

GRE Mathematics Practice Test 49

Bridge to Standard

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A bridge toward standard practice, with routine multi-step questions and a smaller number of challenging concepts. This is cycle 9 of 19, position 4 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Evaluate $\int_C (3x^2 + 10y) dx + (10x + 3y^2) dy$, where C is the line segment from $(0, 0)$ to $(5, 1)$.

- (A) 88
- (B) -176
- (C) 176
- (D) 352
- (E) 177

2. Let $A = \begin{bmatrix} 1 & 45 & 0 \\ 0 & 1 & 8 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 2)$ entry of A^4 ?

- (A) 180
- (B) 178
- (C) 179
- (D) 181
- (E) 182

3. Evaluate $\int_0^1 \frac{x^2}{1 + 10x^3} dx$.

- (A) $-\frac{\log(11)}{30}$
- (B) $\frac{\log(11)}{30}$
- (C) $\frac{\log(11)}{10}$
- (D) $\log(11)$
- (E) $\frac{\log(11)}{3}$

Questions 4–6

4. X, Y are independent Poisson variables with means 21 and 22. Given $X + Y = 23$, find $\text{Var}(X | X + Y)$.

- (A) $\frac{5313}{1849}$
- (B) $\frac{21252}{1849}$
- (C) $\frac{14324}{1849}$
- (D) $\frac{10626}{1849}$
- (E) $\frac{12475}{1849}$

5. Evaluate $\int_0^1 \int_0^x x^{10} y^3 dy dx$.

- (A) $\frac{1}{56}$
- (B) $\frac{1}{75}$
- (C) $\frac{1}{15}$
- (D) $\frac{1}{60}$
- (E) $\frac{1}{44}$

6. An invertible 51×51 matrix A has determinant 3. Find $\det(A^T A)$.

- (A) 10
- (B) 8
- (C) 7
- (D) 11
- (E) 9

Questions 7–9

7. Find the arc length of $r(t) = (10t, 3t)$ for $0 \leq t \leq 1$.

- (A) $2\sqrt{109}$
- (B) $\frac{\sqrt{109}}{2}$
- (C) $1 + \sqrt{109}$
- (D) $-\sqrt{109}$
- (E) $\sqrt{109}$

8. X, Y are independent, with variances 21 and 22. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 13
- (B) 7
- (C) 25
- (D) 19
- (E) 31

9. Find the coefficient of x^3 in the Taylor series of $(1 - 10x)^{-2}$ at zero.

- (A) $\frac{3}{4} (4 \cdot 10^3)$
- (B) $-4 \cdot 10^3$
- (C) $\frac{1}{4} (4 \cdot 10^3)$
- (D) $\frac{1}{10} (4 \cdot 10^3)$
- (E) $4 \cdot 10^3$

Questions 10–12

10. On the space P_{52} of real polynomials of degree at most 52, define $T(p) = p'$. What is the rank of T ?

- (A) 53
- (B) 52
- (C) 50
- (D) 54
- (E) 51

11. Evaluate $\lim_{x \rightarrow 0} (1 + 10x)^{1/(2x)}$.

- (A) $e^{\frac{1}{5}}$
- (B) e^{100}
- (C) 10
- (D) e^5
- (E) e^{11}

12. X has density cx^{21} on $0 < x < 1$ and zero elsewhere. Find the normalizing constant c .

- (A) 36
- (B) 29
- (C) 22
- (D) 15
- (E) 8

Questions 13–15

13. Let $F(x) = \int_0^{x^3} e^{10t} dt$. Find $F'(1)$.

- (A) e^{10}
- (B) $30e^{10}$
- (C) $\frac{e^{10}}{3}$
- (D) $3e^{10}$
- (E) e^{30}

