

GRE Mathematics Practice Test 47

Advanced Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A demanding practice form with frequent multi-step arguments and a mix of computational and conceptual questions. This is cycle 9 of 19, position 2 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Let C be the unit circle $x^2 + y^2 = 1$, $z = 5$, counterclockwise viewed from above. Evaluate $\int_C -10y(x^2 + y^2)^4 dx + (10x(x^2 + y^2)^4) dy + 3z dz$.

- (A) 20π
- (B) -20π
- (C) $1 + 20\pi$
- (D) 40π
- (E) 10π

2. Let $D = \text{diag}(\underbrace{1, \dots, 1}_8, \underbrace{2, \dots, 2}_{41})$. Find the dimension of the space of real matrices X with $XD = DX$.

- (A) 1746
- (B) 1747
- (C) 1745
- (D) 1744
- (E) 1743

3. Let $f(x, y) = x^2 + 20xy + (105)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 20
- (B) 10
- (C) 5
- (D) 420
- (E) 105

Questions 4–6

4. X, Y are independent Poisson variables with means 19 and 20. Given $X + Y = 21$, find $\text{Var}(X | X + Y)$.

- (A) $\frac{3167}{507}$
- (B) $\frac{1330}{507}$
- (C) $\frac{3674}{507}$
- (D) $\frac{5320}{507}$
- (E) $\frac{2660}{507}$

5. Let C be $x^2/25 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C \left(-\frac{10y^3}{3}\right) dx + \left(\frac{x^3}{3}\right) dy$.

- (A) $\frac{175\pi}{4}$
- (B) $\frac{175\pi}{2}$
- (C) $1 + \frac{175\pi}{4}$
- (D) $-\frac{175\pi}{4}$
- (E) $\frac{175\pi}{8}$

6. When $x^3 + 1$ is divided by $(x - 44)(x - 45)$, the remainder is $ux + v$. Find u .

- (A) 5940
- (B) 5939
- (C) 5943
- (D) 5942
- (E) 5941

Questions 7–9

7. For $f(x, y) = x^2 + 10xy + 5y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) 64
- (B) 16
- (C) 8
- (D) -8
- (E) 0

8. X, Y are independent, with variances 19 and 20. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 17
- (B) 12
- (C) 7
- (D) 22
- (E) 27

9. Find the coefficient of x^6 in $\cos(10x)/(1 - 4x^2)$.

- (A) $-\frac{12500}{9}$
- (B) $-\frac{4124}{9}$
- (C) $-\frac{4133}{9}$
- (D) -329920
- (E) $\frac{4124}{9}$

Questions 10–12

10. Let $P = \text{diag}(\underbrace{1, \dots, 1}_4, \underbrace{0, \dots, 0}_{46})$. What is the dimension of the space of 50×50 real matrices X satisfying $PX = X$ and $XP = 0$?

- (A) 184
- (B) 183
- (C) 182
- (D) 186
- (E) 185

11. For $x > 0$, $x^2 y'' + (-24)xy' + 150y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $-\frac{e^{10}}{5} + 1 + \frac{e^{15}}{5}$
- (B) $-\frac{(1 - e^5) e^{10}}{10}$
- (C) $-\frac{(1 - e^5) e^{10}}{5}$
- (D) $\frac{(1 - e^5) e^{10}}{5}$
- (E) $-\frac{2(1 - e^5) e^{10}}{5}$

12. X has density cx^{19} on $0 < x < 1$ and zero elsewhere. Find $E[X^2]$.

- (A) $\frac{5}{11}$
- (B) $\frac{22}{10}$
- (C) $\frac{11}{1}$
- (D) $\frac{1}{11}$
- (E) $\frac{10}{33}$

Questions 13–15

13. Let $f(x, y) = e^{10x+5y}$. Find $\partial^5 f / \partial x^4 \partial y$ at $(0, 0)$.

- (A) $5 \cdot 10^4$
- (B) $\frac{1}{16} (5 \cdot 10^4)$
- (C) $2 (5 \cdot 10^4)$
- (D) $\frac{1}{8} (5 \cdot 10^4)$
- (E) $\frac{1}{5} (5 \cdot 10^4)$

