

GRE Mathematics Practice Test 48

Upper-Intermediate Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

Intermediate reinforcement, combining standard applications with selected more demanding arguments. This is cycle 9 of 19, position 3 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(10x) - \sum_{j=0}^2 (-1)^j (10x)^{2j+1} / (2j+1)!}{x^7}$.

- (A) $-\frac{(-1)^3 10^7}{7!}$
- (B) $7 \left(\frac{(-1)^3 10^7}{7!} \right)$
- (C) $10 \left(\frac{(-1)^3 10^7}{7!} \right)$
- (D) $\frac{1}{10} \left(\frac{(-1)^3 10^7}{7!} \right)$
- (E) $\frac{(-1)^3 10^7}{7!}$

2. Let A be the 50×50 matrix whose diagonal entries are 47 and whose off-diagonal entries are 1. What is its largest eigenvalue?

- (A) 95
- (B) 98
- (C) 94
- (D) 97
- (E) 96

3. For $f(x) = x^4 e^{10x}$, find $f^{(6)}(0)$.

- (A) $\frac{6! 10^2}{2}$
- (B) $\frac{1}{5} \left(\frac{6! 10^2}{2} \right)$
- (C) $\frac{1}{30} \left(\frac{6! 10^2}{2} \right)$
- (D) $\frac{1}{360} \left(\frac{6! 10^2}{2} \right)$
- (E) $2 \left(\frac{6! 10^2}{2} \right)$

Questions 4–6

4. X, Y are independent Poisson variables with means 20 and 21. Given $X + Y = 22$, find $\text{Var}(X | X + Y)$.

- (A) $\frac{12602}{1681}$
- (B) $\frac{9240}{1681}$
- (C) $\frac{4620}{1681}$
- (D) $\frac{18480}{1681}$
- (E) $\frac{10921}{1681}$

5. If $y'' - 20y' + 100y = 4e^{10x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $1 + 2e^{10}$
- (B) e^{10}
- (C) $-2e^{10}$
- (D) $4e^{10}$
- (E) $2e^{10}$

6. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 440$ and $x + y = 54$. Find $x^2 + y^2$.

- (A) 2034
- (B) 2036
- (C) 2037
- (D) 2035
- (E) 2038

Questions 7–9

7. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{10t} \cos(4t)$, and $y = e^{10t} \sin(4t)$, find $d^2 f/dt^2$ at $t = 1$.

- (A) 20
- (B) 232
- (C) 8
- (D) 400
- (E) 0

8. X, Y are independent, with variances 20 and 21. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 6
- (B) 12
- (C) 24
- (D) 30
- (E) 18

9. For $\mathbf{F} = (10x^4y, y^4z, xz^4)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) 116
- (B) 117
- (C) 232
- (D) -116
- (E) 58

Questions 10–12

10. A sequence satisfies $u_0 = 44$ and $u_{k+1} = 2u_k + 10$ for all $k \geq 0$. Find u_7 .

- (A) 6900
- (B) 6901
- (C) 6902
- (D) 6903
- (E) 6904

11. Find the upward flux of $\mathbf{F} = (x, y, z)$ through $z = 10x^2 + 4y^2$, $0 \leq x, y \leq 1$.

- (A) $-\frac{7}{3}$
- (B) $-\frac{28}{3}$
- (C) $-\frac{11}{3}$
- (D) $\frac{14}{3}$
- (E) $-\frac{14}{3}$

12. X has density cx^{20} on $0 < x < 1$ and zero elsewhere. Find the normalizing constant c .

- (A) 28
- (B) 14
- (C) 35
- (D) 21
- (E) 7

Questions 13–15

13. For $x > 0$, $x^2 y'' + (-23)xy' + 140y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $-\frac{(1 - e^4) e^{10}}{8}$
- (B) $-\frac{(1 - e^4) e^{10}}{2}$
- (C) $-\frac{(1 - e^4) e^{10}}{4}$
- (D) $\frac{(1 - e^4) e^{10}}{4}$
- (E) $-\frac{e^{10}}{4} + 1 + \frac{e^{14}}{4}$

14. Linear maps $T : \mathbb{R}^{51} \rightarrow \mathbb{R}^{51}$ and $U : \mathbb{R}^{51} \rightarrow \mathbb{R}^{51}$ have ranks 4 and 2. What is the largest possible rank of UT ?

