

GRE Mathematics Practice Test 51

Advanced Challenge

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

The most demanding form in this five-test cycle. Emphasis on connected ideas, non-routine reasoning, and careful conceptual distinctions. This is cycle 10 of 19, position 1 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

zarimion.com/gre

100 practice tests, complete answer keys, and future updates.

Please retain the source attribution when sharing this material.



This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Evaluate $\iiint_{x^2+y^2+z^2 \leq 11} (x^2 + y^2 + z^2)^5 dV$.

- (A) $13 \left(\frac{4\pi 11^{\frac{13}{2}}}{13} \right)$
- (B) $\frac{1}{3} \left(\frac{4\pi 11^{\frac{13}{2}}}{13} \right)$
- (C) $\frac{13}{121} \left(\frac{4\pi 11^{\frac{13}{2}}}{13} \right)$
- (D) $\frac{1}{2} \left(\frac{4\pi 11^{\frac{13}{2}}}{13} \right)$
- (E) $\frac{4\pi 11^{\frac{13}{2}}}{13}$

2. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 423$ and $x + y = 56$. Find $x^2 + y^2$.

- (A) 2289
- (B) 2290
- (C) 2291
- (D) 2292
- (E) 2288

3. Find the coefficient of x^4 in $e^{5x} \log(1 + 11x)$.

- (A) 1969
- (B) $-\frac{14641}{4}$
- (C) -1958
- (D) -47256
- (E) -1969

Questions 4–6

4. A permutation of 20 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z^2]$.
- (A) 0
(B) 2
(C) 4
(D) 3
(E) 1
5. Evaluate $\int_C (6x^5 + 11y) dx + (11x + 6y^5) dy$, where C is $r(t) = (t, 2t^2)$, $0 \leq t \leq 1$.
- (A) 87
(B) $\frac{87}{2}$
(C) 174
(D) 88
(E) -87
6. A sequence satisfies $u_0 = 47$ and $u_{k+1} = 2u_k + 9$ for all $k \geq 0$. Find u_7 .
- (A) 7160
(B) 7159
(C) 7158
(D) 7157
(E) 7161

Questions 7–9

7. Solve $y' + 11y = x^6 e^{-11x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{1}{6e^{11}}$
- (B) $\frac{e^{11}}{7}$
- (C) $\frac{1}{7e^{11}}$
- (D) e^{-11}
- (E) $\frac{11}{7e^{11}}$

8. A coin success probability p has uniform prior on $[0, 1]$. After observing 18 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?

- (A) $\frac{3}{22}$
- (B) $\frac{19}{66}$
- (C) $\frac{41}{44}$
- (D) $\frac{19}{22}$
- (E) $\frac{19}{44}$

9. If $y' + 11y = 6x^6 e^{-11x}$ and $y(0) = 2$, find $y(1)$.

- (A) $\frac{20}{7e^{11}}$
- (B) $\frac{10}{7e^{11}}$
- (C) $\frac{20}{7e^{11}} + 1$
- (D) $\frac{40}{7e^{11}}$
- (E) $-\frac{20}{7e^{11}}$

Questions 10–12

10. Linear maps $T : \mathbb{R}^{54} \rightarrow \mathbb{R}^{54}$ and $U : \mathbb{R}^{54} \rightarrow \mathbb{R}^{54}$ have ranks 3 and 4. What is the largest possible rank of UT ?

- (A) 3
- (B) 1
- (C) 4
- (D) 5
- (E) 2

11. Let $F(x) = \int_0^{x^6} e^{11t} dt$. Find $F'(1)$.

- (A) e^{66}
- (B) $6e^{11}$
- (C) $66e^{11}$
- (D) $\frac{e^{11}}{6}$
- (E) e^{11}

12. A random variable has mean 0 and variance 18. What upper bound does the Chebyshev inequality give for $P(|X| \geq 36)$?

- (A) $\frac{73}{144}$
- (B) $\frac{1}{72}$
- (C) $\frac{1}{216}$
- (D) $\frac{1}{144}$
- (E) $\frac{71}{72}$

Questions 13–15

13. If $y'' + 36y = 11$ and $y(0) = y'(0) = 0$, find $y(\pi/6)$.

- (A) $-\frac{11}{18}$
- (B) $\frac{29}{18}$
- (C) $\frac{11}{18}$
- (D) $\frac{11}{9}$
- (E) $\frac{11}{36}$

14. Fix a nonzero vector $u \in \mathbb{R}^{53}$. What is the dimension of the space of skew-symmetric 53×53 matrices A satisfying $Au = 0$?