14. Subspaces U, V of $\mathbb{R}^{53}$ have dimensions 5 and 6. What is the largest possible value of $\dim(U \cap V)$?

- (A) 5
- (B) 4
- (C) 7
- (D) 3
- (E) 6

15. Find the total outward flux of $\mathbf{F} = (10x^3, y^3, z^3)$ through the boundary of $[0, 1]^3$.

- (A) 12
- (B) 24
- (C) -12
- (D) 6
- (E) 13

Questions 16–18

16. Independent exponential waiting times X, Y have rates 21 and 23. Find $P(X < Y)$.

- (A) $\frac{65}{88}$
- (B) $\frac{21}{44}$
- (C) $\frac{23}{44}$
- (D) $\frac{7}{44}$
- (E) $\frac{21}{88}$

17. Solve $y' + 10y = x^3e^{-10x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{1}{4e^{10}}$
- (B) e^{-10}
- (C) $\frac{1}{3e^{10}}$
- (D) $\frac{5}{2e^{10}}$
- (E) $\frac{e^{10}}{4}$

18. Let V consist of real polynomials of degree at most 8 that vanish at 45, 46, and 48. Find $\dim V$.

- (A) 5
- (B) 6
- (C) 4
- (D) 8
- (E) 7

Questions 19–21

19. Evaluate $\iiint_B 10(x^2 + y^2 + z^2)^{3/2} dV$, where B is the unit ball.

- (A) $\frac{40\pi}{3}$
- (B) $1 + \frac{20\pi}{3}$
- (C) $\frac{10\pi}{3}$
- (D) $-\frac{20\pi}{3}$
- (E) $\frac{20\pi}{3}$

20. $X_1, \dots, X_{21}$ are independent with common mean and variance 23. With $\bar{X}$ their sample mean, find $E[\sum_i (X_i - \bar{X})^2]$.

- (A) 154
- (B) 460
- (C) 766
- (D) 613
- (E) 307

21. Let C be the unit circle $x^2 + y^2 = 1$, $z = 3$, counterclockwise viewed from above. Evaluate $\int_C -10y(x^2 + y^2)^2 dx + (10x(x^2 + y^2)^2) dy$.

- (A) 10π
- (B) 20π
- (C) $1 + 20\pi$
- (D) 40π
- (E) -20π

Questions 22–24

22. The roots r, s of $x^2 - 53x + 360 = 0$ are counted with multiplicity. Find $r^2 + s^2$.

- (A) 2087
- (B) 2088
- (C) 2090
- (D) 2089
- (E) 2091

23. The solution of $y' = 10x^3y^2$ has $y(0) = 1/11$. Find $y(1)$.

- (A) $\frac{2}{17}$
- (B) $-\frac{2}{17}$
- (C) $\frac{19}{17}$
- (D) $\frac{4}{17}$
- (E) $\frac{1}{17}$

24. A continuous $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(q) = q^{16}$ for every rational q . Find $f(2)$.

- (A) 2^{15}
- (B) 2^{17}
- (C) 2^{18}
- (D) 2^{16}
- (E) 2^{19}

Questions 25–27

25. Evaluate $\int_0^1 \int_x^1 10y^3 \, dy \, dx$.

- (A) $\frac{5}{2}$
- (B) $\frac{1}{2}$
- (C) 2
- (D) 5
- (E) $\frac{10}{3}$

26. For real t , let $M(t) = \begin{pmatrix} t-45 & 8 & 0 \\ 0 & t-45 & 1 \\ 0 & 0 & t-45 \end{pmatrix}$. At $t = 45$, what is $\text{rank } M(t)$?

- (A) 4
- (B) 0
- (C) 2
- (D) 3
- (E) 1

27. For $y' = 10y(1 - y/3)$, what is the derivative of the right-hand side with respect to y at the positive equilibrium?