- (A) 1
- (B) 3
- (C) 4
- (D) 0
- (E) 2

15. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 10x$ and has density $\rho(x, y) = x^2 y$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{5}{6}$
- (B) $\frac{5}{12}$
- (C) $-\frac{5}{6}$
- (D) $\frac{5}{3}$
- (E) $\frac{11}{6}$

Questions 16–18

16. Independent exponential waiting times X, Y have rates 20 and 22. Find $P(X < Y)$.

- (A) $\frac{10}{63}$
- (B) $\frac{31}{42}$
- (C) $\frac{5}{21}$
- (D) $\frac{11}{21}$
- (E) $\frac{10}{21}$

17. Two solutions of $y'' + 10x^4y' + 4y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $e^{-2} + 1$
- (B) e^{-2}
- (C) $\frac{2}{e^2}$
- (D) $\frac{1}{2e^2}$
- (E) $-\frac{1}{e^2}$

18. Fix a nonzero vector $u \in \mathbb{R}^{50}$. What is the dimension of the space of skew-symmetric 50×50 matrices A satisfying $Au = 0$?

- (A) 1174
- (B) 1177
- (C) 1178
- (D) 1176
- (E) 1175

Questions 19–21

19. Let $f(x, y) = e^{10x+4y}$. Find $\partial^4 f / \partial x^3 \partial y$ at $(0, 0)$.

- (A) $\frac{5}{2} (4 \cdot 10^3)$
- (B) $\frac{8}{125} (4 \cdot 10^3)$
- (C) $\frac{4}{25} (4 \cdot 10^3)$
- (D) $4 \cdot 10^3$
- (E) $\frac{1}{4} (4 \cdot 10^3)$

20. $X_1, \dots, X_{20}$ are independent with common mean and variance 22. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 418
- (B) 140
- (C) 557
- (D) 696
- (E) 279

21. Let $\mathbf{F} = (10y^4, z^4, x^4)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (1, 1, 4) / \sqrt{18}$.

- (A) $28\sqrt{2}$
- (B) $-14\sqrt{2}$
- (C) $-56\sqrt{2}$
- (D) $-28\sqrt{2}$
- (E) $1 - 28\sqrt{2}$

Questions 22–24

22. Let $A = \begin{bmatrix} 1 & 44 & 0 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 3)$ entry of A^3 ?

- (A) 1319
- (B) 1321
- (C) 1320
- (D) 1318
- (E) 1322

23. For the curve $x = t^4$, $y = t^5 + 10t$, find d^2y/dx^2 at $t = 1$.

- (A) $-\frac{5}{2}$
- (B) $\frac{35}{16}$
- (C) $-\frac{25}{4}$
- (D) $\frac{15}{4}$
- (E) $-\frac{25}{16}$

24. A continuous $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(q) = q^{15}$ for every rational q . Find $f(2)$.

- (A) 2^{15}
- (B) 2^{18}
- (C) 2^{14}
- (D) 2^{17}
- (E) 2^{16}

Questions 25–27

25. If $y' = 20xy$ and $y(0) = 1$, find the coefficient of x^8 in the Maclaurin series of y .

- (A) $-\frac{1250}{3}$
- (B) $\frac{1250}{3}$
- (C) $\frac{2500}{3}$
- (D) $\frac{1253}{3}$
- (E) $\frac{625}{3}$

26. An invertible 50×50 matrix A has determinant 2. Find $\det(\text{adj } A)$.

- (A) 2^{50}
- (B) 2^{48}
- (C) $2^{49} \cdot 5$
- (D) $2^{49} \cdot 3$
- (E) 2^{49}

27. Under $x = u + 10v$, $y = u - 4v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) 6
- (B) 14
- (C) 40
- (D) $\frac{1}{14}$
- (E) 196

