

GRE Mathematics Practice Test 53

Upper-Intermediate Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

Intermediate reinforcement, combining standard applications with selected more demanding arguments. This is cycle 10 of 19, position 3 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Evaluate $\int_0^{\log 2} e^x(e^x + 11)^4 dx$.

- (A) $5 \left(\frac{13^5 - 12^5}{5} \right)$
- (B) $13^4 - 12^4$
- (C) $-\frac{13^5 - 12^5}{5}$
- (D) $\frac{13^5}{5}$
- (E) $\frac{13^5 - 12^5}{5}$

2. For real t , let $M(t) = \begin{pmatrix} t - 49 & 10 & 0 \\ 0 & t - 49 & 1 \\ 0 & 0 & t - 49 \end{pmatrix}$. At $t = 49$, what is $\dim \ker(M(t)^2) - \dim \ker M(t)$?

- (A) 1
- (B) 3
- (C) 4
- (D) 0
- (E) 2

3. For the curve $x = t^4$, $y = t^5 + 11t$, find d^2y/dx^2 at $t = 1$.

- (A) $-\frac{11}{4}$
- (B) 4
- (C) $\frac{19}{8}$
- (D) -7
- (E) $-\frac{7}{4}$

Questions 4–6

4. A permutation of 22 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z^2]$.

- (A) 1
- (B) 3
- (C) 2
- (D) 0
- (E) 4

5. Solve $y' + 11y = x^4e^{-11x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{e^{11}}{5}$
- (B) $\frac{1}{5e^{11}}$
- (C) $\frac{1}{4e^{11}}$
- (D) $\frac{11}{5e^{11}}$
- (E) e^{-11}

6. Let $u = (10, 4)^T$ and $v = (1, 0)^T$. Find the squared distance from $(10, 0)^T$ to $\text{span}(u)$.

- (A) $\frac{400}{29}$
- (B) $\frac{829}{58}$
- (C) $\frac{771}{58}$
- (D) $\frac{429}{29}$
- (E) $\frac{371}{29}$

Questions 7–9

7. For $f(x) = x^4 e^{11x}$, find $f^{(6)}(0)$.

- (A) $2 \left(\frac{6! 11^2}{2} \right)$
- (B) $\frac{2}{11} \left(\frac{6! 11^2}{2} \right)$
- (C) $\frac{1}{360} \left(\frac{6! 11^2}{2} \right)$
- (D) $\frac{6! 11^2}{2}$
- (E) $\frac{1}{30} \left(\frac{6! 11^2}{2} \right)$

8. A coin success probability p has uniform prior on $[0, 1]$. After observing 20 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?

- (A) $\frac{7}{24}$
- (B) $\frac{7}{16}$
- (C) $\frac{7}{8}$
- (D) $\frac{15}{16}$
- (E) $\frac{1}{8}$

9. Let $f(x) = \log(1 + 11x)$. Find $f^{(4)}(0)$.

- (A) $\frac{1}{4} \left((-1)^3 3! 11^4 \right)$
- (B) $4 \left((-1)^3 3! 11^4 \right)$
- (C) $-(-1)^3 3! 11^4$
- (D) $\frac{1}{11} \left((-1)^3 3! 11^4 \right)$
- (E) $(-1)^3 3! 11^4$

Questions 10–12

10. An idempotent 55×55 real matrix P has rank 3. What is $\det(I + P)$?

- (A) 7
- (B) 6
- (C) 10
- (D) 8
- (E) 9

11. If $y'' - 22y' + 121y = 4e^{11x}$ and $y(0) = y'(0) = 0$, find $y(1)$.

- (A) $1 + 2e^{11}$
- (B) $2e^{11}$
- (C) e^{11}
- (D) $-2e^{11}$
- (E) $4e^{11}$

12. A random variable has mean 0 and variance 20. What upper bound does the Chebyshev inequality give for $P(|X| \geq 40)$?

- (A) $\frac{79}{80}$
- (B) $\frac{1}{80}$
- (C) $\frac{1}{240}$
- (D) $\frac{81}{160}$
- (E) $\frac{1}{160}$

Questions 13–15

13. If $y'' + 16y = 11$ and $y(0) = y'(0) = 0$, find $y(\pi/4)$.

- (A) $\frac{19}{8}$
- (B) $-\frac{11}{8}$
- (C) $\frac{11}{16}$
- (D) $\frac{11}{8}$
- (E) $\frac{11}{4}$

14. For $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 49 \\ 10 \\ 59 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find x_2 .

- (A) 3
- (B) 6
- (C) 5
- (D) 7
- (E) 4

15. Find the curvature of the helix $r(t) = (11 \cos t, 11 \sin t, 4t)$.

- (A) $-\frac{11}{137}$
- (B) $\frac{11}{137}$
- (C) $\frac{148}{137}$
- (D) $\frac{11}{274}$
- (E) $\frac{22}{137}$

Questions 16–18

16. A box is chosen uniformly from two boxes. Box A contains 25 red and 27 blue balls; box B contains 27 red and 25 blue balls. Two balls are drawn without replacement and both are red. What is the conditional probability that A was chosen?