14. A 3×3 matrix A has characteristic polynomial $(t - 43)(t + 1)(t - 2)$. What is $\text{tr}(A^3 - 2A + I)$?

- (A) 79427
- (B) 79431
- (C) 79428
- (D) 79430
- (E) 79429

15. Let $F(x) = \int_0^{x^5} e^{10t} dt$. Find $F'(1)$.

- (A) e^{10}
- (B) $\frac{e^{10}}{5}$
- (C) $50e^{10}$
- (D) $5e^{10}$
- (E) e^{50}

Questions 16–18

16. Independent exponential waiting times X, Y have rates 19 and 21. Find $P(X < Y)$.

- (A) $\frac{19}{120}$
- (B) $\frac{19}{40}$
- (C) $\frac{21}{40}$
- (D) $\frac{59}{80}$
- (E) $\frac{19}{80}$

17. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^{10} y^4 dV$.

- (A) $17 \left(\frac{10! 4!}{17!} \right)$
- (B) $6 \left(\frac{10! 4!}{17!} \right)$
- (C) $\frac{10! 4!}{17!}$
- (D) $\frac{1}{55}$
- (E) $\frac{1}{6} \left(\frac{10! 4!}{17!} \right)$

18. Let N be the 49×49 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\dim(\ker N^{14} / \ker N^7)$.

- (A) 5
- (B) 7
- (C) 9
- (D) 8
- (E) 6

Questions 19–21

19. If $y' = y(1 - y/5)$ and $y(0) = 5/2$, find $y(-\log 10)$.

- (A) $-\frac{5}{11}$
- (B) $\frac{10}{11}$
- (C) $\frac{5}{22}$
- (D) $\frac{16}{11}$
- (E) $\frac{5}{11}$

20. $X_1, \dots, X_{19}$ are independent with common mean and variance 21. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 504
- (B) 630
- (C) 126
- (D) 378
- (E) 252

21. Evaluate $\iiint_{x^2+y^2+z^2 \leq 10} (x^2 + y^2 + z^2)^4 dV$.

- (A) $\frac{1}{3} \left(\frac{4\pi 10^{\frac{11}{2}}}{11} \right)$
- (B) $11 \left(\frac{4\pi 10^{\frac{11}{2}}}{11} \right)$
- (C) $\frac{11}{90} \left(\frac{4\pi 10^{\frac{11}{2}}}{11} \right)$
- (D) $\frac{1}{2} \left(\frac{4\pi 10^{\frac{11}{2}}}{11} \right)$
- (E) $\frac{4\pi 10^{\frac{11}{2}}}{11}$

Questions 22–24

22. Let A be the 49×49 matrix whose diagonal entries are 46 and whose off-diagonal entries are 1. What is its largest eigenvalue?

- (A) 95
- (B) 92
- (C) 94
- (D) 96
- (E) 93

23. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 10x)^{1/(4x)} - e^{10/4}}{x}$.

- (A) $-25e^{\frac{5}{2}}$
- (B) $-\frac{5e^{\frac{5}{2}}}{4}$
- (C) $\frac{25e^{\frac{5}{2}}}{2}$
- (D) $e^{\frac{5}{2}}$
- (E) $-\frac{25e^{\frac{5}{2}}}{2}$

24. A continuous $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(q) = q^{14}$ for every rational q . Find $f(2)$.

- (A) 2^{14}
- (B) 2^{17}
- (C) 2^{13}
- (D) 2^{16}
- (E) 2^{15}

Questions 25–27

25. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 10x$ and has density $\rho(x, y) = x^3y$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{3}{7}$
- (B) $-\frac{6}{7}$
- (C) $\frac{6}{7}$
- (D) $\frac{12}{7}$
- (E) $\frac{13}{7}$

26. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 301$ and $x + y = 50$. Find $x^2 + y^2$.

- (A) 1898
- (B) 1897
- (C) 1896
- (D) 1900
- (E) 1899

27. Find the coefficient of x^5 in the Taylor series of $(1 - 10x)^{-2}$ at zero.

- (A) $6 \cdot 10^5$
- (B) $\frac{1}{10} (6 \cdot 10^5)$
- (C) $\frac{1}{6} (6 \cdot 10^5)$
- (D) $-6 \cdot 10^5$
- (E) $\frac{5}{6} (6 \cdot 10^5)$

Questions 28–30

28. Find the radius of convergence of $\sum_{k \geq 1} k^2 z^{2k} / 14^{2k} + \sum_{k \geq 0} z^{2k+1} / (15)^{2k+1}$.

- (A) 10
- (B) 14
- (C) 22
- (D) 18
- (E) 6

29. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(10j)!}{(j!)^{10}} x^{5j}$?