- (A) 10
- (B) -10
- (C) -20
- (D) -5
- (E) -9

Questions 28–30

28. Find the radius of convergence of $\sum_{k=1}^{\infty} k^{16} x^k / 16^k$.

- (A) 6
- (B) 16
- (C) 26
- (D) 11
- (E) 21

29. Find the directional derivative of $f(x, y, z) = x^2 + 10y^2 + 3z^2$ at $(1, 1, 1)$ in the direction $(1, 5, 0)$.

- (A) $1 + \frac{51\sqrt{26}}{13}$
- (B) $\frac{51\sqrt{26}}{26}$
- (C) $-\frac{51\sqrt{26}}{13}$
- (D) $\frac{102\sqrt{26}}{13}$
- (E) $\frac{51\sqrt{26}}{13}$

30. Let $u = (9, 3)^T$ and $v = (1, 0)^T$. Find the squared norm of the orthogonal projection of v onto $\text{span}(u)$.

- (A) $\frac{2}{5}$
- (B) $\frac{19}{10}$
- (C) $\frac{12}{5}$
- (D) $\frac{7}{5}$
- (E) $\frac{9}{10}$

Questions 31–33

31. If $y' = 10y$, $y(0) = 1$, what is the coefficient of x^3 in the Maclaurin series of y ?

- (A) $-\frac{500}{3}$
- (B) $\frac{1000}{3}$
- (C) $\frac{500}{3}$
- (D) $\frac{250}{3}$
- (E) $\frac{503}{3}$

32. On $[0, 1]$, f is 16 on rationals and $1/16$ on irrationals. Find its upper Darboux integral minus its lower Darboux integral.

- (A) $\frac{255}{32}$
- (B) $\frac{255}{16}$
- (C) $\frac{255}{8}$
- (D) $\frac{287}{16}$
- (E) $\frac{271}{16}$

33. If $y'' + 9y = 0$, $y(0) = 10$, and $y'(0) = 5$, find $y(\pi/6)$.

- (A) $\frac{8}{3}$
- (B) $-\frac{5}{3}$
- (C) $\frac{10}{3}$
- (D) $\frac{5}{6}$
- (E) $\frac{5}{3}$

Questions 34–36

34. An idempotent 51×51 real matrix P has rank 3. What is $\text{tr } P$?

- (A) 3
- (B) 5
- (C) 1
- (D) 4
- (E) 2

35. Let C be $x^2/9 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-y) dx + (10x) dy$.

- (A) 33π
- (B) -33π
- (C) $\frac{33\pi}{2}$
- (D) $1 + 33\pi$
- (E) 66π

36. A set with 16 elements has the discrete topology. How many of its subsets are simultaneously open and closed?

- (A) 2^{16}
- (B) 2^{15}
- (C) 2^{18}
- (D) 2^{19}
- (E) 2^{17}

Questions 37–39

37. For $f(x, y) = x^2 + 10xy + 3y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) 12
- (B) 4
- (C) -8
- (D) $4\sqrt{5}$
- (E) 80

38. For $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $b = \begin{bmatrix} 45 \\ 8 \\ 53 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find x_2 .

- (A) 7
- (B) 6
- (C) 8
- (D) 9
- (E) 10

39. Evaluate $\int_0^{\pi/2} \sin^7 t \cos^{21} t \, dt$.

- (A) $\frac{3! 10!}{2 \cdot 14!}$
- (B) $14 \left(\frac{3! 10!}{2 \cdot 14!} \right)$
- (C) $286 \left(\frac{3! 10!}{2 \cdot 14!} \right)$
- (D) $2 \left(\frac{3! 10!}{2 \cdot 14!} \right)$
- (E) $\frac{1}{2} \left(\frac{3! 10!}{2 \cdot 14!} \right)$

Questions 40–42

40. The circle $|z| = 2$ is traversed counterclockwise. What is $\frac{1}{2\pi i} \int_{|z|=2} \frac{e^z}{z^{17}} dz$?

- (A) $1/16!$
- (B) $16!$
- (C) $1/17!$
- (D) $1/32!$
- (E) $1/15!$

41. What is the maximum of x^3y^{10} over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\frac{30}{169}$
- (B) $\frac{1}{2^{13}}$
- (C) $\left(\frac{3}{13}\right)^{10} \left(\frac{10}{13}\right)^3$
- (D) $13 \left(\left(\frac{3}{13}\right)^3 \left(\frac{10}{13}\right)^{10} \right)$
- (E) $\left(\frac{3}{13}\right)^3 \left(\frac{10}{13}\right)^{10}$

