

GRE Mathematics Practice Test 44

Bridge to Standard

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A bridge toward standard practice, with routine multi-step questions and a smaller number of challenging concepts. This is cycle 8 of 19, position 4 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. For $D = \{(x, y) : x^2 + y^2 \leq 9\}$, evaluate $\iint_D (x^2 + y^2)^2 dA$.
- (A) 243π
(B) 27π
(C) $\frac{729\pi}{5}$
(D) 729π
(E) 486π
2. Let $u = (1, 2, \dots, 46)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $\det B$.
- (A) 1082
(B) 1083
(C) 1084
(D) 1081
(E) 1080
3. The function $y(x)$ satisfies $y^3 + 9y = 2x$ and $y(0) = 0$. Find $y'(0)$.
- (A) 9
(B) 0
(C) $\frac{2}{9}$
(D) $\frac{1}{6}$
(E) $-\frac{2}{9}$

Questions 4–6

4. A random variable has mean 0 and variance 16. What upper bound does the Chebyshev inequality give for $P(|X| \geq 32)$?

- (A) $\frac{1}{64}$
- (B) $\frac{65}{128}$
- (C) $\frac{1}{128}$
- (D) $\frac{63}{64}$
- (E) $\frac{1}{192}$

5. Let $f(x, y) = x^2 + 18xy + (84)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 3
- (B) 336
- (C) 6
- (D) 84
- (E) 12

6. Let D be a diagonal 46×46 real matrix with distinct diagonal entries. What is the dimension of the space of matrices X satisfying $XD = DX$?

- (A) 44
- (B) 45
- (C) 48
- (D) 47
- (E) 46

Questions 7–9

7. Solve $y' + 9y = x^3e^{-9x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{e^9}{4}$
- (B) $\frac{1}{4e^9}$
- (C) e^{-9}
- (D) $\frac{1}{3e^9}$
- (E) $\frac{9}{4e^9}$

8. A box is chosen uniformly from two boxes. Box A contains 21 red and 23 blue balls; box B contains 23 red and 21 blue balls. Two balls are drawn without replacement and both are red. What is the conditional probability that A was chosen?

- (A) $\frac{673}{926}$
- (B) $\frac{70}{463}$
- (C) $\frac{105}{463}$
- (D) $\frac{253}{463}$
- (E) $\frac{210}{463}$

9. If $y'' + 9y = 0$, $y(0) = 9$, and $y'(0) = 5$, find $y(\pi/6)$.

- (A) $\frac{10}{3}$
- (B) $\frac{8}{3}$
- (C) $\frac{5}{6}$
- (D) $\frac{5}{3}$
- (E) $-\frac{5}{3}$

Questions 10–12

10. What is the remainder when $x^2 + 1$ is divided by $x - 41$?
- (A) 1684
(B) 1681
(C) 1680
(D) 1683
(E) 1682
11. For $\mathbf{F} = (9x^3, y^3, z^3)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.
- (A) 42
(B) 43
(C) 84
(D) 21
(E) -42
12. Let $X_1, \dots, X_{21}$ be independent uniform random variables on $[0, 1]$. What is $E[\max_i X_i]$?
- (A) $\frac{7}{22}$
(B) $\frac{43}{44}$
(C) $\frac{1}{22}$
(D) $\frac{21}{44}$
(E) $\frac{21}{22}$

Questions 13–15

13. Evaluate $\int_0^1 \int_x^1 9y^3 \, dy \, dx$.

- (A) $\frac{9}{4}$
- (B) $\frac{9}{5}$
- (C) $\frac{9}{2}$
- (D) $\frac{9}{20}$
- (E) 3

14. Let $P = \text{diag}(\underbrace{1, \dots, 1}_4, \underbrace{0, \dots, 0}_{43})$. What is the dimension of the space of 47×47 real matrices X satisfying $PX = X$?

