

GRE Mathematics Practice Test 45

Standard-Level Target

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A mixed standard-level target, intended to approximate public GRE Mathematics practice in scope and overall demand. This is cycle 8 of 19, position 5 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

zarimion.com/gre

100 practice tests, complete answer keys, and future updates.

Please retain the source attribution when sharing this material.



This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. For $D = \{(x, y) : x^2 + y^2 \leq 9\}$, evaluate $\iint_D (x^2 + y^2)^1 dA$.
- (A) $\frac{9\pi}{2}$
(B) 81π
(C) $\frac{81\pi}{2}$
(D) $\frac{81\pi}{4}$
(E) 27π
2. Let N be the 47×47 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\text{rank } N^6$.
- (A) 41
(B) 42
(C) 39
(D) 40
(E) 43
3. Find the curvature at $t = 0$ of $r(t) = (2t, 9t^2)$.
- (A) $\frac{9}{4}$
(B) $\frac{9}{2}$
(C) $\frac{11}{2}$
(D) $-\frac{9}{2}$
(E) 9

Questions 4–6

4. A random variable has mean 0 and variance 17. What upper bound does the Chebyshev inequality give for $P(|X| \geq 34)$?

- (A) $\frac{67}{68}$
- (B) $\frac{69}{136}$
- (C) $\frac{1}{136}$
- (D) $\frac{1}{68}$
- (E) $\frac{1}{204}$

5. Evaluate $\int_0^1 x^8(\log x)^1 dx$.

- (A) $9 \left(\frac{(-1)^1 1!}{9^2} \right)$
- (B) $\frac{(-1)^1 1!}{9^2}$
- (C) $-\frac{(-1)^1 1!}{9^2}$
- (D) $2 \left(\frac{(-1)^1 1!}{9^2} \right)$
- (E) $\frac{1}{2} \left(\frac{(-1)^1 1!}{9^2} \right)$

6. Let A be the 47×47 matrix whose diagonal entries are 44 and whose off-diagonal entries are 1. What is its smallest eigenvalue?

- (A) 43
- (B) 44
- (C) 41
- (D) 42
- (E) 45

Questions 7–9

7. Evaluate $\int_0^{\log 2} e^x(e^x + 9)^2 dx$.

- (A) $\frac{11^3}{3}$
- (B) $11^2 - 10^2$
- (C) $\frac{11^3 - 10^3}{3}$
- (D) $3 \left(\frac{11^3 - 10^3}{3} \right)$
- (E) $-\frac{11^3 - 10^3}{3}$

8. A box is chosen uniformly from two boxes. Box A contains 22 red and 24 blue balls; box B contains 24 red and 22 blue balls. One ball is drawn and is red. What is the conditional probability that A was chosen?

- (A) $\frac{11}{23}$
- (B) $\frac{17}{23}$
- (C) $\frac{12}{23}$
- (D) $\frac{11}{69}$
- (E) $\frac{11}{46}$

9. Evaluate $\int_C (2x + 9y) dx + (9x + 2y) dy$, where C is the line segment from $(0, 0)$ to $(6, 1)$.

- (A) 182
- (B) -91
- (C) 92
- (D) 91
- (E) $\frac{91}{2}$

Questions 10–12

10. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 205$ and $x + y = 46$. Find xy .

- (A) 206
- (B) 203
- (C) 205
- (D) 207
- (E) 204

11. Let $F(x) = \int_0^x (x - t)^1 e^{9t} dt$. Find $F^{(2)}(0)$.

- (A) 1
- (B) $\frac{1}{2}$
- (C) 2
- (D) 9
- (E) 0

12. Let $X_1, \dots, X_{22}$ be independent uniform random variables on $[0, 1]$. What is $E[\max_i X_i]$?

- (A) $\frac{11}{23}$
- (B) $\frac{23}{22}$
- (C) $\frac{23}{45}$
- (D) $\frac{46}{22}$
- (E) $\frac{1}{23}$