42. What is the coefficient of x^3 in $(x + 9)^5$?

- (A) 811
- (B) 809
- (C) 808
- (D) 812
- (E) 810

Questions 43–45

43. The region $0 \leq x \leq 1$, $0 \leq y \leq 10x^3$ is revolved about the x -axis. Find its volume.

- (A) $\frac{50\pi}{7}$
- (B) $\frac{100\pi}{7}$
- (C) $\frac{200\pi}{7}$
- (D) $-\frac{100\pi}{7}$
- (E) $1 + \frac{100\pi}{7}$

44. Find the residue of $z^{16}/(z-1)^2$ at $z=1$.

- (A) 21
- (B) 26
- (C) 16
- (D) 11
- (E) 6

45. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(10j)!}{(j!)^{10}} x^{3j}$?

- (A) 10^{-10}
- (B) $10^{\frac{10}{3}}$
- (C) $10^{-\frac{10}{3}}$
- (D) $10^{-\frac{3}{10}}$
- (E) $\frac{1}{10}$

Questions 46–48

46. The field K has 3^{10} elements. Find $[K : \mathbb{F}_3]$.

- (A) 8
- (B) 12
- (C) 10
- (D) 9
- (E) 11

47. Evaluate $\int_0^{\log 2} e^x(e^x + 10)^3 dx$.

- (A) $12^3 - 11^3$
- (B) $-\frac{12^4 - 11^4}{4}$
- (C) $4\left(\frac{12^4 - 11^4}{4}\right)$
- (D) $\frac{12^4}{4}$
- (E) $\frac{12^4 - 11^4}{4}$

48. How many integers in $\{1, \dots, 126\}$ are divisible by 2 or 3?

- (A) 112
- (B) 56
- (C) 84
- (D) 28
- (E) 140

Questions 49–51

49. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 10z^2 = x + 3y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) -30
- (B) -20
- (C) -3
- (D) -60
- (E) 60

50. Let $G = \mathbb{Z}/300\mathbb{Z} \times \mathbb{Z}/204\mathbb{Z}$. Find the order of $(1, 1)$.

- (A) 5100
- (B) 5102
- (C) 5099
- (D) 5101
- (E) 5098

51. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^{10}y^2 dV$.

- (A) $\frac{1}{6} \left(\frac{10! 2!}{15!} \right)$
- (B) $15 \left(\frac{10! 2!}{15!} \right)$
- (C) $6 \left(\frac{10! 2!}{15!} \right)$
- (D) $\frac{10! 2!}{15!}$
- (E) $\frac{1}{33}$

Questions 52–54

52. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 21 have no adjacent 1s?

- (A) F_{21}
- (B) F_{24}
- (C) F_{23}
- (D) F_{22}
- (E) F_{25}

53. For $\mathbf{F} = (10x^3, y^3, z^3)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) 45
- (B) -45
- (C) $\frac{45}{2}$
- (D) 90
- (E) 46

54. How many group homomorphisms $\mathbb{Z}/300\mathbb{Z} \rightarrow \mathbb{Z}/204\mathbb{Z}$ are there?

- (A) 10
- (B) 13
- (C) 12
- (D) 11
- (E) 14

Questions 55–57

55. For $f(x) = x^3 e^{10x}$, find $f^{(5)}(0)$.

- (A) $\frac{1}{20} \left(\frac{5! 10^2}{2} \right)$
- (B) $2 \left(\frac{5! 10^2}{2} \right)$
- (C) $\frac{1}{5} \left(\frac{5! 10^2}{2} \right)$
- (D) $\frac{1}{60} \left(\frac{5! 10^2}{2} \right)$
- (E) $\frac{5! 10^2}{2}$

