

GRE Mathematics Practice Test 52

Advanced Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A demanding practice form with frequent multi-step arguments and a mix of computational and conceptual questions. This is cycle 10 of 19, position 2 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Under $x = u + 11v$, $y = u - 5v$, what is the area of the image of $0 \leq u, v \leq 1$?
- (A) 55
(B) $\frac{1}{16}$
(C) 256
(D) 6
(E) 16
2. An invertible 54×54 matrix A has determinant 3. Find $\det(\text{adj } A)$.
- (A) 3^{53}
(B) $3^{53} \cdot 5$
(C) $2 \cdot 3^{53}$
(D) 3^{54}
(E) 3^{52}
3. For $f(x, y) = x^2 + 11xy + 5y^2$, what is its maximum directional derivative at $(1, -1)$?
- (A) $\sqrt{82}$
(B) 82
(C) 10
(D) -9
(E) 1

Questions 4–6

4. A permutation of 21 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z^2]$.
- (A) 3
(B) 4
(C) 0
(D) 2
(E) 1
5. The region $0 \leq x \leq 1$, $0 \leq y \leq 11x^5$ is revolved about the line $x = -3$. Find its volume.
- (A) $\frac{198\pi}{7}$
(B) $\frac{99\pi}{14}$
(C) $-\frac{99\pi}{7}$
(D) $1 + \frac{99\pi}{7}$
(E) $\frac{99\pi}{7}$
6. On P_{55} , the real polynomials of degree at most 55, define $T(p) = x^2p'' - 16xp' + 72p$. Find its rank.
- (A) 53
(B) 52
(C) 54
(D) 55
(E) 56

Questions 7–9

7. If $y' = 22xy$ and $y(0) = 1$, find the coefficient of x^{10} in the Maclaurin series of y .

- (A) $\frac{161171}{120}$
(B) $\frac{161051}{120}$
(C) $\frac{161051}{60}$
(D) $\frac{161051}{240}$
(E) $-\frac{161051}{120}$

8. A coin success probability p has uniform prior on $[0, 1]$. After observing 19 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?

- (A) $\frac{20}{69}$
(B) $\frac{10}{23}$
(C) $\frac{3}{23}$
(D) $\frac{43}{46}$
(E) $\frac{20}{23}$

9. If $y'' - 22y' + 121y = 5e^{11x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $1 + \frac{5e^{11}}{2}$
(B) $-\frac{5e^{11}}{2}$
(C) $\frac{5e^{11}}{4}$
(D) $\frac{5e^{11}}{2}$
(E) $5e^{11}$

Questions 10–12

10. Subspaces U, V of $\mathbb{R}^{56}$ have dimensions 4 and 5. What is the smallest possible value of $\dim(U \cap V)$?

- (A) 2
- (B) 4
- (C) 3
- (D) 0
- (E) 1

11. Evaluate $\int_0^1 x^{10}(\log x)^4 dx$.

- (A) $\frac{(-1)^4 4!}{11^5}$
- (B) $\frac{1}{5} \left(\frac{(-1)^4 4!}{11^5} \right)$
- (C) $-\frac{(-1)^4 4!}{11^5}$
- (D) $\frac{14641}{24} \left(\frac{(-1)^4 4!}{11^5} \right)$
- (E) $11 \left(\frac{(-1)^4 4!}{11^5} \right)$

12. A random variable has mean 0 and variance 19. What upper bound does the Chebyshev inequality give for $P(|X| \geq 38)$?

- (A) $\frac{75}{76}$
- (B) $\frac{1}{228}$
- (C) $\frac{77}{152}$
- (D) $\frac{1}{152}$
- (E) $\frac{1}{76}$

Questions 13–15

13. If $y'' + 25y = 11$ and $y(0) = y'(0) = 0$, find $y(\pi/5)$.

- (A) $\frac{11}{25}$
- (B) $\frac{47}{25}$
- (C) $-\frac{22}{25}$
- (D) $\frac{22}{25}$
- (E) $\frac{44}{25}$

14. Let V consist of real polynomials of degree at most 11 that have both value and first derivative zero at 48, 49, and 51. Find $\dim V$.

