

GRE Mathematics Practice Test 42

Advanced Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A demanding practice form with frequent multi-step arguments and a mix of computational and conceptual questions. This is cycle 8 of 19, position 2 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. For $f(x, y) = x^2 + 9xy + 5y^2$, what is its maximum directional derivative at $(1, -1)$?
- (A) 8
(B) $5\sqrt{2}$
(C) 50
(D) -1
(E) -7
2. Subspaces U, V of $\mathbb{R}^{46}$ have dimensions 5 and 6. What is the smallest possible value of $\dim(U \cap V)$?
- (A) 0
(B) 1
(C) 3
(D) 2
(E) 4
3. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j(x-9)^j}{j^{\frac{4}{3}}(\log j)^m}$ converge absolutely at $x = 8$? Give the least such m , or 0 if none exists.
- (A) 2
(B) 4
(C) 1
(D) 0
(E) 3

Questions 4–6

4. A random variable has mean 0 and variance 14. What upper bound does the Chebyshev inequality give for $P(|X| \geq 28)$?

- (A) $\frac{1}{56}$
- (B) $\frac{56}{55}$
- (C) $\frac{1}{112}$
- (D) $\frac{57}{112}$
- (E) $\frac{1}{168}$

5. Under $x = u + 9v$, $y = u - 5v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) 45
- (B) 14
- (C) 196
- (D) $\frac{1}{14}$
- (E) 4

6. Let V consist of real polynomials of degree at most 9 that have both value and first derivative zero at 38, 39, and 41. Find $\dim V$.

- (A) 4
- (B) 3
- (C) 2
- (D) 5
- (E) 6

Questions 7–9

7. Let C be $x^2/25 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-3y^3) dx + (\frac{x^3}{3}) dy$.

- (A) 85π
- (B) $\frac{85\pi}{4}$
- (C) $1 + \frac{85\pi}{2}$
- (D) $\frac{85\pi}{2}$
- (E) $-\frac{85\pi}{2}$

8. A box is chosen uniformly from two boxes. Box A contains 19 red and 21 blue balls; box B contains 21 red and 19 blue balls. Two balls are drawn without replacement and both are red. What is the conditional probability that A was chosen?

- (A) $\frac{70}{127}$
- (B) $\frac{19}{127}$
- (C) $\frac{92}{127}$
- (D) $\frac{57}{254}$
- (E) $\frac{57}{127}$

9. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(9+5j)(14+5j)}$.

- (A) $\frac{1}{14}$
- (B) $\frac{1}{5}$
- (C) $\frac{1}{45}$
- (D) $\frac{1}{9}$
- (E) $\frac{5}{9}$

Questions 10–12

10. The roots r, s of $x^2 - 44x + 228 = 0$ are counted with multiplicity. Find $r^3 + s^3$.
- (A) 55090
(B) 55086
(C) 55088
(D) 55087
(E) 55089
11. Find the upward flux of $\mathbf{F} = (x, y, z)$ through $z = 9x^2 + 5y^2$, $0 \leq x, y \leq 1$.
- (A) $-\frac{11}{3}$
(B) $-\frac{14}{3}$
(C) $-\frac{7}{3}$
(D) $\frac{14}{3}$
(E) $-\frac{28}{3}$
12. Let $X_1, \dots, X_{19}$ be independent uniform random variables on $[0, 1]$. What is $E[(\max_i X_i)^2]$?
- (A) $\frac{20}{21}$
(B) $\frac{19}{21}$
(C) $\frac{19}{63}$
(D) $\frac{2}{21}$
(E) $\frac{19}{42}$

Questions 13–15

13. Evaluate $\lim_{x \rightarrow 0} \frac{e^{9x} - \sum_{j=0}^4 (9x)^j / j!}{x^5}$.

- (A) $5 \left(\frac{9^5}{5!} \right)$
- (B) $-\frac{9^5}{5!}$
- (C) $\frac{9^5}{5!}$
- (D) $\frac{1}{9} \left(\frac{9^5}{5!} \right)$
- (E) $120 \left(\frac{9^5}{5!} \right)$

14. For real t , let $M(t) = \begin{pmatrix} t-38 & 6 & 0 \\ 0 & t-38 & 1 \\ 0 & 0 & t-38 \end{pmatrix}$. At $t = 38$, what is $\dim \ker(M(t)^2) - \dim \ker M(t)$?