- (A) 10^{-10}
- (B) $10^{-\frac{1}{2}}$
- (C) 10^2
- (D) $10(10^{-2})$
- (E) 10^{-2}

30. A sequence satisfies $u_0 = 43$ and $u_{k+1} = 2u_k + 7$ for all $k \geq 0$. Find u_8 .

- (A) 12792
- (B) 12793
- (C) 12791
- (D) 12795
- (E) 12794

Questions 31–33

31. The local inverse $y(x)$ of $x = 10y + y^3$ satisfies $y(0) = 0$. Find $y^{(5)}(0)$.
- (A) $10 \left(\frac{360}{10^7} \right)$
(B) $\frac{1}{120} \left(\frac{360}{10^7} \right)$
(C) $-\frac{360}{10^7}$
(D) $\frac{1}{10} \left(\frac{360}{10^7} \right)$
(E) $\frac{360}{10^7}$
32. On $[0, 1]$, f is 14 on rationals and $1/14$ on irrationals. Find its upper Darboux integral minus its lower Darboux integral.
- (A) $\frac{209}{14}$
(B) $\frac{195}{28}$
(C) $\frac{223}{14}$
(D) $\frac{195}{14}$
(E) $\frac{195}{7}$
33. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 10z^2 = x + 5y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.
- (A) 100
(B) -50
(C) -5
(D) -20
(E) -100

Questions 34–36

34. Linear maps $T : \mathbb{R}^{50} \rightarrow \mathbb{R}^{50}$ and $U : \mathbb{R}^{50} \rightarrow \mathbb{R}^{50}$ have ranks 5 and 3. What is the largest possible rank of UT ?

- (A) 5
- (B) 3
- (C) 4
- (D) 1
- (E) 2

35. For $f(x) = x^5 e^{10x}$, find $f^{(7)}(0)$.

- (A) $2 \left(\frac{7! 10^2}{2} \right)$
- (B) $\frac{1}{5} \left(\frac{7! 10^2}{2} \right)$
- (C) $\frac{1}{2520} \left(\frac{7! 10^2}{2} \right)$
- (D) $\frac{7! 10^2}{2}$
- (E) $\frac{1}{42} \left(\frac{7! 10^2}{2} \right)$

36. A set with 14 elements has the discrete topology. How many of its subsets are simultaneously open and closed?

- (A) 2^{17}
- (B) 2^{13}
- (C) 2^{14}
- (D) 2^{15}
- (E) 2^{16}

Questions 37–39

37. Evaluate $\lim_{x \rightarrow 0} \frac{e^{10x} - \sum_{j=0}^4 (10x)^j / j!}{x^5}$.

- (A) $-\frac{10^5}{5!}$
- (B) $\frac{10^5}{5!}$
- (C) $5 \left(\frac{10^5}{5!} \right)$
- (D) $\frac{1}{10} \left(\frac{10^5}{5!} \right)$
- (E) $120 \left(\frac{10^5}{5!} \right)$

38. Fix a nonzero vector $u \in \mathbb{R}^{49}$. What is the dimension of the space of skew-symmetric 49×49 matrices A satisfying $Au = 0$?

- (A) 1126
- (B) 1127
- (C) 1129
- (D) 1128
- (E) 1130

39. Evaluate $\int_0^1 \int_0^x x^{10} y^5 dy dx$.

- (A) $\frac{1}{119}$
- (B) $\frac{1}{96}$
- (C) $\frac{1}{102}$
- (D) $\frac{1}{66}$
- (E) $\frac{1}{17}$

Questions 40–42

40. The circle $|z| = 2$ is traversed counterclockwise. What is $\frac{1}{2\pi i} \int_{|z|=2} \frac{e^z}{z^{15}} dz$?

- (A) $1/14!$
- (B) $14!$
- (C) $1/28!$
- (D) $1/15!$
- (E) $1/13!$

41. If $y'' - 20y' + 100y = 5e^{10x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $\frac{5e^{10}}{2}$
- (B) $\frac{5e^{10}}{4}$
- (C) $1 + \frac{5e^{10}}{2}$
- (D) $5e^{10}$
- (E) $-\frac{5e^{10}}{2}$