- (A) 6
- (B) 5
- (C) 7
- (D) 4
- (E) 8

15. Evaluate $\int_0^1 \int_x^1 11y^5 \, dy \, dx$.

- (A) $\frac{11}{5}$
- (B) $\frac{11}{2}$
- (C) $\frac{11}{7}$
- (D) $\frac{11}{6}$
- (E) $\frac{11}{42}$

Questions 16–18

16. A box is chosen uniformly from two boxes. Box A contains 24 red and 26 blue balls; box B contains 26 red and 24 blue balls. Two balls are drawn without replacement and both are red. What is the conditional probability that A was chosen?

- (A) $\frac{325}{601}$
- (B) $\frac{276}{601}$
- (C) $\frac{138}{601}$
- (D) $\frac{92}{601}$
- (E) $\frac{877}{1202}$

17. Let $f(x) = \log(1 + 11x)$. Find $f^{(5)}(0)$.

- (A) $5 \left((-1)^4 4! 11^5 \right)$
- (B) $-(-1)^4 4! 11^5$
- (C) $\frac{1}{5} \left((-1)^4 4! 11^5 \right)$
- (D) $(-1)^4 4! 11^5$
- (E) $\frac{1}{11} \left((-1)^4 4! 11^5 \right)$

18. The roots r, s of $x^2 - 55x + 336 = 0$ are counted with multiplicity. Find $r^3 + s^3$.

- (A) 110935
- (B) 110934
- (C) 110933
- (D) 110937
- (E) 110936

Questions 19–21

19. The solution of $y' = 11x^4y^2$, $y(0) = 1$, has its first positive singularity at which x ?
- (A) $\frac{5}{11}$
(B) $\frac{11^{\frac{4}{5}}}{11}$
(C) $\frac{11^{\frac{4}{5}}\sqrt[5]{5}}{11}$
(D) $\frac{22^{\frac{4}{5}}\sqrt[5]{5}}{22}$
(E) $\frac{\sqrt[5]{11} \cdot 5^{\frac{4}{5}}}{5}$
20. Let $X_1, \dots, X_{24}$ be independent uniform random variables on $[0, 1]$. What is $E[(\max_i X_i)^2]$?
- (A) $\frac{12}{13}$
(B) $\frac{25}{26}$
(C) $\frac{6}{13}$
(D) $\frac{4}{13}$
(E) $\frac{1}{13}$
21. For which positive integer m does $\sum_{j=2}^{\infty} \frac{(-1)^j(x-11)^j}{j^{\frac{4}{3}}(\log j)^m}$ converge absolutely at $x = 10$? Give the least such m , or 0 if none exists.
- (A) 0
(B) 3
(C) 4
(D) 2
(E) 1

Questions 22–24

22. For real t , let $M(t) = \begin{pmatrix} t-48 & 7 & 0 \\ 0 & t-48 & 1 \\ 0 & 0 & t-48 \end{pmatrix}$. At $t = 48$, what is $\dim \ker(M(t)^2) - \dim \ker M(t)$?

- (A) 0
- (B) 2
- (C) 3
- (D) 1
- (E) 4

23. For the curve $x = t^5$, $y = t^6 + 11t$, find d^2y/dx^2 at $t = 1$.

- (A) 2
- (B) $\frac{17}{5}$
- (C) $-\frac{38}{25}$
- (D) $-\frac{38}{5}$
- (E) $-\frac{11}{5}$

24. For which real a does $f_m(x) = m^a x^{19}(1-x)^m$ converge uniformly to zero on $[0, 1]$?

- (A) $a \leq 19$
- (B) $a < 19$
- (C) All real a
- (D) $a < 0$
- (E) $a > 19$

Questions 25–27

25. Evaluate $\int_0^{\pi/2} \sin^{11} t \cos^{23} t \, dt$.

- (A) $4368 \left(\frac{5! 11!}{2 \cdot 17!} \right)$
- (B) $\frac{5! 11!}{2 \cdot 17!}$
- (C) $2 \left(\frac{5! 11!}{2 \cdot 17!} \right)$
- (D) $17 \left(\frac{5! 11!}{2 \cdot 17!} \right)$
- (E) $\frac{1}{2} \left(\frac{5! 11!}{2 \cdot 17!} \right)$

26. Let $u = (10, 5)^T$ and $v = (1, 0)^T$. Find the squared distance from $(10, 0)^T$ to $\text{span}(u)$.

- (A) 18
- (B) 22
- (C) 21
- (D) 19
- (E) 20

27. Evaluate $\int_0^\infty x^5 e^{-11x} \, dx$.

- (A) 120
- (B) $\frac{5!}{11^6}$
- (C) $\frac{1}{5} \left(\frac{5!}{11^6} \right)$
- (D) $11 \left(\frac{5!}{11^6} \right)$
- (E) $\frac{1}{11} \left(\frac{5!}{11^6} \right)$

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{19p}]$ converge?