56. Apply one Newton iteration to solve $x^2 - 21 = 0$, starting at $x_0 = 1$. What is x_1 ?

- (A) 17
- (B) 14
- (C) 11
- (D) 5
- (E) 8

57. Evaluate $\int_0^1 x^{10}(1-x)^2 dx$.

- (A) $26 \left(\frac{10! 2!}{13!} \right)$
- (B) $\frac{10! 2!}{13!}$
- (C) $\frac{1}{3} \left(\frac{10! 2!}{13!} \right)$
- (D) $11 \left(\frac{10! 2!}{13!} \right)$
- (E) $13 \left(\frac{10! 2!}{13!} \right)$

Questions 58–60

58. How many 1-dimensional subspaces does $\mathbb{F}_{23}^4$ have?

- (A) $\frac{23^5 - 1}{23 - 1}$
- (B) $\frac{23^4 - 1}{23 - 1}$
- (C) $\frac{23^6 - 1}{23 - 1}$
- (D) $\frac{23^7 - 1}{23 - 1}$
- (E) $\frac{23^3 - 1}{23 - 1}$

59. Let $f(x, y) = e^{10x+3y}$. Find $\partial^3 f / \partial x^2 \partial y$ at $(0, 0)$.

- (A) $3 \cdot 10^2$
- (B) $\frac{1}{3} (3 \cdot 10^2)$
- (C) $\frac{9}{100} (3 \cdot 10^2)$
- (D) $\frac{10}{3} (3 \cdot 10^2)$
- (E) $\frac{3}{10} (3 \cdot 10^2)$

60. Bisection starts with an interval of length 2^{21} . After 23 interval halvings, what is the maximum distance from the final midpoint to a root remaining in the interval?

- (A) $\frac{9}{16}$
- (B) $\frac{1}{24}$
- (C) $\frac{1}{16}$
- (D) $\frac{7}{8}$
- (E) $\frac{1}{8}$

Questions 61–63

61. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j (x-10)^j}{j^{\frac{2}{3}} (\log j)^m}$ converge absolutely at $x = 9$? Give the least such m , or 0 if none exists.

- (A) 2
- (B) 3
- (C) 4
- (D) 1
- (E) 0

62. How many monic irreducible polynomials of degree 2 are there over $\mathbb{F}_{23}$?

- (A) 251
- (B) 254
- (C) 252
- (D) 253
- (E) 255

63. Find the coefficient of x^3 in $e^{2x} \log(1 + 10x)$.

- (A) $-\frac{760}{3}$
- (B) $\frac{760}{3}$
- (C) 1520
- (D) $\frac{1000}{3}$
- (E) $\frac{790}{3}$

Questions 64–66

64. What is the least positive multiplicative inverse of 2 modulo 43?

- (A) 36
- (B) 15
- (C) 29
- (D) 22
- (E) 8

65. Evaluate $\int_0^1 x^9(\log x)^2 dx$.

- (A) $10 \left(\frac{(-1)^2 2!}{10^3} \right)$
- (B) $\frac{(-1)^2 2!}{10^3}$
- (C) $\frac{1}{3} \left(\frac{(-1)^2 2!}{10^3} \right)$
- (D) $50 \left(\frac{(-1)^2 2!}{10^3} \right)$
- (E) $-\frac{(-1)^2 2!}{10^3}$

66. How many elements z of $\mathbb{F}_{310}$ satisfy $z^3 = z$?

- (A) 5
- (B) 4
- (C) 1
- (D) 3
- (E) 2

Answer Key

Practice Test 49 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	C	23	A	45	C
2	A	24	D	46	C
3	B	25	C	47	E
4	D	26	C	48	C
5	D	27	B	49	D
6	E	28	B	50	A
7	E	29	E	51	D
8	D	30	E	52	C
9	E	31	C	53	A
10	B	32	B	54	C
11	D	33	E	55	E
12	C	34	A	56	C
13	D	35	A	57	B
14	A	36	A	58	B
15	A	37	D	59	A
16	B	38	C	60	E
17	A	39	A	61	E
18	B	40	A	62	D
19	E	41	E	63	B
20	B	42	E	64	D
21	B	43	B	65	B
22	D	44	C	66	D

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