42. How many generators does the cyclic group of order 6436343 have?

- (A) $23^5(23 - 1)$
- (B) $23^6(23 - 1)$
- (C) $23^3(23 - 1)$
- (D) $23^4(23 - 1)$
- (E) $23^7(23 - 1)$

Questions 43–45

43. The solution of $y' = 10x^4y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{10^{\frac{4}{5}}}{10}$
- (B) $\frac{2^{\frac{3}{5}}}{2}$
- (C) $\frac{1}{2}$
- (D) $\sqrt[5]{2}$
- (E) $\frac{2^{\frac{4}{5}}}{2}$

44. Find the residue of $z^{14}/(z-1)^3$ at $z = 1$.

- (A) 151
- (B) 91
- (C) 31
- (D) 121
- (E) 61

45. What is the maximum of x^5y^{10} over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\frac{2}{9}$
- (B) $\frac{1}{2^{15}}$
- (C) $15 \left(\left(\frac{5}{15} \right)^5 \left(\frac{10}{15} \right)^{10} \right)$
- (D) $\left(\frac{5}{15} \right)^5 \left(\frac{10}{15} \right)^{10}$
- (E) $\left(\frac{5}{15} \right)^{10} \left(\frac{10}{15} \right)^5$

Questions 46–48

46. Let $R = (\mathbb{Z}/96\mathbb{Z})/([12])$, where $([12])$ is the ideal generated by $[12]$. How many units does R have?

- (A) 5
- (B) 2
- (C) 4
- (D) 3
- (E) 6

47. Evaluate $\int_0^\infty x^5 e^{-10x} dx$.

- (A) $\frac{1}{5} \left(\frac{5!}{10^6} \right)$
- (B) $10 \left(\frac{5!}{10^6} \right)$
- (C) 120
- (D) $\frac{5!}{10^6}$
- (E) $\frac{1}{10} \left(\frac{5!}{10^6} \right)$

48. How many integers in $\{1, \dots, 114\}$ are divisible by 2 or 3?

- (A) 26
- (B) 126
- (C) 101
- (D) 51
- (E) 76

Questions 49–51

49. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(10x) - \sum_{j=0}^3 (-1)^j (10x)^{2j+1} / (2j+1)!}{x^9}$.

- (A) $9 \left(\frac{(-1)^4 10^9}{9!} \right)$
- (B) $\frac{(-1)^4 10^9}{9!}$
- (C) $\frac{1}{10} \left(\frac{(-1)^4 10^9}{9!} \right)$
- (D) $10 \left(\frac{(-1)^4 10^9}{9!} \right)$
- (E) $-\frac{(-1)^4 10^9}{9!}$

50. What is the largest integer k such that 3^k divides $52!$?

- (A) 25
- (B) 21
- (C) 24
- (D) 22
- (E) 23

51. Evaluate $\int_0^1 \frac{x^4}{1 + 10x^5} dx$.

- (A) $\frac{\log(11)}{50}$
- (B) $-\frac{\log(11)}{50}$
- (C) $\frac{\log(11)}{5}$
- (D) $\frac{\log(11)}{10}$
- (E) $\log(11)$

Questions 52–54

52. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 19 have no adjacent 1s?

- (A) F_{23}
- (B) F_{19}
- (C) F_{22}
- (D) F_{21}
- (E) F_{20}

53. Find the directional derivative of $f(x, y, z) = x^2y + 5xz^2 + 10y^2z$ at $(1, 1, 1)$ in the direction $(1, 3, 0)$.

- (A) $7\sqrt{10}$
- (B) $1 + 7\sqrt{10}$
- (C) $14\sqrt{10}$
- (D) $-7\sqrt{10}$
- (E) $\frac{7\sqrt{10}}{2}$

54. In the ring $\mathbb{Z}/282\mathbb{Z}$, how many elements satisfy $x^2 = 1$?

- (A) 6
- (B) 4
- (C) 5
- (D) 2
- (E) 3

Questions 55–57

55. If $y'' + 25y = 10$ and $y(0) = y'(0) = 0$, find $y(\pi/5)$.

- (A) $-\frac{4}{5}$
- (B) $\frac{8}{5}$
- (C) $\frac{4}{5}$
- (D) $\frac{9}{5}$
- (E) $\frac{2}{5}$

56. For Newton iteration on $f(x) = x^2 - 361$ near the root $r = 19$, find $\lim_{k \rightarrow \infty} (x_{k+1} - r)/(x_k - r)^2$ for nonterminating convergent iterates.

