

GRE Mathematics Practice Test 43

Upper-Intermediate Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

Intermediate reinforcement, combining standard applications with selected more demanding arguments. This is cycle 8 of 19, position 3 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Let $F(x) = \int_0^{x^4} e^{9t} dt$. Find $F'(1)$.

- (A) e^{36}
- (B) $36e^9$
- (C) e^9
- (D) $4e^9$
- (E) $\frac{e^9}{4}$

2. An idempotent 45×45 real matrix P has rank 1. What is $\det(I + P)$?

- (A) 2
- (B) 1
- (C) 4
- (D) 3
- (E) 0

3. Evaluate $\iiint_{x^2+y^2+z^2 \leq 9} (x^2 + y^2 + z^2)^3 dV$.

- (A) $\frac{4\pi 9^{\frac{9}{2}}}{9}$
- (B) $9 \left(\frac{4\pi 9^{\frac{9}{2}}}{9} \right)$
- (C) $\frac{1}{3} \left(\frac{4\pi 9^{\frac{9}{2}}}{9} \right)$
- (D) $\frac{1}{7} \left(\frac{4\pi 9^{\frac{9}{2}}}{9} \right)$
- (E) $\frac{1}{2} \left(\frac{4\pi 9^{\frac{9}{2}}}{9} \right)$

Questions 4–6

4. A random variable has mean 0 and variance 15. What upper bound does the Chebyshev inequality give for $P(|X| \geq 30)$?

- (A) $\frac{59}{60}$
- (B) $\frac{61}{120}$
- (C) $\frac{1}{180}$
- (D) $\frac{1}{120}$
- (E) $\frac{1}{60}$

5. For the curve $x = t^4$, $y = t^5 + 9t$, find d^2y/dx^2 at $t = 1$.

- (A) $\frac{7}{2}$
- (B) $-\frac{11}{8}$
- (C) 2
- (D) $-\frac{11}{2}$
- (E) $-\frac{9}{4}$

6. For $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 39 \\ 9 \\ 48 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find x_2 .

- (A) $\frac{9}{2}$
- (B) $\frac{11}{2}$
- (C) 5
- (D) $\frac{7}{2}$
- (E) 4

Questions 7–9

7. For $x > 0$, $x^2y'' + (-21)xy' + 117y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $-\frac{e^9}{4} + 1 + \frac{e^{13}}{4}$
- (B) $\frac{(1 - e^4)e^9}{4}$
- (C) $-\frac{(1 - e^4)e^9}{2}$
- (D) $-\frac{(1 - e^4)e^9}{8}$
- (E) $-\frac{(1 - e^4)e^9}{4}$

8. A box is chosen uniformly from two boxes. Box A contains 20 red and 22 blue balls; box B contains 22 red and 20 blue balls. Two balls are drawn without replacement and both are red. What is the conditional probability that A was chosen?

- (A) $\frac{190}{1263}$
- (B) $\frac{231}{421}$
- (C) $\frac{95}{421}$
- (D) $\frac{611}{842}$
- (E) $\frac{190}{421}$

9. Evaluate $\int_0^\infty x^4 e^{-9x} dx$.

- (A) $\frac{1}{9} \left(\frac{4!}{9^5} \right)$
- (B) $59049 \left(\frac{4!}{9^5} \right)$
- (C) $\frac{1}{4} \left(\frac{4!}{9^5} \right)$
- (D) $\frac{4!}{9^5}$
- (E) $9 \left(\frac{4!}{9^5} \right)$

Questions 10–12

10. What is the coefficient of x^7 in $(x + 8)^6(1 - x)^3$?

- (A) -820
- (B) -821
- (C) -818
- (D) -819
- (E) -817

11. The solution of $y' = 9x^3y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{\sqrt{6}}{3}$
- (B) $\frac{\sqrt{3}}{3}$
- (C) $\frac{4}{9}$
- (D) $\frac{\sqrt{6}}{2}$
- (E) $\frac{\sqrt[4]{2}\sqrt{3}}{3}$

12. Let $X_1, \dots, X_{20}$ be independent uniform random variables on $[0, 1]$. What is $E[(\max_i X_i)^2]$?

- (A) $\frac{1}{11}$
- (B) $\frac{10}{11}$
- (C) $\frac{5}{11}$
- (D) $\frac{21}{22}$
- (E) $\frac{10}{33}$

Questions 13–15

13. Two solutions of $y'' + 9x^4y' + 4y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $\frac{2}{e^{\frac{9}{5}}}$
- (B) $\frac{1}{2e^{\frac{9}{5}}}$
- (C) $e^{-\frac{9}{5}}$
- (D) $e^{-\frac{9}{5}} + 1$
- (E) $-\frac{1}{e^{\frac{9}{5}}}$

14. A matrix A satisfies $(A - 40I)(A - 42I) = 0$, and its minimal polynomial has degree two. If $A^4 = uA + vI$, find u .