- (A) 1325
- (B) 1327
- (C) 1328
- (D) 1326
- (E) 1324

15. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(11+6j)(17+6j)}$.

- (A) $\frac{6}{11}$
- (B) $\frac{1}{66}$
- (C) $\frac{1}{6}$
- (D) $\frac{1}{17}$
- (E) $\frac{1}{11}$

Questions 16–18

16. A box is chosen uniformly from two boxes. Box A contains 23 red and 25 blue balls; box B contains 25 red and 23 blue balls. Two balls are drawn without replacement and both are red. What is the conditional probability that A was chosen?

- (A) $\frac{253}{1659}$
- (B) $\frac{403}{553}$
- (C) $\frac{253}{1106}$
- (D) $\frac{300}{553}$
- (E) $\frac{253}{553}$

17. For the curve $x = t^6$, $y = t^7 + 11t$, find d^2y/dx^2 at $t = 1$.

- (A) -8
- (B) $-\frac{4}{3}$
- (C) $\frac{31}{18}$
- (D) 3
- (E) $-\frac{11}{6}$

18. Let $A = \begin{bmatrix} 1 & 47 & 0 \\ 0 & 1 & 9 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 3)$ entry of A^2 ?

- (A) 425
- (B) 424
- (C) 421
- (D) 422
- (E) 423

Questions 19–21

19. The region $0 \leq x \leq 1$, $0 \leq y \leq 11x^6$ is revolved about the line $x = -2$. Find its volume.
- (A) $1 + \frac{253\pi}{28}$
(B) $-\frac{253\pi}{28}$
(C) $\frac{253\pi}{56}$
(D) $\frac{253\pi}{28}$
(E) $\frac{253\pi}{14}$
20. Let $X_1, \dots, X_{23}$ be independent uniform random variables on $[0, 1]$. What is $E[(\max_i X_i)^2]$?
- (A) $\frac{23}{25}$
(B) $\frac{23}{50}$
(C) $\frac{24}{25}$
(D) $\frac{2}{25}$
(E) $\frac{23}{75}$
21. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^{11}y^5 dV$.
- (A) $\frac{1}{6} \left(\frac{11!5!}{19!} \right)$
(B) $6 \left(\frac{11!5!}{19!} \right)$
(C) $\frac{1}{72}$
(D) $19 \left(\frac{11!5!}{19!} \right)$
(E) $\frac{11!5!}{19!}$

Questions 22–24

22. An invertible 53×53 matrix A has determinant 2. Find $\det(\text{adj } A)$.

- (A) 2^{51}
- (B) $2^{52} \cdot 5$
- (C) 2^{53}
- (D) 2^{52}
- (E) $2^{52} \cdot 3$

23. Find the arc length of $r(t) = (11(t - t^3/3), 11t^2)$ for $0 \leq t \leq 1/6$.

- (A) $\frac{1199}{1296}$
- (B) $-\frac{1199}{648}$
- (C) $\frac{1199}{648}$
- (D) $\frac{1847}{648}$
- (E) $\frac{1199}{324}$

24. For which real a does $f_m(x) = m^a x^{18}(1 - x)^m$ converge uniformly to zero on $[0, 1]$?

- (A) $a < 18$
- (B) $a \leq 18$
- (C) $a < 0$
- (D) All real a
- (E) $a > 18$

Questions 25–27

25. Evaluate $\int_0^1 \int_0^x x^{11} y^6 dy dx$.

- (A) $\frac{1}{152}$
- (B) $\frac{1}{126}$
- (C) $\frac{1}{19}$
- (D) $\frac{1}{84}$
- (E) $\frac{1}{133}$

26. On P_{54} , the real polynomials of degree at most 54, define $T(p) = x^2 p'' - 16xp' + 72p$. Find its rank.

- (A) 55
- (B) 54
- (C) 51
- (D) 53
- (E) 52

27. Under $x = u + 11v$, $y = u - 6v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) 289
- (B) $\frac{1}{17}$
- (C) 66
- (D) 5
- (E) 17

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{18p}]$ converge?