- (A) 189
- (B) 187
- (C) 190
- (D) 188
- (E) 186

15. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-9)^j}{j^{\frac{2}{3}} (\log j)^m}$ converge absolutely at $x = 8$? Give the least such m , or 0 if none exists.

- (A) 2
- (B) 4
- (C) 1
- (D) 0
- (E) 3

Questions 16–18

16. Independent trials have success probability $1/21$. Let T count trials up to and including the first success. Find $E[T]$.

- (A) 35
- (B) 14
- (C) 21
- (D) 7
- (E) 28

17. If $y' + 9y = 3e^{-9x}$ and $y(0) = 5$, find $y(1)$.

- (A) $\frac{8}{e^9}$
- (B) $\frac{4}{e^9}$
- (C) $\frac{8}{e^9} + 1$
- (D) $\frac{16}{e^9}$
- (E) $-\frac{8}{e^9}$

18. A 3×3 matrix A has characteristic polynomial $(t - 40)(t + 1)(t - 2)$. What is $\text{tr}(A^2)$?

- (A) 1603
- (B) 1606
- (C) 1607
- (D) 1604
- (E) 1605

Questions 19–21

19. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^9 y^2 dV$.

- (A) $14 \left(\frac{9! 2!}{14!} \right)$
- (B) $\frac{1}{30}$
- (C) $6 \left(\frac{9! 2!}{14!} \right)$
- (D) $\frac{9! 2!}{14!}$
- (E) $\frac{1}{6} \left(\frac{9! 2!}{14!} \right)$

20. A uniformly random subset of 21 objects is selected from a collection with 22 red and 23 blue objects. If X is the number of red objects selected, find $E[X]$.

- (A) $\frac{184}{15}$
- (B) $\frac{308}{15}$
- (C) $\frac{77}{15}$
- (D) $\frac{169}{15}$
- (E) $\frac{154}{15}$

21. For $x > 0$, $xy' = 9y$ and $y(1) = 1$. Find $y(e^3)$.

- (A) $2e^{27}$
- (B) $-e^{27}$
- (C) $1 + e^{27}$
- (D) e^{27}
- (E) $\frac{e^{27}}{2}$

Questions 22–24

22. Let N be the 46×46 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\text{rank } N^6$.

- (A) 40
- (B) 42
- (C) 41
- (D) 38
- (E) 39

23. Evaluate $\int_0^\infty x^3 e^{-9x} dx$.

- (A) $\frac{3!}{9^4}$
- (B) $\frac{1}{9} \left(\frac{3!}{9^4} \right)$
- (C) $\frac{1}{3} \left(\frac{3!}{9^4} \right)$
- (D) $9 \left(\frac{3!}{9^4} \right)$
- (E) $6561 \left(\frac{3!}{9^4} \right)$

24. Which subset of $\mathbb{R}$ is compact, with 16 fixed as displayed?

- (A) $\{16/k : k \geq 1\}$
- (B) $16\mathbb{Z}$
- (C) $(0, 16)$
- (D) $\{0\} \cup \{16/k : k \geq 1\}$
- (E) $\mathbb{Q} \cap [0, 16]$

Questions 25–27

25. Evaluate $\int_0^{\log 2} e^x(e^x + 9)^3 dx$.

- (A) $4 \left(\frac{11^4 - 10^4}{4} \right)$
- (B) $-\frac{11^4 - 10^4}{4}$
- (C) $\frac{11^4 - 10^4}{4}$
- (D) $11^3 - 10^3$
- (E) $\frac{11^4}{4}$

26. Let A be the 46×46 matrix whose diagonal entries are 43 and whose off-diagonal entries are 1. What is its smallest eigenvalue?