- (A) All real p
- (B) $p > 0$
- (C) $p < 1/19$
- (D) $p \geq 1/19$
- (E) $p > 1/19$

29. Let $\mathbf{F} = (11y^5, z^5, x^5)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (1, 1, 3)/\sqrt{11}$.

- (A) $-\frac{175\sqrt{11}}{22}$
- (B) $-\frac{175\sqrt{11}}{11}$
- (C) $-\frac{350\sqrt{11}}{11}$
- (D) $\frac{175\sqrt{11}}{11}$
- (E) $1 - \frac{175\sqrt{11}}{11}$

30. An idempotent 54×54 real matrix P has rank 2. What is $\det(I + P)$?

- (A) 5
- (B) 3
- (C) 2
- (D) 4
- (E) 6

Questions 31–33

31. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(11+5j)(16+5j)}$.

- (A) $\frac{1}{11}$
- (B) $\frac{1}{5}$
- (C) $\frac{1}{16}$
- (D) $\frac{5}{11}$
- (E) $\frac{1}{55}$

32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{19 + 1/k : k \geq 1\}$ have?

- (A) 2
- (B) 0
- (C) 3
- (D) 4
- (E) 1

33. For $D = \{(x, y) : x^2 + y^2 \leq 11\}$, evaluate $\iint_D (x^2 + y^2)^4 dA$.

- (A) $\frac{161051\pi}{9}$
- (B) $\frac{161051\pi}{5}$
- (C) $\frac{322102\pi}{5}$
- (D) $\frac{14641\pi}{5}$
- (E) 161051π

Questions 34–36

34. For $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 48 \\ 7 \\ 55 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find the minimum squared residual.

- (A) $\frac{3959}{3}$
- (B) $\frac{7921}{6}$
- (C) $\frac{3962}{3}$
- (D) $\frac{7927}{6}$
- (E) $\frac{7915}{6}$

35. Evaluate $\int_0^{\log 2} e^x(e^x + 11)^5 dx$.

- (A) $13^5 - 12^5$
- (B) $\frac{13^6}{6}$
- (C) $-\frac{13^6 - 12^6}{6}$
- (D) $\frac{13^6 - 12^6}{6}$
- (E) $6 \left(\frac{13^6 - 12^6}{6} \right)$

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 19\}$ have?

- (A) 20
- (B) 32
- (C) 8
- (D) 14
- (E) 26

Questions 37–39

37. Find the total outward flux of $\mathbf{F} = (11x^5y, y^5z, xz^5)$ through the boundary of $[0, 1]^3$.

- (A) $\frac{15}{2}$
- (B) $\frac{13}{2}$
- (C) $\frac{13}{4}$
- (D) $\frac{13}{2}$
- (E) $-\frac{13}{2}$

38. What is the coefficient of x^7 in $(x + 10)^7(1 - x)^3$?

- (A) -28910
- (B) -28907
- (C) -28909
- (D) -28908
- (E) -28911

39. For $\mathbf{F} = (11x^5y, y^5z, xz^5)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) $\frac{195}{2}$
- (B) 196
- (C) 195
- (D) -195
- (E) 390

Questions 40–42

40. How many zeros, counted with multiplicity, does $z^{14} + 17z + 1$ have in $|z| < 1$?

- (A) 2
- (B) 0
- (C) 4
- (D) 3
- (E) 1

41. What is the maximum of x^5y^{11} over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\left(\frac{5}{16}\right)^{11} \left(\frac{11}{16}\right)^5$
- (B) $\left(\frac{5}{16}\right)^5 \left(\frac{11}{16}\right)^{11}$
- (C) $\frac{55}{256}$
- (D) $16 \left(\left(\frac{5}{16}\right)^5 \left(\frac{11}{16}\right)^{11} \right)$
- (E) $\frac{1}{2^{16}}$