- (A) All real p
- (B) $p \geq 1/18$
- (C) $p > 1/18$
- (D) $p > 0$
- (E) $p < 1/18$

29. Find the curvature of the helix $r(t) = (11 \cos t, 11 \sin t, 6t)$.

- (A) $\frac{11}{157}$
- (B) $-\frac{11}{157}$
- (C) $\frac{11}{314}$
- (D) $\frac{168}{157}$
- (E) $\frac{22}{157}$

30. Subspaces U, V of $\mathbb{R}^{55}$ have dimensions 4 and 5. What is the smallest possible value of $\dim(U \cap V)$?

- (A) 4
- (B) 2
- (C) 0
- (D) 1
- (E) 3

Questions 31–33

31. Evaluate $\int_0^1 \int_x^1 11y^6 \, dy \, dx$.

- (A) $\frac{11}{2}$
- (B) $\frac{11}{8}$
- (C) $\frac{11}{6}$
- (D) $\frac{11}{7}$
- (E) $\frac{11}{56}$

32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{18 + 1/k : k \geq 1\}$ have?

- (A) 1
- (B) 2
- (C) 4
- (D) 0
- (E) 3

33. Let $\mathbf{F} = (11y^6, z^6, x^6)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (1, 1, 2)/\sqrt{6}$.

- (A) $-24\sqrt{6}$
- (B) $-48\sqrt{6}$
- (C) $24\sqrt{6}$
- (D) $1 - 24\sqrt{6}$
- (E) $-12\sqrt{6}$

Questions 34–36

34. Let V consist of real polynomials of degree at most 12 that have both value and first derivative zero at 47, 48, and 50. Find $\dim V$.

- (A) 5
- (B) 9
- (C) 8
- (D) 7
- (E) 6

35. For $x > 0$, $x^2y'' + (1 - 17)xy' + 66y = 0$, $y(1) = 0$, $y'(1) = 11$. Find $y(e)$.

- (A) $\frac{11(1 - e^5)e^6}{5}$
- (B) $\frac{e^{11}}{17}$
- (C) $-\frac{11(1 - e^5)e^6}{5}$
- (D) $e^6 + e^{11}$
- (E) $-e^6 + e^{11}$

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 18\}$ have?

- (A) 13
- (B) 31
- (C) 19
- (D) 7
- (E) 25

Questions 37–39

37. For $f(x) = x^6 e^{11x}$, find $f^{(8)}(0)$.

- (A) $\frac{1}{56} \left(\frac{8! 11^2}{2} \right)$
- (B) $\frac{2}{11} \left(\frac{8! 11^2}{2} \right)$
- (C) $\frac{8! 11^2}{2}$
- (D) $\frac{1}{20160} \left(\frac{8! 11^2}{2} \right)$
- (E) $2 \left(\frac{8! 11^2}{2} \right)$

38. The roots r, s of $x^2 - 56x + 423 = 0$ are counted with multiplicity. Find $r^4 + s^4$.

- (A) $56^4 - 4(423)(56^2) + 2(423^2)$
- (B) $56^4 - 4(423)(56^2) + 423^2$
- (C) $56^4 - 4(423)(56^2)$
- (D) $56^4 + 2(423^2)$
- (E) $56^4 - 2(423)(56^2) + 2(423^2)$

39. The local inverse $y(x)$ of $x = 11y + y^3$ satisfies $y(0) = 0$. Find $y^{(7)}(0)$.

- (A) $\frac{60480}{11^{10}}$
- (B) $\frac{1}{5040} \left(-\frac{60480}{11^{10}} \right)$
- (C) $\frac{1}{11} \left(-\frac{60480}{11^{10}} \right)$
- (D) $11 \left(-\frac{60480}{11^{10}} \right)$
- (E) $-\frac{60480}{11^{10}}$

Questions 40–42

40. How many zeros, counted with multiplicity, does $z^{13} + 16z + 1$ have in $|z| < 1$?

- (A) 1
- (B) 2
- (C) 3
- (D) 0
- (E) 4

41. Evaluate $\int_0^{\log 2} e^x (e^x + 11)^6 dx$.

- (A) $13^6 - 12^6$
- (B) $7 \left(\frac{13^7 - 12^7}{7} \right)$
- (C) $\frac{13^7}{7}$
- (D) $\frac{13^7 - 12^7}{7}$
- (E) $-\frac{13^7 - 12^7}{7}$

42. How many group homomorphisms $\mathbb{Z}/312\mathbb{Z} \rightarrow \mathbb{Z}/212\mathbb{Z}$ have image of order 4?

- (A) 0
- (B) 4
- (C) 3
- (D) 2
- (E) 1

Questions 43–45

43. For $\mathbf{F} = (11x^6y, y^6z, xz^6)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) 330
- (B) 165
- (C) 331
- (D) -330
- (E) 660

44. A meromorphic function has 15 zeros and 2 poles inside a positively oriented simple closed contour, counted with multiplicity, and no zeros or poles on it. Find $\frac{1}{2\pi i} \int f'(z)/f(z) dz$.