- (A) 40
- (B) 42
- (C) 41
- (D) 43
- (E) 44

27. Find the coefficient of x^3 in the Taylor series of $(1 - 9x)^{-2}$ at zero.

- (A) $\frac{1}{9} (4 \cdot 9^3)$
- (B) $\frac{1}{4} (4 \cdot 9^3)$
- (C) $\frac{3}{4} (4 \cdot 9^3)$
- (D) $-4 \cdot 9^3$
- (E) $4 \cdot 9^3$

Questions 28–30

28. For $a_k = (-1)^k + 16/k$, find $\limsup_{k \rightarrow \infty} a_k - \liminf_{k \rightarrow \infty} a_k$.

- (A) 1
- (B) 3
- (C) 4
- (D) 2
- (E) 0

29. Find the upward flux of $\mathbf{F} = (0, 0, z)$ through $z = 9x^2 + 3y^2$, $0 \leq x, y \leq 1$.

- (A) -4
- (B) 4
- (C) 8
- (D) 5
- (E) 2

30. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 280$ and $x + y = 47$. Find xy .

- (A) 282
- (B) 279
- (C) 278
- (D) 281
- (E) 280

Questions 31–33

31. Let C be $x^2/9 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-y) dx + (9x) dy$.
- (A) -30π
(B) $1 + 30\pi$
(C) 30π
(D) 15π
(E) 60π
32. For which real a is $f(x) = x^a$ in $L^{16}(0, 1)$?
- (A) $a > 0$
(B) $a > -1/16$
(C) All real a
(D) $a \geq -1/16$
(E) $a < 1/16$
33. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 9x$ and has density $\rho(x, y) = x$. Find its centroid coordinate $\bar{x}$.
- (A) $\frac{3}{8}$
(B) $\frac{3}{2}$
(C) $-\frac{3}{4}$
(D) $\frac{7}{4}$
(E) $\frac{3}{4}$

Questions 34–36

34. A sequence satisfies $u_0 = 40$ and $u_{k+1} = 1u_k + 7$ for all $k \geq 0$. Find u_5 .

- (A) 74
- (B) 75
- (C) 73
- (D) 76
- (E) 77

35. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(9+3j)(12+3j)}$.

- (A) $\frac{1}{9}$
- (B) $\frac{2}{27}$
- (C) $\frac{1}{27}$
- (D) $\frac{1}{12}$
- (E) $\frac{1}{3}$

36. A map $T : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $|T(x) - T(y)| \leq |x - y|/16$. If $T(x_*) = x_*$ and $|x_0 - x_*| \leq 1$, what is the smallest guaranteed upper bound for $|T^3(x_0) - x_*|$?

- (A) $\frac{4097}{8192}$
- (B) $\frac{1}{12288}$
- (C) $\frac{1}{4096}$
- (D) $\frac{4095}{4096}$
- (E) $\frac{1}{8192}$

Questions 37–39

37. If $y' = 9y$, $y(0) = 1$, what is the coefficient of x^3 in the Maclaurin series of y ?

- (A) $-\frac{243}{2}$
- (B) $\frac{243}{4}$
- (C) $\frac{243}{2}$
- (D) $\frac{245}{2}$
- (E) 243

38. A linear map $T : \mathbb{R}^{47} \rightarrow \mathbb{R}^{49}$ has rank 3. What is its nullity?

- (A) 44
- (B) 45
- (C) 43
- (D) 42
- (E) 46

39. Under $x = u + 9v$, $y = u - 3v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) 27
- (B) $\frac{1}{12}$
- (C) 6
- (D) 12
- (E) 144

Questions 40–42

40. What is the coefficient of z^{-1} in the Laurent expansion of e^{11z}/z^3 about zero?

- (A) $\frac{123}{2}$
- (B) $\frac{121}{2}$
- (C) 121
- (D) $\frac{121}{4}$
- (E) $\frac{125}{2}$

41. Evaluate $\iiint_{x^2+y^2+z^2 \leq 9} (x^2 + y^2 + z^2)^2 dV$.

- (A) $\frac{1}{2} \left(\frac{4\pi 9^{\frac{7}{2}}}{7} \right)$
- (B) $\frac{7}{45} \left(\frac{4\pi 9^{\frac{7}{2}}}{7} \right)$
- (C) $7 \left(\frac{4\pi 9^{\frac{7}{2}}}{7} \right)$
- (D) $\frac{4\pi 9^{\frac{7}{2}}}{7}$
- (E) $\frac{1}{3} \left(\frac{4\pi 9^{\frac{7}{2}}}{7} \right)$

