683\pi}{2}$
- (E) $\frac{177147\pi}{2}$

Questions 52–54

52. How many functions from a set of 18 elements onto a set of 3 elements exist?

- (A) $3^{18} - 3(2^{18})$
- (B) $3^{18} - 2^{18}$
- (C) $3^{18} - 3(2^{18}) + 3$
- (D) $3^{18} - 3(2^{18}) - 3$
- (E) 3^{18}

53. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 9z^2 = x + 6y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) -18
- (B) -108
- (C) -6
- (D) -54
- (E) 108

54. How many residues x modulo 60 satisfy $2x \equiv 14 \pmod{60}$ and $2x \equiv 0 \pmod{60}$?

- (A) 4
- (B) 1
- (C) 2
- (D) 0
- (E) 3

Questions 55–57

55. If $y' = 18xy$ and $y(0) = 1$, find the coefficient of x^{12} in the Maclaurin series of y .

- (A) $\frac{59049}{160}$
- (B) $\frac{59049}{40}$
- (C) $-\frac{59049}{80}$
- (D) $\frac{59049}{80}$
- (E) $\frac{59129}{80}$

56. How many nonnegative integer triples sum to 36, if each coordinate is at most 18?

- (A) 253
- (B) 190
- (C) 316
- (D) 64
- (E) 127

57. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(9j)!}{(j!)^9} x^{6j}$?

- (A) $3\left(9^{-\frac{3}{2}}\right)$
- (B) 9^{-9}
- (C) $9^{-\frac{2}{3}}$
- (D) $9^{-\frac{3}{2}}$
- (E) $9^{\frac{3}{2}}$

Questions 58–60

58. The field K has 11^9 elements. Find the largest possible multiplicative order of a nonzero element of K .

- (A) $11^8 - 1$
- (B) 11^9
- (C) $11^9 + 1$
- (D) $11^{10} - 1$
- (E) $11^9 - 1$

59. Two solutions of $y'' + 9x^6y' + 6y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $e^{-\frac{9}{7}} + 1$
- (B) $\frac{1}{2e^{\frac{9}{7}}}$
- (C) $-\frac{1}{e^{\frac{9}{7}}}$
- (D) $e^{-\frac{9}{7}}$
- (E) $\frac{2}{e^{\frac{9}{7}}}$

60. A tree has 18 vertices of degree 3; every other vertex has degree 1. How many leaves does it have?

- (A) 26
- (B) 32
- (C) 8
- (D) 14
- (E) 20

Questions 61–63

61. What is the maximum of x^6y^9 over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\frac{6}{25}$
- (B) $\left(\frac{6}{15}\right)^6 \left(\frac{9}{15}\right)^9$
- (C) $15 \left(\left(\frac{6}{15}\right)^6 \left(\frac{9}{15}\right)^9 \right)$
- (D) $\frac{1}{2^{15}}$
- (E) $\left(\frac{6}{15}\right)^9 \left(\frac{9}{15}\right)^6$

62. Let $G = \mathbb{Z}/252\mathbb{Z} \times \mathbb{Z}/172\mathbb{Z}$. Find the order of G/H , where $H = \{g \in G : 4g = 0\}$.

- (A) 2710
- (B) 2709
- (C) 2708
- (D) 2711
- (E) 2707

63. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 9x)^{1/(5x)} - e^{9/5}}{x}$.

- (A) $-\frac{81e^{\frac{9}{5}}}{10}$
- (B) $\frac{81e^{\frac{9}{5}}}{10}$
- (C) $e^{\frac{9}{5}}$
- (D) $-\frac{9e^{\frac{9}{5}}}{10}$
- (E) $-\frac{81e^{\frac{9}{5}}}{5}$

Questions 64–66

64. A connected planar graph has 21 vertices and 37 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 30
- (B) 24
- (C) 18
- (D) 6
- (E) 12

65. Let C be the unit circle $x^2 + y^2 = 1$, $z = 6$, counterclockwise viewed from above. Evaluate $\int_C -9y(x^2 + y^2)^5 dx + (9x(x^2 + y^2)^5) dy + 2z dz$.

- (A) $1 + 18\pi$
- (B) 9π
- (C) 36π
- (D) 18π
- (E) -18π

66. How many group homomorphisms $\mathbb{Z}/252\mathbb{Z} \rightarrow \mathbb{Z}/172\mathbb{Z}$ have image of order 4?

- (A) 0
- (B) 4
- (C) 1
- (D) 3
- (E) 2

Answer Key

Practice Test 41 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	D	23	A	45	A
2	D	24	A	46	A
3	A	25	E	47	B
4	A	26	A	48	C
5	D	27	E	49	D
6	A	28	C	50	A
7	C	29	D	51	E
8	A	30	E	52	C
9	C	31	C	53	B
10	E	32	E	54	D
11	E	33	C	55	D
12	C	34	D	56	B
13	C	35	B	57	D
14	A	36	E	58	E
15	B	37	E	59	D
16	A	38	B	60	E
17	B	39	C	61	B
18	B	40	C	62	B
19	D	41	B	63	A
20	E	42	C	64	C
21	B	43	D	65	D
22	C	44	B	66	E

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