- (A) 21
- (B) 13
- (C) 5
- (D) 17
- (E) 9

45. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(11x) - \sum_{j=0}^4 (-1)^j (11x)^{2j+1} / (2j+1)!}{x^{11}}$.

- (A) $-\frac{(-1)^5 11^{11}}{11!}$
- (B) $\frac{1}{11} \left(\frac{(-1)^5 11^{11}}{11!} \right)$
- (C) $11 \left(\frac{(-1)^5 11^{11}}{11!} \right)$
- (D) $2 \left(\frac{(-1)^5 11^{11}}{11!} \right)$
- (E) $\frac{(-1)^5 11^{11}}{11!}$

Questions 46–48

46. How many 2-dimensional subspaces does $\mathbb{F}_{29}^7$ have?

- (A) $\frac{(29^6 - 1)(29^6 - 29)}{(29^2 - 1)(29^2 - 29)}$
- (B) $\frac{(29^9 - 1)(29^9 - 29)}{(29^2 - 1)(29^2 - 29)}$
- (C) $\frac{(29^8 - 1)(29^8 - 29)}{(29^2 - 1)(29^2 - 29)}$
- (D) $\frac{(29^7 - 1)(29^7 - 29)}{(29^2 - 1)(29^2 - 29)}$
- (E) $\frac{(29^{10} - 1)(29^{10} - 29)}{(29^2 - 1)(29^2 - 29)}$

47. Let C be $x^2/36 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C \left(-\frac{11y^3}{3}\right) dx + \left(\frac{x^3}{3}\right) dy$.

- (A) $\frac{141\pi}{4}$
- (B) $-\frac{141\pi}{2}$
- (C) 141π
- (D) $1 + \frac{141\pi}{2}$
- (E) $\frac{141\pi}{2}$

48. How many truth assignments to $p, q, r_1, \dots, r_{16}$ make $p \Rightarrow q$ false?

- (A) 2^{15}
- (B) 2^{17}
- (C) 2^{16}
- (D) 2^{19}
- (E) 2^{18}

Questions 49–51

49. For $f(x, y) = x^2 + 11xy + 6y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) -1
- (B) -9
- (C) $\sqrt{82}$
- (D) 82
- (E) 10

50. How many monic irreducible polynomials of degree 5 are there over $\mathbb{F}_{29}$?

- (A) $2(29^5 - 29)$
- (B) $\frac{2(29^5 - 29)}{5}$
- (C) $29^5 - 29$
- (D) $\frac{29^5 - 29}{5}$
- (E) $\frac{3(29^5 - 29)}{5}$

51. Two solutions of $y'' + 11x^6y' + 6y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $\frac{2}{e^{\frac{11}{7}}}$
- (B) $e^{-\frac{11}{7}} + 1$
- (C) $e^{-\frac{11}{7}}$
- (D) $-\frac{1}{e^{\frac{11}{7}}}$
- (E) $\frac{1}{2e^{\frac{11}{7}}}$

Questions 52–54

52. Find the coefficient of x^3 in $(1+x)^{18}(1-x)^{18}(1+2x)^2$.

- (A) -96
- (B) -120
- (C) -24
- (D) -72
- (E) -48

53. Near $(0,0)$, $z = z(x,y)$ satisfies $z + 11z^2 = x + 6y$ and $z(0,0) = 0$. Find $z_{xy}(0,0)$.

- (A) -132
- (B) -22
- (C) -6
- (D) 132
- (E) -66

54. How many elements z of $\mathbb{F}_{11^{11}}$ satisfy $z^{121} = z$?

- (A) 11
- (B) 10
- (C) 9
- (D) 12
- (E) 13

Questions 55–57

55. Evaluate $\int_0^\infty x^6 e^{-11x} dx$.

- (A) $\frac{1}{6} \left(\frac{6!}{11^7} \right)$
- (B) 720
- (C) $11 \left(\frac{6!}{11^7} \right)$
- (D) $\frac{1}{11} \left(\frac{6!}{11^7} \right)$
- (E) $\frac{6!}{11^7}$