- (A) 275850
- (B) 275848
- (C) 275847
- (D) 275849
- (E) 275846

15. Find the coefficient of x^6 in $\cos(9x)/(1 - 3x^2)$.

- (A) $\frac{20439}{80}$
- (B) -183951
- (C) $-\frac{59049}{80}$
- (D) $-\frac{20439}{80}$
- (E) $-\frac{20519}{80}$

Questions 16–18

16. Independent trials have success probability $1/20$. Let T count trials up to and including the first success. Find $E[T]$.

- (A) 32
- (B) 8
- (C) 20
- (D) 26
- (E) 14

17. Let $\mathbf{F} = (9y^4, z^4, x^4)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (1, 1, 4)/\sqrt{18}$.

- (A) $-\frac{152\sqrt{2}}{3}$
- (B) $1 - \frac{76\sqrt{2}}{3}$
- (C) $\frac{76\sqrt{2}}{3}$
- (D) $-\frac{76\sqrt{2}}{3}$
- (E) $-\frac{38\sqrt{2}}{3}$

18. Let $V = \begin{bmatrix} 1 & 39 & 1521 \\ 1 & 41 & 1681 \\ 1 & 44 & 1936 \end{bmatrix}$. Find $\det(V^T V)$.

- (A) 899
- (B) 901
- (C) 900
- (D) 902
- (E) 898

Questions 19–21

19. Let C be the unit circle $x^2 + y^2 = 1$, $z = 4$, counterclockwise viewed from above. Evaluate $\int_C -9y(x^2 + y^2)^3 dx + (9x(x^2 + y^2)^3) dy + 4z dz$.

- (A) $1 + 18\pi$
- (B) -18π
- (C) 36π
- (D) 18π
- (E) 9π

20. A uniformly random subset of 20 objects is selected from a collection with 21 red and 22 blue objects. If X is the number of red objects selected, find $E[X]$.

- (A) $\frac{420}{43}$
- (B) $\frac{840}{43}$
- (C) $\frac{210}{43}$
- (D) $\frac{506}{43}$
- (E) $\frac{463}{43}$

21. What is the maximum of $x^4 y^9$ over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\left(\frac{4}{13}\right)^9 \left(\frac{9}{13}\right)^4$
- (B) $\frac{1}{2^{13}}$
- (C) $\left(\frac{4}{13}\right)^4 \left(\frac{9}{13}\right)^9$
- (D) $13 \left(\left(\frac{4}{13}\right)^4 \left(\frac{9}{13}\right)^9 \right)$
- (E) $\frac{36}{169}$

Questions 22–24

22. Let $u = (1, 2, \dots, 45)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $\det B$.

- (A) 1038
- (B) 1037
- (C) 1036
- (D) 1035
- (E) 1034

23. Let $f(x, y) = e^{9x+4y}$. Find $\partial^4 f / \partial x^3 \partial y$ at $(0, 0)$.

- (A) $4 \cdot 9^3$
- (B) $\frac{64}{729} (4 \cdot 9^3)$
- (C) $\frac{1}{4} (4 \cdot 9^3)$
- (D) $\frac{9}{4} (4 \cdot 9^3)$
- (E) $\frac{16}{81} (4 \cdot 9^3)$

24. Which subset of $\mathbb{R}$ is compact, with 15 fixed as displayed?

- (A) $15\mathbb{Z}$
- (B) $\{15/k : k \geq 1\}$
- (C) $(0, 15)$
- (D) $\mathbb{Q} \cap [0, 15]$
- (E) $\{0\} \cup \{15/k : k \geq 1\}$

Questions 25–27

25. Find the arc length of $r(t) = (9(t - t^3/3), 9t^2)$ for $0 \leq t \leq 1/4$.

- (A) $\frac{147}{32}$
- (B) $\frac{211}{64}$
- (C) $\frac{147}{64}$
- (D) $\frac{147}{128}$
- (E) $-\frac{147}{64}$

26. Let $D = \text{diag}(\underbrace{1, \dots, 1}_7, \underbrace{2, \dots, 2}_{38})$. Find the dimension of the space of real matrices X with $XD = DX$.