42. What is the dimension of the real vector space of 46×46 symmetric matrices?

- (A) 1079
- (B) 1082
- (C) 1080
- (D) 1081
- (E) 1083

Questions 43–45

43. Evaluate $\int_C (3x^2 + 9y) dx + (9x + 3y^2) dy$, where C is the line segment from $(0, 0)$ to $(5, 1)$.

- (A) -171
- (B) 171
- (C) 342
- (D) 172
- (E) $\frac{171}{2}$

44. An entire function satisfies $|f(z)| \leq C(1 + |z|^{11})$ for some $C > 0$. What is the largest possible degree of f ?

- (A) 11
- (B) 17
- (C) 14
- (D) 8
- (E) 5

45. For the curve $x = t^3$, $y = t^4 + 9t$, find d^2y/dx^2 at $t = 1$.

- (A) $-\frac{14}{3}$
- (B) $\frac{22}{9}$
- (C) $-\frac{14}{9}$
- (D) $\frac{13}{3}$
- (E) -3

Questions 46–48

46. How many permutations in S_{46} consist of one 4-cycle and 42 fixed points?
- (A) 979111
 - (B) 979109
 - (C) 979112
 - (D) 979110
 - (E) 979108
47. For $f(x, y) = x^2 + 9xy + 3y^2$, what is its maximum directional derivative at $(1, -1)$?
- (A) -7
 - (B) 3
 - (C) $\sqrt{58}$
 - (D) 10
 - (E) 58
48. How many edges does the complete bipartite graph $K_{16,18}$ have?
- (A) 288
 - (B) 480
 - (C) 384
 - (D) 96
 - (E) 192

Questions 49–51

49. Find the directional derivative of $f(x, y, z) = x^2 + 9y^2 + 3z^2$ at $(1, 1, 1)$ in the direction $(1, 5, 0)$.

- (A) $1 + \frac{46\sqrt{26}}{13}$
- (B) $\frac{46\sqrt{26}}{13}$
- (C) $\frac{23\sqrt{26}}{13}$
- (D) $\frac{92\sqrt{26}}{13}$
- (E) $-\frac{46\sqrt{26}}{13}$

50. How many subgroups does the cyclic group of order 528 have?

- (A) 22
- (B) 20
- (C) 21
- (D) 18
- (E) 19

51. For $f(x) = x^3e^{9x}$, find $f^{(5)}(0)$.

- (A) $\frac{1}{60} \left(\frac{5! 9^2}{2} \right)$
- (B) $\frac{2}{9} \left(\frac{5! 9^2}{2} \right)$
- (C) $\frac{1}{20} \left(\frac{5! 9^2}{2} \right)$
- (D) $2 \left(\frac{5! 9^2}{2} \right)$
- (E) $\frac{5! 9^2}{2}$

Questions 52–54

52. How many functions from a set of 21 elements onto a set of 3 elements exist?

- (A) $3^{21} - 3(2^{21}) + 3$
- (B) $3^{21} - 3(2^{21}) - 3$
- (C) $3^{21} - 2^{21}$
- (D) $3^{21} - 3(2^{21})$
- (E) 3^{21}

53. The solution of $y' = 9x^2y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\sqrt[3]{3}$
- (B) $\frac{1}{3}$
- (C) $\frac{3^{\frac{2}{3}}}{3}$
- (D) $\frac{6^{\frac{2}{3}}}{6}$
- (E) $\frac{\sqrt[3]{3}}{3}$

54. How many residues x modulo 54 satisfy $5x \equiv 15 \pmod{54}$?

- (A) 2
- (B) 1
- (C) 3
- (D) 0
- (E) 4

Questions 55–57

55. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{9t} \cos(3t)$, and $y = e^{9t} \sin(3t)$, find d^2f/dt^2 at $t = 1$.