- (A) 0
- (B) 2
- (C) 4
- (D) 3
- (E) 1

15. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 9z^2 = x + 5y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) -18
- (B) 90
- (C) -45
- (D) -90
- (E) -5

Questions 16–18

16. Independent trials have success probability $1/19$. Let T count trials up to and including the first success. Find $E[T^2]$.

- (A) 1171
- (B) 703
- (C) 469
- (D) 235
- (E) 937

17. Evaluate $\int_0^1 \frac{x^4}{1+9x^5} dx$.

- (A) $\frac{\log(10)}{45}$
- (B) $\frac{\log(10)}{5}$
- (C) $-\frac{\log(10)}{45}$
- (D) $\log(10)$
- (E) $\frac{\log(10)}{9}$

18. Let $u = (8, 5)^T$ and $v = (1, 0)^T$. Find the squared distance from $(8, 0)^T$ to $\text{span}(u)$.

- (A) $\frac{1600}{89}$
- (B) $\frac{1511}{89}$
- (C) $\frac{1689}{89}$
- (D) $\frac{3289}{178}$
- (E) $\frac{3111}{178}$

Questions 19–21

19. If $y' = 18xy$ and $y(0) = 1$, find the coefficient of x^{10} in the Maclaurin series of y .

- (A) $\frac{19683}{40}$
- (B) $\frac{19683}{80}$
- (C) $\frac{19683}{20}$
- (D) $-\frac{19683}{40}$
- (E) $\frac{19723}{40}$

20. A uniformly random subset of 19 objects is selected from a collection with 20 red and 21 blue objects. If X is the number of red objects selected, find $E[X]$.

- (A) $\frac{380}{41}$
- (B) $\frac{421}{41}$
- (C) $\frac{462}{41}$
- (D) $\frac{190}{41}$
- (E) $\frac{760}{41}$

21. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(9j)!}{(j!)^9} x^{5j}$?

- (A) $9^{-\frac{5}{9}}$
- (B) $\frac{1}{9}$
- (C) $9^{\frac{9}{5}}$
- (D) 9^{-9}
- (E) $9^{-\frac{9}{5}}$

Questions 22–24

22. An idempotent 44×44 real matrix P has rank 5. What is $\det(I + P)$?

- (A) 33
- (B) 32
- (C) 31
- (D) 30
- (E) 34

23. The local inverse $y(x)$ of $x = 9y + y^3$ satisfies $y(0) = 0$. Find $y^{(5)}(0)$.

- (A) $9 \left(\frac{360}{9^7} \right)$
- (B) $\frac{1}{120} \left(\frac{360}{9^7} \right)$
- (C) $-\frac{360}{9^7}$
- (D) $\frac{360}{9^7}$
- (E) $\frac{1}{9} \left(\frac{360}{9^7} \right)$

24. Which subset of $\mathbb{R}$ is compact, with 14 fixed as displayed?

- (A) $\mathbb{Q} \cap [0, 14]$
- (B) $\{14/k : k \geq 1\}$
- (C) $\{0\} \cup \{14/k : k \geq 1\}$
- (D) $14\mathbb{Z}$
- (E) $(0, 14)$

Questions 25–27

25. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 9x)^{1/(4x)} - e^{9/4}}{x}$.

- (A) $\frac{81e^{\frac{9}{4}}}{8}$
- (B) $-\frac{81e^{\frac{9}{4}}}{8}$
- (C) $-\frac{9e^{\frac{9}{4}}}{8}$
- (D) $e^{\frac{9}{4}}$
- (E) $-\frac{81e^{\frac{9}{4}}}{4}$

26. For $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 38 \\ 6 \\ 44 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find the minimum squared residual.

- (A) $\frac{4903}{6}$
- (B) $\frac{2450}{3}$
- (C) $\frac{4897}{6}$
- (D) $\frac{2453}{3}$
- (E) $\frac{2447}{3}$

27. Let $F(x) = \int_0^{x^5} e^{9t} dt$. Find $F'(1)$.

- (A) e^{45}
- (B) e^9
- (C) $45e^9$
- (D) $5e^9$
- (E) $\frac{e^9}{5}$

Questions 28–30

28. For $a_k = (-1)^k + 14/k$, find $\limsup_{k \rightarrow \infty} a_k - \liminf_{k \rightarrow \infty} a_k$.

- (A) 3
- (B) 4
- (C) 1
- (D) 0
- (E) 2

29. Find the arc length of $r(t) = (9(t - t^3/3), 9t^2)$ for $0 \leq t \leq 1/5$.

- (A) $\frac{456}{125}$
- (B) $\frac{353}{125}$
- (C) $\frac{228}{125}$
- (D) $\frac{114}{125}$
- (E) $-\frac{228}{125}$