- (A) 1491
- (B) 1493
- (C) 1495
- (D) 1494
- (E) 1492

27. Evaluate $\lim_{x \rightarrow 0} \frac{e^{9x} - \sum_{j=0}^3 (9x)^j / j!}{x^4}$.

- (A) $\frac{1}{9} \left(\frac{9^4}{4!} \right)$
- (B) $\frac{9^4}{4!}$
- (C) $4 \left(\frac{9^4}{4!} \right)$
- (D) $24 \left(\frac{9^4}{4!} \right)$
- (E) $-\frac{9^4}{4!}$

Questions 28–30

28. For $a_k = (-1)^k + 15/k$, find $\limsup_{k \rightarrow \infty} a_k - \liminf_{k \rightarrow \infty} a_k$.

- (A) 0
- (B) 3
- (C) 4
- (D) 1
- (E) 2

29. Find the total outward flux of $\mathbf{F} = (9x^4y, y^4z, xz^4)$ through the boundary of $[0, 1]^3$.

- (A) $\frac{13}{2}$
- (B) $\frac{11}{4}$
- (C) 11
- (D) $\frac{11}{2}$
- (E) $-\frac{11}{2}$

30. When $x^3 + 1$ is divided by $(x - 40)(x - 41)$, the remainder is $ux + v$. Find u .

- (A) 4922
- (B) 4920
- (C) 4921
- (D) 4919
- (E) 4923

Questions 31–33

31. If $y'' - 18y' + 81y = 4e^{9x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) e^9
- (B) $1 + 2e^9$
- (C) $2e^9$
- (D) $4e^9$
- (E) $-2e^9$

32. For which real a is $f(x) = x^a$ in $L^{15}(0, 1)$?

- (A) $a \geq -1/15$
- (B) All real a
- (C) $a < 1/15$
- (D) $a > 0$
- (E) $a > -1/15$

33. For $x > 0$, $x^2y'' + (1 - 13x)y' + 36y = 0$, $y(1) = 0$, $y'(1) = 9$. Find $y(e)$.

- (A) $e^4 + e^9$
- (B) $\frac{9(1 - e^5)e^4}{5}$
- (C) $\frac{e^9}{13}$
- (D) $-\frac{9(1 - e^5)e^4}{5}$
- (E) $-e^4 + e^9$

Questions 34–36

34. Let $P = \text{diag}(\underbrace{1, \dots, 1}_3, \underbrace{0, \dots, 0}_{43})$. What is the dimension of the space of 46×46 real matrices X satisfying $PX = X$ and $XP = 0$?

- (A) 130
- (B) 129
- (C) 131
- (D) 128
- (E) 127

35. The region $0 \leq x \leq 1$, $0 \leq y \leq 9x^4$ is revolved about the line $x = -4$. Find its volume.

- (A) $-\frac{87\pi}{5}$
- (B) $\frac{87\pi}{10}$
- (C) $1 + \frac{87\pi}{5}$
- (D) $\frac{87\pi}{5}$
- (E) $\frac{174\pi}{5}$

36. A map $T : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $|T(x) - T(y)| \leq |x - y|/15$. If $T(x_*) = x_*$ and $|x_0 - x_*| \leq 1$, what is the smallest guaranteed upper bound for $|T^3(x_0) - x_*|$?

- (A) $\frac{3374}{3375}$
- (B) $\frac{1}{6750}$
- (C) $\frac{1688}{3375}$
- (D) $\frac{1}{3375}$
- (E) $\frac{1}{10125}$

Questions 37–39

37. For $f(x) = x^4 e^{9x}$, find $f^{(6)}(0)$.

- (A) $\frac{1}{30} \left(\frac{6! 9^2}{2} \right)$
- (B) $\frac{2}{9} \left(\frac{6! 9^2}{2} \right)$
- (C) $\frac{1}{360} \left(\frac{6! 9^2}{2} \right)$
- (D) $2 \left(\frac{6! 9^2}{2} \right)$
- (E) $\frac{6! 9^2}{2}$

38. A 3×3 matrix A has characteristic polynomial $(t - 39)(t + 1)(t - 2)$. What is $\text{tr}(A^3)$?

- (A) 59324
- (B) 59326
- (C) 59327
- (D) 59328
- (E) 59325

39. If $y' = 18xy$ and $y(0) = 1$, find the coefficient of x^8 in the Maclaurin series of y .

- (A) $\frac{2187}{16}$
- (B) $\frac{2187}{8}$
- (C) $\frac{2187}{4}$
- (D) $\frac{2195}{8}$
- (E) $-\frac{2187}{8}$