Questions 28–30

28. Find the radius of convergence of $\sum_{k \geq 1} k^2 z^{2k} / 15^{2k} + \sum_{k \geq 0} z^{2k+1} / (16)^{2k+1}$.

- (A) 25
- (B) 10
- (C) 20
- (D) 5
- (E) 15

29. For $x > 0$, $x^2 y'' + (1 - 14)xy' + 40y = 0$, $y(1) = 0$, $y'(1) = 10$. Find $y(e)$.

- (A) $\frac{e^{10}}{14}$
- (B) $-\frac{5(1 - e^6)e^4}{3}$
- (C) $-e^4 + e^{10}$
- (D) $e^4 + e^{10}$
- (E) $\frac{5(1 - e^6)e^4}{3}$

30. On P_{51} , the real polynomials of degree at most 51, define $T(p) = x^2 p'' - 42p$. Find its rank.

- (A) 51
- (B) 53
- (C) 52
- (D) 50
- (E) 49

Questions 31–33

31. Evaluate $\iiint_B 10z^2(x^2 + y^2 + z^2)^4 dV$, where B is the unit ball.

- (A) $\frac{20\pi}{39}$
- (B) $-\frac{40\pi}{39}$
- (C) $1 + \frac{40\pi}{39}$
- (D) $\frac{40\pi}{39}$
- (E) $\frac{80\pi}{39}$

32. On $[0, 1]$, f is 15 on rationals and $1/15$ on irrationals. Find its upper Darboux integral minus its lower Darboux integral.

- (A) $\frac{254}{15}$
- (B) $\frac{112}{15}$
- (C) $\frac{224}{15}$
- (D) $\frac{448}{15}$
- (E) $\frac{239}{15}$

33. Let C be $x^2/16 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-\frac{10y^3}{3}) dx + (\frac{x^3}{3}) dy$.

- (A) 26π
- (B) -26π
- (C) 52π
- (D) $1 + 26\pi$
- (E) 13π

Questions 34–36

34. Subspaces U, V of $\mathbb{R}^{52}$ have dimensions 5 and 6. What is the smallest possible value of $\dim(U \cap V)$?

- (A) 2
- (B) 0
- (C) 4
- (D) 3
- (E) 1

35. Evaluate $\int_0^1 \int_x^1 10y^4 \, dy \, dx$.

- (A) $\frac{1}{3}$
- (B) 2
- (C) $\frac{5}{3}$
- (D) $\frac{5}{2}$
- (E) 5

36. A set with 15 elements has the discrete topology. How many of its subsets are simultaneously open and closed?

- (A) 2^{15}
- (B) 2^{16}
- (C) 2^{14}
- (D) 2^{18}
- (E) 2^{17}

Questions 37–39

37. If $y' = y(1 - y/4)$ and $y(0) = 2$, find $y(-\log 10)$.

- (A) $-\frac{4}{11}$
- (B) $\frac{8}{11}$
- (C) $\frac{2}{11}$
- (D) $\frac{4}{11}$
- (E) $\frac{15}{11}$

38. Let V consist of real polynomials of degree at most 9 that vanish at 44, 45, and 47. Find $\dim V$.

- (A) 7
- (B) 5
- (C) 6
- (D) 9
- (E) 8

39. Evaluate $\int_0^1 \int_0^x x^{10} y^4 dy dx$.

- (A) $\frac{1}{75}$
- (B) $\frac{1}{96}$
- (C) $\frac{1}{80}$
- (D) $\frac{1}{55}$
- (E) $\frac{1}{16}$

Questions 40–42

40. The circle $|z| = 2$ is traversed counterclockwise. What is $\frac{1}{2\pi i} \int_{|z|=2} \frac{e^z}{z^{16}} dz$?

- (A) $1/16!$
- (B) $1/30!$
- (C) $1/14!$
- (D) $1/15!$
- (E) $15!$

41. Evaluate $\int_C (4x^3 + 10y) dx + (10x + 4y^3) dy$, where C is $r(t) = (t, 4t^2)$, $0 \leq t \leq 1$.

- (A) 594
- (B) 298
- (C) 297
- (D) $\frac{297}{2}$
- (E) -297

42. The symmetric group S_{50} acts on the two-element subsets of $\{1, \dots, 50\}$. What is the index of the setwise stabilizer of $\{1, 2\}$?