56. How many distinct unoriented cycles of length 4 are in $K_{18,20}$?

- (A) 9690
- (B) 19380
- (C) 29070
- (D) 38760
- (E) 48450

57. Evaluate $\iiint_B 11z^2(x^2 + y^2 + z^2)^6 dV$, where B is the unit ball.

- (A) $-\frac{44\pi}{51}$
- (B) $\frac{22\pi}{51}$
- (C) $1 + \frac{44\pi}{51}$
- (D) $\frac{44\pi}{51}$
- (E) $\frac{88\pi}{51}$

Questions 58–60

58. How many generators does the cyclic group of order 594823321 have?

- (A) $29^8(29 - 1)$
- (B) $29^6(29 - 1)$
- (C) $29^4(29 - 1)$
- (D) $29^7(29 - 1)$
- (E) $29^5(29 - 1)$

59. Evaluate $\lim_{x \rightarrow 0} \frac{e^{11x} - \sum_{j=0}^5 (11x)^j / j!}{x^6}$.

- (A) $720 \left(\frac{11^6}{6!} \right)$
- (B) $6 \left(\frac{11^6}{6!} \right)$
- (C) $\frac{11^6}{6!}$
- (D) $\frac{1}{11} \left(\frac{11^6}{6!} \right)$
- (E) $-\frac{11^6}{6!}$

60. How many functions from a set of 23 elements onto a set of 3 elements exist?

- (A) $3^{23} - 3(2^{23}) - 3$
- (B) 3^{23}
- (C) $3^{23} - 3(2^{23}) + 3$
- (D) $3^{23} - 3(2^{23})$
- (E) $3^{23} - 2^{23}$

Questions 61–63

61. Let C be the unit circle $x^2 + y^2 = 1$, $z = 6$, counterclockwise viewed from above. Evaluate $\int_C -11y(x^2 + y^2)^5 dx + (11x(x^2 + y^2)^5) dy + 2z dz$.

- (A) 22π
- (B) 11π
- (C) 44π
- (D) -22π
- (E) $1 + 22\pi$

62. Let $R = (\mathbb{Z}/72\mathbb{Z})/([12])$, where $([12])$ is the ideal generated by $[12]$. How many units does R have?

- (A) 3
- (B) 4
- (C) 6
- (D) 5
- (E) 2

63. Evaluate $\int_0^1 x^{10}(\log x)^5 dx$.

- (A) $-\frac{1}{11}$
- (B) $\frac{(-1)^5 5!}{11^6}$
- (C) $11 \left(\frac{(-1)^5 5!}{11^6} \right)$
- (D) $\frac{1}{6} \left(\frac{(-1)^5 5!}{11^6} \right)$
- (E) $-\frac{(-1)^5 5!}{11^6}$

Questions 64–66

64. How many nonnegative integer triples sum to 46, if each coordinate is at most 23?

- (A) 200
- (B) 400
- (C) 500
- (D) 100
- (E) 300

65. If $y' = y(1 - y/6)$ and $y(0) = 3$, find $y(-\log 11)$.

- (A) $-\frac{1}{2}$
- (B) $\frac{1}{2}$
- (C) 1
- (D) $\frac{1}{4}$
- (E) $\frac{3}{2}$

66. What is the largest integer k such that 3^k divides $56!$?

- (A) 24
- (B) 28
- (C) 25
- (D) 26
- (E) 27

Answer Key

Practice Test 51 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	E	23	C	45	E
2	B	24	A	46	D
3	E	25	E	47	E
4	B	26	D	48	C
5	A	27	E	49	C
6	B	28	C	50	D
7	C	29	A	51	C
8	D	30	C	52	D
9	A	31	B	53	A
10	A	32	B	54	A
11	B	33	A	55	E
12	B	34	D	56	C
13	C	35	C	57	D
14	D	36	C	58	E
15	B	37	C	59	C
16	E	38	A	60	C
17	B	39	E	61	A
18	E	40	A	62	B
19	D	41	D	63	B
20	A	42	D	64	E
21	E	43	A	65	B
22	D	44	B	66	D

zarimion.com/gre

100 practice tests, complete answer keys, and future updates.
Please retain the source attribution when sharing this material.