Questions 40–42

40. What is the coefficient of z^{-1} in the Laurent expansion of e^{10z}/z^3 about zero?

- (A) 66
- (B) 34
- (C) 82
- (D) 18
- (E) 50

41. Under $x = u + 9v$, $y = u - 4v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) 5
- (B) 169
- (C) 13
- (D) $\frac{1}{13}$
- (E) 36

42. A permutation consists of disjoint cycles of lengths 3 and 6, together with a disjoint transposition when the sum of these two lengths is odd. It fixes all other points. What is its order?

- (A) 7
- (B) 5
- (C) 4
- (D) 8
- (E) 6

Questions 43–45

43. If $y' = y(1 - y/4)$ and $y(0) = 2$, find $y(-\log 9)$.

- (A) $-\frac{2}{5}$
- (B) $\frac{1}{5}$
- (C) $\frac{4}{5}$
- (D) $\frac{7}{5}$
- (E) $\frac{2}{5}$

44. An entire function satisfies $|f(z)| \leq C(1 + |z|^{10})$ for some $C > 0$. What is the largest possible degree of f ?

- (A) 10
- (B) 13
- (C) 7
- (D) 4
- (E) 16

45. Evaluate $\int_0^1 x^9(1 - x)^3 dx$.

- (A) $13 \left(\frac{9! 3!}{13!} \right)$
- (B) $10 \left(\frac{9! 3!}{13!} \right)$
- (C) $\frac{1}{4} \left(\frac{9! 3!}{13!} \right)$
- (D) $\frac{143}{2} \left(\frac{9! 3!}{13!} \right)$
- (E) $\frac{9! 3!}{13!}$

Questions 46–48

46. In $R = \mathbb{F}_5[x]/(x^9)$, how many elements are units?

- (A) $5^{10} - 5^8$
- (B) $5^{11} - 5^{10}$
- (C) $5^9 - 5^0$
- (D) $5^{10} - 5^9$
- (E) $5^9 - 5^8$

47. Evaluate $\int_0^1 \int_0^x x^9 y^4 dy dx$.

- (A) $\frac{1}{70}$
- (B) $\frac{1}{90}$
- (C) $\frac{1}{75}$
- (D) $\frac{1}{15}$
- (E) $\frac{1}{50}$

48. How many edges does the complete bipartite graph $K_{15,17}$ have?

- (A) 85
- (B) 170
- (C) 255
- (D) 340
- (E) 425

Questions 49–51

49. Evaluate $\int_0^{\pi/2} \sin^9 t \cos^{19} t \, dt$.

- (A) $\frac{4! 9!}{2 \cdot 14!}$
- (B) $715 \left(\frac{4! 9!}{2 \cdot 14!} \right)$
- (C) $\frac{1}{2} \left(\frac{4! 9!}{2 \cdot 14!} \right)$
- (D) $2 \left(\frac{4! 9!}{2 \cdot 14!} \right)$
- (E) $14 \left(\frac{4! 9!}{2 \cdot 14!} \right)$

50. Integers x satisfy $x \equiv 1 \pmod{7}$ and $x \equiv 4 \pmod{9}$. Find the least nonnegative solution.

- (A) 22
- (B) 21
- (C) 24
- (D) 20
- (E) 23

51. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-9)^j}{j^1 (\log j)^m}$ converge absolutely at $x = 8$? Give the least such m , or 0 if none exists.

- (A) 3
- (B) 2
- (C) 4
- (D) 1
- (E) 0

Questions 52–54

52. How many functions from a set of 20 elements onto a set of 3 elements exist?

- (A) $3^{20} - 3(2^{20}) + 3$
- (B) 3^{20}
- (C) $3^{20} - 3(2^{20})$
- (D) $3^{20} - 2^{20}$
- (E) $3^{20} - 3(2^{20}) - 3$

53. Let C be $x^2/16 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-3y^3) dx + (\frac{x^3}{3}) dy$.

- (A) -25π
- (B) 25π
- (C) 50π
- (D) $1 + 25\pi$
- (E) $\frac{25\pi}{2}$

54. The symmetric group S_{45} acts on the two-element subsets of $\{1, \dots, 45\}$. What is the index of the setwise stabilizer of $\{1, 2\}$?

- (A) 992
- (B) 988
- (C) 989
- (D) 991
- (E) 990

Questions 55–57

55. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^9 y^3 dV$.