- (A) 1223
- (B) 1225
- (C) 1227
- (D) 1224
- (E) 1226

Questions 43–45

43. For $f(x, y) = x^2 + 10xy + 4y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) 2
- (B) -8
- (C) $2\sqrt{17}$
- (D) 68
- (E) 10

44. Find the residue of $z^{15}/(z - 1)^3$ at $z = 1$.

- (A) 105
- (B) 175
- (C) 140
- (D) 35
- (E) 70

45. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^{10}y^3 dV$.

- (A) $\frac{1}{44}$
- (B) $\frac{10! 3!}{16!}$
- (C) $\frac{1}{6} \left(\frac{10! 3!}{16!} \right)$
- (D) $16 \left(\frac{10! 3!}{16!} \right)$
- (E) $6 \left(\frac{10! 3!}{16!} \right)$

Questions 46–48

46. In $\mathbb{Z}/1362\mathbb{Z}$, how many residues are not idempotent ($x^2 \neq x$)?

- (A) 1355
- (B) 1353
- (C) 1356
- (D) 1354
- (E) 1352

47. If $y' + 10y = 4x^4e^{-10x}$ and $y(0) = 4$, find $y(1)$.

- (A) $\frac{24}{5e^{10}} + 1$
- (B) $\frac{24}{5e^{10}}$
- (C) $\frac{12}{5e^{10}}$
- (D) $-\frac{24}{5e^{10}}$
- (E) $\frac{48}{5e^{10}}$

48. How many integers in $\{1, \dots, 120\}$ are divisible by 2 or 3?

- (A) 132
- (B) 28
- (C) 54
- (D) 80
- (E) 106

Questions 49–51

49. The solution of $y' = 10x^4y^3$ has $y(0) = 1/11$. Find $y(1)$.

- (A) $-\frac{\sqrt{13}}{39}$
- (B) $\frac{\sqrt{13}}{39} + 1$
- (C) $\frac{\sqrt{13}}{78}$
- (D) $\frac{\sqrt{13}}{39}$
- (E) $\frac{2\sqrt{13}}{39}$

50. In the additive group $\mathbb{Z}/60\mathbb{Z}$, find the index of the subgroup generated by $[6]$.

- (A) 6
- (B) 5
- (C) 4
- (D) 8
- (E) 7

51. Let $f(x, y) = x^2 + 20xy + (104)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 416
- (B) 8
- (C) 16
- (D) 104
- (E) 4

Questions 52–54

52. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 20 have no adjacent 1s?

- (A) F_{21}
- (B) F_{22}
- (C) F_{20}
- (D) F_{23}
- (E) F_{24}

53. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-10)^j}{j^1 (\log j)^m}$ converge absolutely at $x = 9$? Give the least such m , or 0 if none exists.

- (A) 0
- (B) 1
- (C) 3
- (D) 4
- (E) 2

54. How many invertible 3×3 matrices over $\mathbb{F}_{23}$ are there whose first column is $(1, 0, 0)^T$?

- (A) $\frac{23^3 23^3 23^3}{23^3 - 1}$
- (B) $\frac{(23^3 - 23^0)(23^3 - 23^1)}{23^3 - 1}$
- (C) $\frac{(23^3 - 23^0)(23^3 - 23^0)(23^3 - 23^0)}{23^3 - 1}$
- (D) $\frac{(23^3 - 23^0)(23^3 - 23^1)(23^3 - 23^2)}{23^3 - 1}$
- (E) $\frac{(23^3 - 23^1)(23^3 - 23^2)(23^3 - 23^3)}{23^3 - 1}$

Questions 55–57

55. Solve $y' + 10y = x^4e^{-10x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{e^{10}}{5}$
- (B) $\frac{1}{4e^{10}}$
- (C) $\frac{1}{5e^{10}}$
- (D) e^{-10}
- (E) $\frac{2}{e^{10}}$

56. Apply one Newton iteration to solve $x^2 - 20 = 0$, starting at $x_0 = 1$. What is x_1 ?

- (A) $\frac{21}{4}$
- (B) $\frac{23}{2}$
- (C) 21
- (D) $\frac{25}{2}$
- (E) $\frac{21}{2}$