30. What is the coefficient of x^7 in $(x + 8)^7(1 - x)^3$?

- (A) -14053
- (B) -14054
- (C) -14057
- (D) -14056
- (E) -14055

Questions 31–33

31. Evaluate $\iiint_{x^2+y^2+z^2 \leq 9} (x^2 + y^2 + z^2)^4 dV$.

- (A) $11 \left(\frac{4\pi 9^{\frac{11}{2}}}{11} \right)$
- (B) $\frac{1}{2} \left(\frac{4\pi 9^{\frac{11}{2}}}{11} \right)$
- (C) $\frac{1}{3} \left(\frac{4\pi 9^{\frac{11}{2}}}{11} \right)$
- (D) $\frac{11}{81} \left(\frac{4\pi 9^{\frac{11}{2}}}{11} \right)$
- (E) $\frac{4\pi 9^{\frac{11}{2}}}{11}$

32. For which real a is $f(x) = x^a$ in $L^{14}(0, 1)$?

- (A) $a < 1/14$
- (B) $a > 0$
- (C) $a \geq -1/14$
- (D) $a > -1/14$
- (E) All real a

33. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(9x) - \sum_{j=0}^3 (-1)^j (9x)^{2j+1} / (2j+1)!}{x^9}$.

- (A) $\frac{1}{9} \left(\frac{(-1)^4 9^9}{9!} \right)$
- (B) $2 \left(\frac{(-1)^4 9^9}{9!} \right)$
- (C) $\frac{(-1)^4 9^9}{9!}$
- (D) $9 \left(\frac{(-1)^4 9^9}{9!} \right)$
- (E) $-\frac{(-1)^4 9^9}{9!}$

Questions 34–36

34. A matrix A satisfies $(A - 39I)(A - 41I) = 0$, and its minimal polynomial has degree two. If $A^5 = uA + vI$, find u .

- (A) $2 \cdot 11 \cdot 761 \cdot 1531$
- (B) $11 \cdot 761 \cdot 1531$
- (C) $3 \cdot 11 \cdot 761 \cdot 1531$
- (D) $761 \cdot 1531$
- (E) $5 \cdot 11 \cdot 761 \cdot 1531$

35. Two solutions of $y'' + 9x^5y' + 5y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $e^{-\frac{3}{2}}$
- (B) $e^{-\frac{3}{2}} + 1$
- (C) $-\frac{1}{e^{\frac{3}{2}}}$
- (D) $\frac{1}{2e^{\frac{3}{2}}}$
- (E) $\frac{2}{e^{\frac{3}{2}}}$

36. $T : \mathbb{R} \rightarrow \mathbb{R}$ has Lipschitz constant at most $1/14$. If $x_1 = T(x_0)$ and $|x_1 - x_0| \leq 1$, what is the sharp upper bound on $|T^3(x_0) - x_*|$, where x_* is its fixed point?

- (A) $\frac{1}{7644}$
- (B) $\frac{1}{5096}$
- (C) $\frac{1}{2548}$
- (D) $\frac{2548}{2547}$
- (E) $\frac{2548}{5096}$

Questions 37–39

37. Let $f(x) = \log(1 + 9x)$. Find $f^{(5)}(0)$.

- (A) $\frac{1}{5} \left((-1)^4 4! 9^5 \right)$
- (B) $(-1)^4 4! 9^5$
- (C) $\frac{1}{9} \left((-1)^4 4! 9^5 \right)$
- (D) $5 \left((-1)^4 4! 9^5 \right)$
- (E) $-(-1)^4 4! 9^5$

38. Let $V = \begin{bmatrix} 1 & 38 & 1444 \\ 1 & 40 & 1600 \\ 1 & 43 & 1849 \end{bmatrix}$. Find $\det(V^T V)$.

- (A) 899
- (B) 900
- (C) 902
- (D) 901
- (E) 898

39. Evaluate $\int_C (5x^4 + 9y) dx + (9x + 5y^4) dy$, where C is $r(t) = (t, 3t^2)$, $0 \leq t \leq 1$.

- (A) 271
- (B) $\frac{271}{2}$
- (C) 542
- (D) 272
- (E) -271

Questions 40–42

40. What is the coefficient of z^{-1} in the Laurent expansion of e^{9z}/z^3 about zero?

- (A) $\frac{81}{4}$
- (B) $\frac{81}{2}$
- (C) 81
- (D) $\frac{85}{2}$
- (E) $\frac{83}{2}$