Questions 13–15

13. The function $y(x)$ satisfies $y^3 + 9y = 1x$ and $y(0) = 0$. Find $y'(0)$.

- (A) $\frac{1}{9}$
- (B) 9
- (C) $-\frac{1}{9}$
- (D) 0
- (E) $\frac{1}{12}$

14. A sequence satisfies $u_0 = 41$ and $u_{k+1} = 1u_k + 5$ for all $k \geq 0$. Find u_4 .

- (A) 60
- (B) 62
- (C) 59
- (D) 61
- (E) 63

15. Find the directional derivative of $f(x, y, z) = x^2 + 9y^2 + 2z^2$ at $(1, 1, 1)$ in the direction $(1, 6, 0)$.

- (A) $-\frac{110\sqrt{37}}{37}$
- (B) $\frac{55\sqrt{37}}{37}$
- (C) $\frac{110\sqrt{37}}{37}$
- (D) $\frac{220\sqrt{37}}{37}$
- (E) $1 + \frac{110\sqrt{37}}{37}$

Questions 16–18

16. Independent trials have success probability $1/22$. Let T count trials up to and including the first success. Find $E[T]$.

- (A) 22
- (B) 36
- (C) 15
- (D) 29
- (E) 8

17. For $x > 0$, $xy' = 9y$ and $y(1) = 1$. Find $y(e^2)$.

- (A) $1 + e^{18}$
- (B) $\frac{e^{18}}{2}$
- (C) e^{18}
- (D) $-e^{18}$
- (E) $2e^{18}$

18. A linear map $T : \mathbb{R}^{48} \rightarrow \mathbb{R}^{50}$ has rank 4. What is its nullity?

- (A) 45
- (B) 43
- (C) 42
- (D) 46
- (E) 44

Questions 19–21

19. The solution of $y' = 9x^2y^2$ has $y(0) = 1/10$. Find $y(1)$.

- (A) $\frac{1}{14}$
- (B) $\frac{1}{7}$
- (C) $\frac{8}{7}$
- (D) $-\frac{1}{7}$
- (E) $\frac{2}{7}$

20. A uniformly random subset of 22 objects is selected from a collection with 23 red and 24 blue objects. If X is the number of red objects selected, find $E[X]$.

- (A) $\frac{553}{47}$
- (B) $\frac{253}{47}$
- (C) $\frac{506}{47}$
- (D) $\frac{600}{47}$
- (E) $\frac{1012}{47}$

21. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^9 y^1 dV$.

- (A) $\frac{9! 1!}{13!}$
- (B) $6 \left(\frac{9! 1!}{13!} \right)$
- (C) $\frac{1}{20}$
- (D) $13 \left(\frac{9! 1!}{13!} \right)$
- (E) $\frac{1}{6} \left(\frac{9! 1!}{13!} \right)$

Questions 22–24

22. What is the dimension of the real vector space of 47×47 symmetric matrices?

- (A) 1128
- (B) 1129
- (C) 1130
- (D) 1127
- (E) 1126

23. For $f(x) = x^2 e^{9x}$, find $f^{(4)}(0)$.

- (A) $\frac{2}{9} \left(\frac{4! 9^2}{2} \right)$
- (B) $\frac{1}{2} \left(\frac{4! 9^2}{2} \right)$
- (C) $\frac{4! 9^2}{2}$
- (D) $2 \left(\frac{4! 9^2}{2} \right)$
- (E) $\frac{1}{12} \left(\frac{4! 9^2}{2} \right)$

24. Which subset of $\mathbb{R}$ is compact, with 17 fixed as displayed?

- (A) $\{0\} \cup \{17/k : k \geq 1\}$
- (B) $(0, 17)$
- (C) $17\mathbb{Z}$
- (D) $\{17/k : k \geq 1\}$
- (E) $\mathbb{Q} \cap [0, 17]$

Questions 25–27

25. Two solutions of $y'' + 9x^1y' + 2y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $e^{-\frac{9}{2}}$
- (B) $\frac{1}{2e^{\frac{9}{2}}}$
- (C) $\frac{\frac{9}{2}}{e^2}$
- (D) $-\frac{1}{e^{\frac{9}{2}}}$
- (E) $e^{-\frac{9}{2}} + 1$