57. Find the coefficient of x^6 in $\cos(10x)/(1 - 3x^2)$.

- (A) $-\frac{5066}{9}$
- (B) $-\frac{5057}{9}$
- (C) $\frac{5057}{9}$
- (D) $-\frac{12500}{9}$
- (E) -404560

Questions 58–60

58. How many permutations in S_{50} consist of one 6-cycle and 44 fixed points?

- (A) $2^5 \cdot 3^2 \cdot 5^4 \cdot 7^2 \cdot 23 \cdot 47$
- (B) $2^4 \cdot 3^2 \cdot 5^3 \cdot 7^2 \cdot 23 \cdot 47$
- (C) $2^5 \cdot 3^2 \cdot 5^3 \cdot 7^2 \cdot 23 \cdot 47$
- (D) $2^6 \cdot 3^2 \cdot 5^3 \cdot 7^2 \cdot 23 \cdot 47$
- (E) $2^5 \cdot 3^3 \cdot 5^3 \cdot 7^2 \cdot 23 \cdot 47$

59. Evaluate $\int_0^1 \frac{x^3}{1+10x^4} dx$.

- (A) $\frac{\log(11)}{4}$
- (B) $\log(11)$
- (C) $\frac{\log(11)}{40}$
- (D) $-\frac{\log(11)}{40}$
- (E) $\frac{\log(11)}{10}$

60. Bisection starts with an interval of length 2^{20} . After 22 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{1}{8}$
- (B) $\frac{1}{16}$
- (C) $\frac{7}{8}$
- (D) $\frac{9}{16}$
- (E) $\frac{1}{24}$

Questions 61–63

61. Evaluate $\int_0^{\log 2} e^x (e^x + 10)^4 dx$.

- (A) $\frac{12^5 - 11^5}{5}$
- (B) $-\frac{12^5 - 11^5}{5}$
- (C) $\frac{12^5}{5}$
- (D) $5 \left(\frac{12^5 - 11^5}{5} \right)$
- (E) $12^4 - 11^4$

62. How many maximal proper subgroups does the cyclic group of order 576 have?

- (A) 3
- (B) 2
- (C) 1
- (D) 4
- (E) 0

63. Find the curvature of the helix $r(t) = (10 \cos t, 10 \sin t, 4t)$.

- (A) $-\frac{5}{58}$
- (B) $\frac{5}{116}$
- (C) $\frac{5}{29}$
- (D) $\frac{5}{58}$
- (E) $\frac{63}{58}$

Questions 64–66

64. What is the least positive multiplicative inverse of 2 modulo 41?

- (A) 14
- (B) 7
- (C) 21
- (D) 35
- (E) 28

65. Evaluate $\int_0^1 x^{10}(1-x)^3 dx$.

- (A) $14 \left(\frac{10! 3!}{14!} \right)$
- (B) $91 \left(\frac{10! 3!}{14!} \right)$
- (C) $11 \left(\frac{10! 3!}{14!} \right)$
- (D) $\frac{10! 3!}{14!}$
- (E) $\frac{1}{4} \left(\frac{10! 3!}{14!} \right)$

66. How many residues x modulo 60 satisfy $2x \equiv 2 \pmod{60}$ and $2x \equiv 0 \pmod{60}$?

- (A) 1
- (B) 2
- (C) 3
- (D) 4
- (E) 0

Answer Key

Practice Test 48 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	E	23	E	45	B
2	E	24	A	46	D
3	A	25	B	47	B
4	B	26	E	48	D
5	E	27	B	49	D
6	B	28	E	50	A
7	E	29	B	51	C
8	E	30	A	52	B
9	A	31	D	53	E
10	C	32	C	54	D
11	E	33	A	55	C
12	D	34	B	56	E
13	C	35	C	57	B
14	E	36	A	58	C
15	A	37	D	59	C
16	E	38	A	60	A
17	B	39	C	61	A
18	D	40	D	62	B
19	D	41	C	63	D
20	A	42	B	64	C
21	D	43	C	65	D
22	C	44	A	66	E

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