- (A) $6 \left(\frac{9! 3!}{15!} \right)$
- (B) $15 \left(\frac{9! 3!}{15!} \right)$
- (C) $\frac{1}{6} \left(\frac{9! 3!}{15!} \right)$
- (D) $\frac{9! 3!}{15!}$
- (E) $\frac{1}{40}$

56. How many nonnegative integer triples sum to 40, if each coordinate is at most 20?

- (A) 308
- (B) 77
- (C) 385
- (D) 231
- (E) 154

57. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{9t} \cos(4t)$, and $y = e^{9t} \sin(4t)$, find $d^2 f / dt^2$ at $t = 1$.

- (A) 8
- (B) 324
- (C) 18
- (D) 0
- (E) 194

Questions 58–60

58. In $\mathbb{Z}/1158\mathbb{Z}$, how many residues are not idempotent ($x^2 \neq x$)?

- (A) 1150
- (B) 1149
- (C) 1151
- (D) 1152
- (E) 1148

59. Let $F(x) = \int_0^x (x-t)^3 e^{9t} dt$. Find $F^{(4)}(0)$.

- (A) 6
- (B) 54
- (C) 0
- (D) 24
- (E) $\frac{1}{6}$

60. A tree has 24 vertices. What is the sum of its vertex degrees?

- (A) 61
- (B) 76
- (C) 46
- (D) 31
- (E) 16

Questions 61–63

61. Let $f(x) = \log(1 + 9x)$. Find $f^{(4)}(0)$.

- (A) $4 \left((-1)^3 3! 9^4 \right)$
- (B) $\frac{1}{9} \left((-1)^3 3! 9^4 \right)$
- (C) $\frac{1}{4} \left((-1)^3 3! 9^4 \right)$
- (D) $(-1)^3 3! 9^4$
- (E) $-(-1)^3 3! 9^4$

62. In the additive group $\mathbb{Z}/72\mathbb{Z}$, find the index of the subgroup generated by $[6]$.

- (A) 7
- (B) 6
- (C) 8
- (D) 5
- (E) 4

63. Evaluate $\int_0^1 x^8 (\log x)^3 dx$.

- (A) $9 \left(\frac{(-1)^3 3!}{9^4} \right)$
- (B) $-\frac{(-1)^3 3!}{9^4}$
- (C) $\frac{(-1)^3 3!}{9^4}$
- (D) $\frac{1}{4} \left(\frac{(-1)^3 3!}{9^4} \right)$
- (E) $\frac{243}{2} \left(\frac{(-1)^3 3!}{9^4} \right)$

Questions 64–66

64. A connected planar graph has 23 vertices and 41 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 20
- (B) 26
- (C) 32
- (D) 8
- (E) 14

65. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(9j)!}{(j!)^9} x^{4j}$?

- (A) $9^{\frac{9}{4}}$
- (B) $9^{-\frac{9}{4}}$
- (C) $\frac{1}{9}$
- (D) 9^{-9}
- (E) $9^{-\frac{4}{9}}$

66. How many invertible 3×3 matrices over $\mathbb{F}_{19}$ are there whose first column is $(1, 0, 0)^T$?

- (A) $\frac{19^3 19^3 19^3}{19^3 - 1}$
- (B) $\frac{(19^3 - 19^0)(19^3 - 19^1)(19^3 - 19^2)}{19^3 - 1}$
- (C) $\frac{(19^3 - 19^1)(19^3 - 19^2)(19^3 - 19^3)}{19^3 - 1}$
- (D) $\frac{(19^3 - 19^0)(19^3 - 19^1)}{19^3 - 1}$
- (E) $\frac{(19^3 - 19^0)(19^3 - 19^0)(19^3 - 19^0)}{19^3 - 1}$

Answer Key

Practice Test 43 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	D	23	A	45	E
2	A	24	E	46	E
3	A	25	C	47	C
4	E	26	B	48	C
5	B	27	B	49	A
6	A	28	E	50	A
7	E	29	D	51	B
8	E	30	C	52	A
9	D	31	C	53	B
10	D	32	E	54	E
11	A	33	D	55	D
12	B	34	B	56	D
13	C	35	D	57	D
14	B	36	D	58	A
15	D	37	E	59	A
16	C	38	B	60	C
17	D	39	B	61	D
18	C	40	E	62	B
19	D	41	C	63	C
20	A	42	E	64	A
21	C	43	E	65	B
22	C	44	A	66	B

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