26. Let $A = \begin{bmatrix} 1 & 41 & 0 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 2)$ entry of A^4 ?

- (A) 162
- (B) 165
- (C) 163
- (D) 166
- (E) 164

27. For $\mathbf{F} = (9x^2, y^2, z^2)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) 12
- (B) -24
- (C) 25
- (D) 48
- (E) 24

Questions 28–30

28. For $a_k = (-1)^k + 17/k$, find $\limsup_{k \rightarrow \infty} a_k - \liminf_{k \rightarrow \infty} a_k$.

- (A) 4
- (B) 3
- (C) 1
- (D) 2
- (E) 0

29. If $y'' + 4y = 0$, $y(0) = 9$, and $y'(0) = 6$, find $y(\pi/4)$.

- (A) 6
- (B) 4
- (C) -3
- (D) 3
- (E) $\frac{3}{2}$

30. An invertible 47×47 matrix A has determinant 2. Find $\det(A^T A)$.

- (A) 5
- (B) 2
- (C) 4
- (D) 3
- (E) 6

Questions 31–33

31. Evaluate $\int_0^1 x^9(1-x)^1 dx$.

- (A) $\frac{1}{2} \left(\frac{9! 1!}{11!} \right)$
- (B) $\frac{9! 1!}{11!}$
- (C) $11 \left(\frac{9! 1!}{11!} \right)$
- (D) $\frac{11}{2} \left(\frac{9! 1!}{11!} \right)$
- (E) $10 \left(\frac{9! 1!}{11!} \right)$

32. For which real a is $f(x) = x^a$ in $L^{17}(0, 1)$?

- (A) $a > -1/17$
- (B) $a < 1/17$
- (C) All real a
- (D) $a > 0$
- (E) $a \geq -1/17$

33. Evaluate $\lim_{x \rightarrow 0} (1 + 9x)^{1/(1x)}$.

- (A) e^{10}
- (B) $e^{\frac{1}{9}}$
- (C) 9
- (D) e^{81}
- (E) e^9

Questions 34–36

34. On the space P_{48} of real polynomials of degree at most 48, define $T(p) = p'$. What is the rank of T ?

- (A) 47
- (B) 49
- (C) 50
- (D) 48
- (E) 46

35. Solve $y'' - 20y' + 99y = 0$ with $y(0) = 1$, $y'(0) = 0$. Find $y(1)$.

- (A) $(11 - 9e^2)e^9$
- (B) $\frac{(11 - 9e^2)e^9}{2}$
- (C) $\frac{(-11 + 9e^2)e^9}{2}$
- (D) $\frac{(11 - 9e^2)e^9}{4}$
- (E) $\frac{(11 - 9e^2)e^9}{2} + 1$

36. A map $T : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $|T(x) - T(y)| \leq |x - y|/17$. If $T(x_*) = x_*$ and $|x_0 - x_*| \leq 1$, what is the smallest guaranteed upper bound for $|T^3(x_0) - x_*|$?

- (A) $\frac{1}{9826}$
- (B) $\frac{1}{4913}$
- (C) $\frac{4913}{4912}$
- (D) $\frac{4913}{2457}$
- (E) $\frac{1}{14739}$

Questions 37–39

37. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 9x$ and has density $\rho(x, y) = 1$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{1}{3}$
- (B) $\frac{2}{3}$
- (C) $-\frac{2}{3}$
- (D) $\frac{5}{3}$
- (E) $\frac{4}{3}$

38. Subspaces U, V of $\mathbb{R}^{49}$ have dimensions 3 and 4. What is the largest possible value of $\dim(U \cap V)$?

- (A) 5
- (B) 4
- (C) 3
- (D) 1
- (E) 2

39. Let $f(x, y) = x^2 + 18xy + (83)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 332
- (B) 2
- (C) 4
- (D) 8
- (E) 83