- (A) $\frac{39}{76}$
- (B) $\frac{1}{76}$
- (C) $\frac{37}{38}$
- (D) $\frac{1}{38}$
- (E) $\frac{1}{114}$

57. The solution of $y' = 10x^5y^3$ has $y(0) = 1/11$. Find $y(1)$.

- (A) $-\frac{\sqrt{1059}}{353}$
- (B) $\frac{\sqrt{1059}}{353} + 1$
- (C) $\frac{\sqrt{1059}}{353}$
- (D) $\frac{\sqrt{1059}}{706}$
- (E) $\frac{2\sqrt{1059}}{353}$

Questions 58–60

58. A permutation consists of disjoint cycles of lengths 8 and 3, together with a disjoint transposition when the sum of these two lengths is odd. It fixes all other points. What is its order?

- (A) 26
- (B) 22
- (C) 23
- (D) 24
- (E) 25

59. Find the arc length of $r(t) = (10(t - t^3/3), 10t^2)$ for $0 \leq t \leq 1/5$.

- (A) $\frac{76}{75}$
- (B) $\frac{304}{75}$
- (C) $\frac{152}{75}$
- (D) $\frac{227}{75}$
- (E) $-\frac{152}{75}$

60. Bisection starts with an interval of length 2^{19} . After 21 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{1}{8}$
- (B) $\frac{1}{16}$
- (C) $\frac{7}{8}$
- (D) $\frac{9}{16}$
- (E) $\frac{1}{24}$

Questions 61–63

61. Evaluate $\iiint_B 10z^2(x^2 + y^2 + z^2)^5 dV$, where B is the unit ball.

- (A) $-\frac{8\pi}{9}$
- (B) $\frac{16\pi}{9}$
- (C) $1 + \frac{8\pi}{9}$
- (D) $\frac{8\pi}{9}$
- (E) $\frac{4\pi}{9}$

62. In $R = \mathbb{F}_7[x]/(x^{10})$, how many elements are units?

- (A) $7^{11} - 7^9$
- (B) $7^{11} - 7^{10}$
- (C) $7^{10} - 7^0$
- (D) $7^{12} - 7^{11}$
- (E) $7^{10} - 7^9$

63. Let $f(x) = \log(1 + 10x)$. Find $f^{(5)}(0)$.

- (A) $5 \left((-1)^4 4! 10^5 \right)$
- (B) $(-1)^4 4! 10^5$
- (C) $-(-1)^4 4! 10^5$
- (D) $\frac{1}{10} \left((-1)^4 4! 10^5 \right)$
- (E) $\frac{1}{5} \left((-1)^4 4! 10^5 \right)$

Questions 64–66

64. What is the least positive multiplicative inverse of 2 modulo 39?

- (A) 14
- (B) 8
- (C) 26
- (D) 20
- (E) 32

65. If $y' + 10y = 5x^5e^{-10x}$ and $y(0) = 3$, find $y(1)$.

- (A) $\frac{23}{3e^{10}}$
- (B) $\frac{23}{6e^{10}}$
- (C) $\frac{23}{12e^{10}}$
- (D) $-\frac{23}{6e^{10}}$
- (E) $\frac{23}{6e^{10}} + 1$

66. Integers x satisfy $x \equiv 0 \pmod{11}$ and $x \equiv 1 \pmod{16}$. Find the sum of all such x with $0 \leq x < 352$.

- (A) 241
- (B) 242
- (C) 243
- (D) 244
- (E) 240

Answer Key

Practice Test 47 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	A	23	E	45	D
2	C	24	A	46	C
3	A	25	C	47	D
4	E	26	A	48	E
5	A	27	A	49	B
6	E	28	B	50	E
7	C	29	E	51	A
8	A	30	B	52	D
9	B	31	E	53	A
10	A	32	D	54	B
11	C	33	E	55	C
12	C	34	B	56	D
13	A	35	D	57	C
14	E	36	C	58	D
15	D	37	B	59	C
16	B	38	D	60	A
17	C	39	C	61	D
18	B	40	A	62	E
19	E	41	A	63	B
20	D	42	D	64	D
21	E	43	E	65	B
22	C	44	B	66	B

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