41. If $y' + 9y = 5x^5e^{-9x}$ and $y(0) = 3$, find $y(1)$.

- (A) $\frac{23}{3e^9}$
- (B) $-\frac{23}{6e^9}$
- (C) $\frac{23}{12e^9}$
- (D) $\frac{23}{6e^9}$
- (E) $\frac{23}{6e^9} + 1$

42. How many 2-dimensional subspaces does $\mathbb{F}_{19}^6$ have?

- (A) $\frac{(19^5 - 1)(19^5 - 19)}{(19^2 - 1)(19^2 - 19)}$
- (B) $\frac{(19^9 - 1)(19^9 - 19)}{(19^2 - 1)(19^2 - 19)}$
- (C) $\frac{(19^6 - 1)(19^6 - 19)}{(19^2 - 1)(19^2 - 19)}$
- (D) $\frac{(19^8 - 1)(19^8 - 19)}{(19^2 - 1)(19^2 - 19)}$
- (E) $\frac{(19^7 - 1)(19^7 - 19)}{(19^2 - 1)(19^2 - 19)}$

Questions 43–45

43. Evaluate $\int_0^1 x^8(\log x)^4 dx$.

- (A) $9 \left(\frac{(-1)^4 4!}{9^5} \right)$
- (B) $\frac{2187}{8} \left(\frac{(-1)^4 4!}{9^5} \right)$
- (C) $-\frac{(-1)^4 4!}{9^5}$
- (D) $\frac{(-1)^4 4!}{9^5}$
- (E) $\frac{1}{5} \left(\frac{(-1)^4 4!}{9^5} \right)$

44. An entire function satisfies $|f(z)| \leq C(1 + |z|^9)$ for some $C > 0$. What is the largest possible degree of f ?

- (A) 3
- (B) 12
- (C) 15
- (D) 6
- (E) 9

45. Find the coefficient of x^6 in $\cos(9x)/(1 - 4x^2)$.

- (A) $-\frac{59049}{80}$
- (B) $-\frac{18369}{80}$
- (C) $\frac{18289}{80}$
- (D) $-\frac{18289}{80}$
- (E) -164601

Questions 46–48

46. How many monic irreducible polynomials of degree 4 are there over $\mathbb{F}_{19}$?

- (A) $\frac{19^4 - 19^2}{4}$
- (B) $\frac{3(19^4 - 19^2)}{4}$
- (C) $19^4 - 19^2$
- (D) $2(19^4 - 19^2)$
- (E) $\frac{2(19^4 - 19^2)}{4}$

47. For $x > 0$, $x^2 y'' + (-22)xy' + 126y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $-\frac{e^9}{5} + 1 + \frac{e^{14}}{5}$
- (B) $\frac{(1 - e^5)e^9}{5}$
- (C) $-\frac{2(1 - e^5)e^9}{5}$
- (D) $-\frac{(1 - e^5)e^9}{5}$
- (E) $-\frac{(1 - e^5)e^9}{10}$

48. How many edges does the complete bipartite graph $K_{14,16}$ have?

- (A) 372
- (B) 76
- (C) 150
- (D) 224
- (E) 298

Questions 49–51

49. For $D = \{(x, y) : x^2 + y^2 \leq 9\}$, evaluate $\iint_D (x^2 + y^2)^4 dA$.

- (A) $\frac{6561\pi}{5}$
- (B) $\frac{118098\pi}{5}$
- (C) 6561π
- (D) 59049π
- (E) $\frac{59049\pi}{5}$

50. How many elements z of $\mathbb{F}_{7^9}$ satisfy $z^{49} = z$?

- (A) 8
- (B) 5
- (C) 6
- (D) 9
- (E) 7

51. Find the curvature of the helix $r(t) = (9 \cos t, 9 \sin t, 5t)$.

- (A) $\frac{9}{212}$
- (B) $\frac{115}{106}$
- (C) $\frac{9}{53}$
- (D) $-\frac{9}{106}$
- (E) $\frac{9}{106}$

Questions 52–54

52. How many functions from a set of 19 elements onto a set of 3 elements exist?

- (A) $3^{19} - 3(2^{19}) - 3$
- (B) $3^{19} - 2^{19}$
- (C) $3^{19} - 3(2^{19}) + 3$
- (D) 3^{19}
- (E) $3^{19} - 3(2^{19})$