- (A) 180
- (B) 6
- (C) 18
- (D) 324
- (E) 0

56. How many nonnegative integer triples (a, b, c) satisfy $a + b + c = 21$?

- (A) 85
- (B) 421
- (C) 253
- (D) 337
- (E) 169

57. Evaluate $\lim_{x \rightarrow 0} (1 + 9x)^{1/(2x)}$.

- (A) $e^{\frac{9}{2}}$
- (B) 9
- (C) e^{81}
- (D) e^{10}
- (E) $e^{\frac{2}{9}}$

Questions 58–60

58. The field K has 3^9 elements. Find $[K : \mathbb{F}_3]$.

- (A) 7
- (B) 10
- (C) 9
- (D) 8
- (E) 11

59. Find the curvature at $t = 0$ of $r(t) = (3t, 9t^2)$.

- (A) 3
- (B) 1
- (C) -2
- (D) 2
- (E) 4

60. A tree has 25 vertices. What is the sum of its vertex degrees?

- (A) 32
- (B) 64
- (C) 16
- (D) 48
- (E) 80

Questions 61–63

61. Evaluate $\int_0^{\pi/2} \sin^7 t \cos^{19} t \, dt$.

- (A) $\frac{1}{2} \left(\frac{3!9!}{2 \cdot 13!} \right)$
- (B) $13 \left(\frac{3!9!}{2 \cdot 13!} \right)$
- (C) $2 \left(\frac{3!9!}{2 \cdot 13!} \right)$
- (D) $220 \left(\frac{3!9!}{2 \cdot 13!} \right)$
- (E) $\frac{3!9!}{2 \cdot 13!}$

62. Let $G = \mathbb{Z}/270\mathbb{Z} \times \mathbb{Z}/184\mathbb{Z}$. Find the order of $(1, 1)$.

- (A) 24839
- (B) 24841
- (C) 24842
- (D) 24840
- (E) 24838

63. The region $0 \leq x \leq 1$, $0 \leq y \leq 9x^3$ is revolved about the x -axis. Find its volume.

- (A) $-\frac{81\pi}{7}$
- (B) $1 + \frac{81\pi}{7}$
- (C) $\frac{81\pi}{7}$
- (D) $\frac{162\pi}{7}$
- (E) $\frac{81\pi}{14}$

Questions 64–66

64. A connected planar graph has 24 vertices and 43 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 28
- (B) 21
- (C) 14
- (D) 7
- (E) 35

65. Solve $y'' - 21y' + 108y = 0$ with $y(0) = 1$, $y'(0) = 0$. Find $y(1)$.

- (A) $\frac{(4 - 3e^3)e^9}{2}$
- (B) $(4 - 3e^3)e^9$
- (C) $(4 - 3e^3)e^9 + 1$
- (D) $(-4 + 3e^3)e^9$
- (E) $(8 - 6e^3)e^9$

66. How many group homomorphisms $\mathbb{Z}/270\mathbb{Z} \rightarrow \mathbb{Z}/184\mathbb{Z}$ are there?

- (A) 2
- (B) 3
- (C) 4
- (D) 0
- (E) 1

Answer Key

Practice Test 44 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	A	23	A	45	C
2	A	24	D	46	D
3	C	25	C	47	C
4	A	26	B	48	A
5	E	27	E	49	B
6	E	28	D	50	B
7	B	29	B	51	E
8	E	30	E	52	A
9	D	31	C	53	C
10	E	32	B	54	B
11	A	33	E	55	E
12	E	34	B	56	C
13	B	35	C	57	A
14	D	36	C	58	C
15	D	37	C	59	D
16	C	38	A	60	D
17	A	39	D	61	E
18	E	40	B	62	D
19	D	41	D	63	C
20	E	42	D	64	B
21	D	43	B	65	B
22	A	44	A	66	A

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