Questions 40–42

40. What is the coefficient of z^{-1} in the Laurent expansion of e^{12z}/z^3 about zero?

- (A) 120
- (B) 96
- (C) 48
- (D) 24
- (E) 72

41. Evaluate $\iiint_{x^2+y^2+z^2 \leq 9} (x^2 + y^2 + z^2)^1 dV$.

- (A) $\frac{5}{27} \left(\frac{4\pi 9^{\frac{5}{2}}}{5} \right)$
- (B) $\frac{1}{3} \left(\frac{4\pi 9^{\frac{5}{2}}}{5} \right)$
- (C) $\frac{1}{2} \left(\frac{4\pi 9^{\frac{5}{2}}}{5} \right)$
- (D) $5 \left(\frac{4\pi 9^{\frac{5}{2}}}{5} \right)$
- (E) $\frac{4\pi 9^{\frac{5}{2}}}{5}$

42. Let V consist of real polynomials of degree at most 6 that vanish at 41, 42, and 44. Find $\dim V$.

- (A) 2
- (B) 6
- (C) 5
- (D) 4
- (E) 3

Questions 43–45

43. Let C be $x^2/4 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-y) dx + (9x) dy$.

- (A) 40π
- (B) $1 + 20\pi$
- (C) 10π
- (D) 20π
- (E) -20π

44. An entire function satisfies $|f(z)| \leq 12$ for all complex z . What is $f'(1)$?

- (A) 2
- (B) 0
- (C) 1
- (D) 4
- (E) 3

45. What is the maximum of x^2y^9 over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\frac{18}{121}$
- (B) $\left(\frac{2}{11}\right)^2 \left(\frac{9}{11}\right)^9$
- (C) $11 \left(\left(\frac{2}{11}\right)^2 \left(\frac{9}{11}\right)^9 \right)$
- (D) $\frac{1}{2^{11}}$
- (E) $\left(\frac{2}{11}\right)^9 \left(\frac{9}{11}\right)^2$

Questions 46–48

46. The roots r, s of $x^2 - 46x + 205 = 0$ are counted with multiplicity. Find $r^2 + s^2$.

- (A) 1707
- (B) 1705
- (C) 1704
- (D) 1708
- (E) 1706

47. Find the coefficient of x^4 in $\cos(9x)/(1 - 1x^2)$.

- (A) $\frac{2187}{8}$
- (B) $-\frac{1871}{8}$
- (C) $\frac{1871}{8}$
- (D) 5613
- (E) $\frac{1863}{8}$

48. How many edges does the complete bipartite graph $K_{17,19}$ have?

- (A) 537
- (B) 109
- (C) 430
- (D) 216
- (E) 323

Questions 49–51

49. For the curve $x = t^2$, $y = t^3 + 9t$, find d^2y/dx^2 at $t = 1$.

- (A) 6
- (B) $-\frac{9}{2}$
- (C) 3
- (D) $-\frac{3}{2}$
- (E) -3

50. How many monic irreducible polynomials of degree 2 are there over $\mathbb{F}_{19}$?

- (A) 173
- (B) 172
- (C) 171
- (D) 170
- (E) 169

51. Let $F(x) = \int_0^{x^2} e^{9t} dt$. Find $F'(1)$.

- (A) $\frac{e^9}{2}$
- (B) $18e^9$
- (C) e^9
- (D) e^{18}
- (E) $2e^9$

Questions 52–54

52. How many functions from a set of 22 elements onto a set of 2 elements exist?

- (A) $2^{22} - 3$
- (B) $2^{22} - 2$
- (C) 2^{22}
- (D) $2^{22} - 1$
- (E) 2^{21}

53. The solution of $y' = 9x^1y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{2\sqrt{2}}{3}$
- (B) $\frac{3\sqrt{2}}{2}$
- (C) $\frac{2}{9}$
- (D) $\frac{\sqrt{2}}{3}$
- (E) $\frac{1}{3}$

54. How many elements z of $\mathbb{F}_{2^9}$ satisfy $z^2 = z$?

- (A) 2
- (B) 1
- (C) 4
- (D) 3
- (E) 0

Questions 55–57

55. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j(x-9)^j}{j^{\frac{1}{3}}(\log j)^m}$ converge absolutely at $x = 8$? Give the least such m , or 0 if none exists.