42. In the ring $\mathbb{Z}/312\mathbb{Z}$, how many elements satisfy $x^2 = 1$?

- (A) 16
- (B) 14
- (C) 17
- (D) 15
- (E) 18

Questions 43–45

43. Let C be $x^2/25 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C \left(-\frac{11y^3}{3}\right) dx + \left(\frac{x^3}{3}\right) dy$.
- (A) 90π
(B) 45π
(C) -45π
(D) $1 + 45\pi$
(E) $\frac{45\pi}{2}$
44. A meromorphic function has 16 zeros and 2 poles inside a positively oriented simple closed contour, counted with multiplicity, and no zeros or poles on it. Find $\frac{1}{2\pi i} \int f'(z)/f(z) dz$.
- (A) 6
(B) 10
(C) 14
(D) 18
(E) 22
45. Find the directional derivative of $f(x, y, z) = x^2y + 5xz^2 + 11y^2z$ at $(1, 1, 1)$ in the direction $(1, 3, 0)$.
- (A) $1 + \frac{38\sqrt{10}}{5}$
(B) $\frac{19\sqrt{10}}{5}$
(C) $\frac{38\sqrt{10}}{5}$
(D) $\frac{76\sqrt{10}}{5}$
(E) $-\frac{38\sqrt{10}}{5}$

Questions 46–48

46. A permutation consists of disjoint cycles of lengths 7 and 10, together with a disjoint transposition when the sum of these two lengths is odd. It fixes all other points. What is its order?

- (A) 69
- (B) 72
- (C) 68
- (D) 70
- (E) 71

47. If $y' + 11y = 5x^5e^{-11x}$ and $y(0) = 3$, find $y(1)$.

- (A) $\frac{23}{12e^{11}}$
- (B) $\frac{23}{6e^{11}} + 1$
- (C) $-\frac{23}{6e^{11}}$
- (D) $\frac{23}{6e^{11}}$
- (E) $\frac{23}{3e^{11}}$

48. How many truth assignments to $p, q, r_1, \dots, r_{17}$ make $p \Rightarrow q$ false?

- (A) 2^{18}
- (B) 2^{16}
- (C) 2^{19}
- (D) 2^{20}
- (E) 2^{17}

Questions 49–51

49. Find the coefficient of x^6 in $\cos(11x)/(1 - 4x^2)$.

- (A) $\frac{665521}{720}$
- (B) $-\frac{1771561}{720}$
- (C) $-\frac{665521}{720}$
- (D) $-\frac{666241}{720}$
- (E) -665521

50. In $R = \mathbb{F}_7[x]/(x^{11})$, how many elements are units?

- (A) $7^{12} - 7^{10}$
- (B) $7^{11} - 7^0$
- (C) $7^{11} - 7^{10}$
- (D) $7^{12} - 7^{11}$
- (E) $7^{13} - 7^{12}$

51. Let C be the unit circle $x^2 + y^2 = 1$, $z = 5$, counterclockwise viewed from above. Evaluate $\int_C -11y(x^2 + y^2)^4 dx + (11x(x^2 + y^2)^4) dy + 3z dz$.

- (A) 22π
- (B) 11π
- (C) -22π
- (D) $1 + 22\pi$
- (E) 44π

Questions 52–54

52. Find the coefficient of x^3 in $(1+x)^{19}(1-x)^{19}(1+2x)^2$.

- (A) -51
- (B) -126
- (C) -26
- (D) -101
- (E) -76

53. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{11t} \cos(5t)$, and $y = e^{11t} \sin(5t)$, find d^2f/dt^2 at $t = 1$.

- (A) 484
- (B) 0
- (C) 22
- (D) 10
- (E) 292

54. Integers x satisfy $x \equiv 5 \pmod{7}$ and $x \equiv 0 \pmod{9}$. Find the sum of all such x with $0 \leq x < 126$.