53. Let $F(x) = \int_0^x (x - t)^4 e^{9t} dt$. Find $F^{(5)}(0)$.

- (A) 216
- (B) 0
- (C) $\frac{1}{24}$
- (D) 120
- (E) 24

54. How many generators does the cyclic group of order 2476099 have?

- (A) $19^4(19 - 1)$
- (B) $19^6(19 - 1)$
- (C) $19^3(19 - 1)$
- (D) $19^7(19 - 1)$
- (E) $19^5(19 - 1)$

Questions 55–57

55. The region $0 \leq x \leq 1$, $0 \leq y \leq 9x^5$ is revolved about the line $x = -3$. Find its volume.

- (A) $-\frac{81\pi}{7}$
- (B) $\frac{81\pi}{14}$
- (C) $\frac{162\pi}{7}$
- (D) $1 + \frac{81\pi}{7}$
- (E) $\frac{81\pi}{7}$

56. How many nonnegative integer triples sum to 38, if each coordinate is at most 19?

- (A) 140
- (B) 280
- (C) 70
- (D) 210
- (E) 350

57. Let $\mathbf{F} = (9y^5, z^5, x^5)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (1, 1, 3)/\sqrt{11}$.

- (A) $1 - \frac{145\sqrt{11}}{11}$
- (B) $\frac{145\sqrt{11}}{11}$
- (C) $-\frac{145\sqrt{11}}{11}$
- (D) $-\frac{145\sqrt{11}}{22}$
- (E) $-\frac{290\sqrt{11}}{11}$

Questions 58–60

58. Let $R = (\mathbb{Z}/22\mathbb{Z})/([11])$, where $([11])$ is the ideal generated by $[11]$. How many units does R have?

- (A) 10
- (B) 11
- (C) 12
- (D) 8
- (E) 9

59. For $x > 0$, $x^2y'' + (1 - 14x)y' + 45y = 0$, $y(1) = 0$, $y'(1) = 9$. Find $y(e)$.

- (A) $\frac{e^9}{14}$
- (B) $\frac{9(1 - e^4)e^5}{4}$
- (C) $-\frac{9(1 - e^4)e^5}{4}$
- (D) $-e^5 + e^9$
- (E) $e^5 + e^9$

60. A tree has 19 vertices of degree 3; every other vertex has degree 1. How many leaves does it have?

- (A) 7
- (B) 14
- (C) 28
- (D) 35
- (E) 21

Questions 61–63

61. Solve $y' + 9y = x^5e^{-9x}$, $y(0) = 0$, and find $y(1)$.

- (A) e^{-9}
- (B) $\frac{3}{2e^9}$
- (C) $\frac{e^9}{6}$
- (D) $\frac{1}{6e^9}$
- (E) $\frac{1}{5e^9}$

62. What is the largest integer k such that 3^k divides $47!$?

- (A) 22
- (B) 20
- (C) 23
- (D) 21
- (E) 19

63. Find the total outward flux of $\mathbf{F} = (9x^5y, y^5z, xz^5)$ through the boundary of $[0, 1]^3$.

- (A) $\frac{11}{2}$
- (B) $-\frac{11}{2}$
- (C) 11
- (D) $\frac{11}{4}$
- (E) $\frac{13}{2}$

Questions 64–66

64. A connected planar graph has 22 vertices and 39 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 13
- (B) 25
- (C) 19
- (D) 7
- (E) 31

65. If $y'' - 18y' + 81y = 5e^{9x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $1 + \frac{5e^9}{2}$
- (B) $\frac{5e^9}{4}$
- (C) $\frac{5e^9}{2}$
- (D) $-\frac{5e^9}{2}$
- (E) $5e^9$

66. In the ring $\mathbb{Z}/252\mathbb{Z}$, how many elements satisfy $x^2 = 1$?

- (A) 6
- (B) 8
- (C) 10
- (D) 7
- (E) 9

Answer Key

Practice Test 42 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	B	23	D	45	D
2	A	24	C	46	A
3	C	25	B	47	D
4	A	26	B	48	D
5	B	27	D	49	E
6	A	28	E	50	E
7	D	29	C	51	E
8	E	30	E	52	C
9	C	31	E	53	E
10	C	32	D	54	A
11	B	33	C	55	E
12	B	34	B	56	D
13	C	35	A	57	C
14	E	36	C	58	A
15	D	37	B	59	C
16	B	38	B	60	E
17	A	39	A	61	D
18	A	40	B	62	D
19	A	41	D	63	A
20	A	42	C	64	C
21	E	43	D	65	C
22	B	44	E	66	B

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