- (A) 4
- (B) 1
- (C) 3
- (D) 2
- (E) 0

56. How many nonnegative integer triples (a, b, c) satisfy $a + b + c = 22$?

- (A) 368
- (B) 184
- (C) 276
- (D) 460
- (E) 92

57. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{9t} \cos(2t)$, and $y = e^{9t} \sin(2t)$, find d^2f/dt^2 at $t = 1$.

- (A) 324
- (B) 18
- (C) 0
- (D) 170
- (E) 4

Questions 58–60

58. How many generators does the cyclic group of order 19 have?

- (A) 16
- (B) 20
- (C) 19
- (D) 18
- (E) 17

59. Evaluate $\int_0^1 \int_0^x x^9 y^2 \, dy \, dx$.

- (A) $\frac{1}{52}$
- (B) $\frac{1}{36}$
- (C) $\frac{1}{30}$
- (D) $\frac{1}{13}$
- (E) $\frac{1}{39}$

60. A tree has 26 vertices. What is the sum of its vertex degrees?

- (A) 50
- (B) 66
- (C) 82
- (D) 18
- (E) 34

Questions 61–63

61. Find the arc length of $r(t) = (9t, 2t)$ for $0 \leq t \leq 1$.

- (A) $1 + \sqrt{85}$
- (B) $\frac{\sqrt{85}}{2}$
- (C) $2\sqrt{85}$
- (D) $-\sqrt{85}$
- (E) $\sqrt{85}$

62. Let $R = (\mathbb{Z}/45\mathbb{Z})/([9])$, where $([9])$ is the ideal generated by $[9]$. How many elements does R have?

- (A) 8
- (B) 9
- (C) 7
- (D) 10
- (E) 11

63. Find the coefficient of x^2 in the Taylor series of $(1 - 9x)^{-2}$ at zero.

- (A) $\frac{1}{9} (3 \cdot 9^2)$
- (B) $2 (3 \cdot 9^2)$
- (C) $\frac{2}{3} (3 \cdot 9^2)$
- (D) $3 \cdot 9^2$
- (E) $\frac{1}{3} (3 \cdot 9^2)$

Questions 64–66

64. A connected planar graph has 25 vertices and 45 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 15
- (B) 22
- (C) 29
- (D) 8
- (E) 36

65. Let $f(x) = \log(1 + 9x)$. Find $f^{(2)}(0)$.

- (A) $\frac{1}{9} \left((-1)^1 1! 9^2 \right)$
- (B) $\frac{1}{2} \left((-1)^1 1! 9^2 \right)$
- (C) $-(-1)^1 1! 9^2$
- (D) $2 \left((-1)^1 1! 9^2 \right)$
- (E) $(-1)^1 1! 9^2$

66. What is the largest integer k such that 2^k divides $50!$?

- (A) 47
- (B) 48
- (C) 46
- (D) 49
- (E) 45

Answer Key

Practice Test 45 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	C	23	C	45	B
2	A	24	A	46	E
3	B	25	A	47	C
4	D	26	E	48	E
5	B	27	E	49	D
6	A	28	D	50	C
7	C	29	D	51	E
8	A	30	C	52	B
9	D	31	B	53	D
10	C	32	A	54	A
11	A	33	E	55	E
12	B	34	D	56	C
13	A	35	B	57	C
14	D	36	B	58	D
15	C	37	B	59	E
16	A	38	C	60	A
17	C	39	D	61	E
18	E	40	E	62	B
19	B	41	E	63	D
20	C	42	D	64	B
21	A	43	D	65	E
22	A	44	B	66	A

zarimion.com/gre

100 practice tests, complete answer keys, and future updates.
Please retain the source attribution when sharing this material.