- (A) 170
- (B) 171
- (C) 173
- (D) 169
- (E) 172

Questions 55–57

55. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 11z^2 = x + 5y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) -110
- (B) -5
- (C) -55
- (D) -22
- (E) 110

56. How many edges does the complete bipartite graph $K_{19,21}$ have?

- (A) 133
- (B) 266
- (C) 399
- (D) 665
- (E) 532

57. Two solutions of $y'' + 11x^5y' + 5y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $e^{-\frac{11}{6}} + 1$
- (B) $-\frac{1}{e^{\frac{11}{6}}}$
- (C) $\frac{2}{e^{\frac{11}{6}}}$
- (D) $\frac{1}{2e^{\frac{11}{6}}}$
- (E) $e^{-\frac{11}{6}}$

Questions 58–60

58. The symmetric group S_{54} acts on the two-element subsets of $\{1, \dots, 54\}$. What is the index of the setwise stabilizer of $\{1, 2\}$?

- (A) 1432
- (B) 1433
- (C) 1431
- (D) 1430
- (E) 1429

59. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 11x$ and has density $\rho(x, y) = x^3y$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{3}{7}$
- (B) $-\frac{6}{7}$
- (C) $\frac{13}{7}$
- (D) $\frac{12}{7}$
- (E) $\frac{6}{7}$

60. How many functions from a set of 24 elements onto a set of 3 elements exist?

- (A) $3^{24} - 3(2^{24})$
- (B) $3^{24} - 3(2^{24}) - 3$
- (C) $3^{24} - 3(2^{24}) + 3$
- (D) $3^{24} - 2^{24}$
- (E) 3^{24}

Questions 61–63

61. The solution of $y' = 11x^5y^3$ has $y(0) = 1/12$. Find $y(1)$.

- (A) $\frac{2\sqrt{1263}}{421}$
- (B) $\frac{\sqrt{1263}}{842}$
- (C) $\frac{\sqrt{1263}}{421} + 1$
- (D) $\frac{\sqrt{1263}}{421}$
- (E) $-\frac{\sqrt{1263}}{421}$

62. In $\mathbb{Z}/1446\mathbb{Z}$, how many residues are not idempotent ($x^2 \neq x$)?

- (A) 1438
- (B) 1437
- (C) 1439
- (D) 1440
- (E) 1436

63. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(11x) - \sum_{j=0}^3 (-1)^j (11x)^{2j+1} / (2j+1)!}{x^9}$.

- (A) $11 \left(\frac{(-1)^4 11^9}{9!} \right)$
- (B) $\frac{1}{11} \left(\frac{(-1)^4 11^9}{9!} \right)$
- (C) $\frac{(-1)^4 11^9}{9!}$
- (D) $-\frac{(-1)^4 11^9}{9!}$
- (E) $9 \left(\frac{(-1)^4 11^9}{9!} \right)$

Questions 64–66

64. How many nonnegative integer triples sum to 48, if each coordinate is at most 24?

- (A) 325
- (B) 217
- (C) 433
- (D) 541
- (E) 109

65. The local inverse $y(x)$ of $x = 11y + y^3$ satisfies $y(0) = 0$. Find $y^{(5)}(0)$.

- (A) $-\frac{360}{11^7}$
- (B) $11 \left(\frac{360}{11^7} \right)$
- (C) $\frac{1}{11} \left(\frac{360}{11^7} \right)$
- (D) $\frac{360}{11^7}$
- (E) $\frac{1}{120} \left(\frac{360}{11^7} \right)$

66. In the additive group $\mathbb{Z}/66\mathbb{Z}$, find the index of the subgroup generated by $[3]$.

- (A) 1
- (B) 3
- (C) 5
- (D) 4
- (E) 2

Answer Key

Practice Test 52 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	E	23	C	45	C
2	A	24	B	46	D
3	A	25	B	47	D
4	D	26	E	48	E
5	E	27	B	49	C
6	C	28	E	50	C
7	B	29	B	51	A
8	E	30	D	52	E
9	D	31	E	53	B
10	D	32	A	54	B
11	A	33	B	55	A
12	E	34	B	56	C
13	D	35	D	57	E
14	A	36	A	58	C
15	C	37	D	59	E
16	B	38	C	60	C
17	D	39	C	61	D
18	A	40	E	62	A
19	C	41	B	63	C
20	A	42	A	64	A
21	E	43	B	65	D
22	D	44	C	66	B

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