- (A) $\frac{317}{434}$
- (B) $\frac{100}{651}$
- (C) $\frac{50}{217}$
- (D) $\frac{100}{217}$
- (E) $\frac{117}{217}$

17. Evaluate $\int_0^1 \int_0^x x^{11} y^4 dy dx$.

- (A) $\frac{1}{102}$
- (B) $\frac{1}{60}$
- (C) $\frac{1}{85}$
- (D) $\frac{1}{80}$
- (E) $\frac{1}{17}$

18. What is the coefficient of x^7 in $(x + 10)^6(1 - x)^3$?

- (A) -1325
- (B) -1322
- (C) -1324
- (D) -1321
- (E) -1323

Questions 19–21

19. Evaluate $\int_0^1 x^{10}(\log x)^3 dx$.

- (A) $\frac{1331}{6} \left(\frac{(-1)^3 3!}{11^4} \right)$
(B) $\frac{1}{4} \left(\frac{(-1)^3 3!}{11^4} \right)$
(C) $\frac{(-1)^3 3!}{11^4}$
(D) $11 \left(\frac{(-1)^3 3!}{11^4} \right)$
(E) $-\frac{(-1)^3 3!}{11^4}$

20. Let $X_1, \dots, X_{25}$ be independent uniform random variables on $[0, 1]$. What is $E[(\max_i X_i)^2]$?

- (A) $\frac{25}{81}$
(B) $\frac{2}{27}$
(C) $\frac{26}{27}$
(D) $\frac{25}{27}$
(E) $\frac{25}{54}$

21. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(11x) - \sum_{j=0}^2 (-1)^j (11x)^{2j+1} / (2j+1)!}{x^7}$.

- (A) $\frac{(-1)^3 11^7}{7!}$
(B) $-\frac{(-1)^3 11^7}{7!}$
(C) $\frac{1}{11} \left(\frac{(-1)^3 11^7}{7!} \right)$
(D) $7 \left(\frac{(-1)^3 11^7}{7!} \right)$
(E) $11 \left(\frac{(-1)^3 11^7}{7!} \right)$

Questions 22–24

22. A matrix A satisfies $(A - 50I)(A - 52I) = 0$, and its minimal polynomial has degree two. If $A^4 = uA + vI$, find u .

- (A) 530809
- (B) 530806
- (C) 530807
- (D) 530808
- (E) 530810

23. For $x > 0$, $x^2y'' + (-25)xy' + 165y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $-\frac{(1 - e^4)e^{11}}{8}$
- (B) $-\frac{(1 - e^4)e^{11}}{4}$
- (C) $\frac{(1 - e^4)e^{11}}{4}$
- (D) $-\frac{(1 - e^4)e^{11}}{2}$
- (E) $-\frac{e^{11}}{4} + 1 + \frac{e^{15}}{4}$

24. For which real a does $f_m(x) = m^a x^{20}(1 - x)^m$ converge uniformly to zero on $[0, 1]$?

- (A) $a \leq 20$
- (B) $a > 20$
- (C) $a < 0$
- (D) All real a
- (E) $a < 20$

Questions 25–27

25. What is the maximum of x^4y^{11} over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\frac{1}{2^{15}}$
- (B) $15 \left(\left(\frac{4}{15} \right)^4 \left(\frac{11}{15} \right)^{11} \right)$
- (C) $\left(\frac{4}{15} \right)^4 \left(\frac{11}{15} \right)^{11}$
- (D) $\left(\frac{4}{15} \right)^{11} \left(\frac{11}{15} \right)^4$
- (E) $\frac{44}{225}$

26. Let $V = \begin{bmatrix} 1 & 49 & 2401 \\ 1 & 51 & 2601 \\ 1 & 54 & 2916 \end{bmatrix}$. Find $\det(V^T V)$.

- (A) 900
- (B) 898
- (C) 902
- (D) 899
- (E) 901

27. Under $x = u + 11v$, $y = u - 4v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) 44
- (B) 225
- (C) 15
- (D) $\frac{1}{15}$
- (E) 7

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{20p}]$ converge?

- (A) $p > 0$
- (B) All real p
- (C) $p < 1/20$
- (D) $p > 1/20$
- (E) $p \geq 1/20$

29. The solution of $y' = 11x^3y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{11^{\frac{3}{4}}\sqrt[4]{2}}{11}$
- (B) $\frac{11^{\frac{3}{4}}\sqrt{2}}{11}$
- (C) $\frac{\sqrt[4]{11}\sqrt{2}}{2}$
- (D) $\frac{4}{11}$
- (E) $\frac{11^{\frac{3}{4}}}{11}$

30. Let $u = (1, 2, \dots, 55)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $\det B$.

- (A) 1539
- (B) 1542
- (C) 1540
- (D) 1543
- (E) 1541

Questions 31–33

31. Evaluate $\iiint_{x^2+y^2+z^2 \leq 11} (x^2 + y^2 + z^2)^3 dV$.

- (A) $9 \left(\frac{4\pi 11^{\frac{9}{2}}}{9} \right)$
- (B) $\frac{9}{77} \left(\frac{4\pi 11^{\frac{9}{2}}}{9} \right)$
- (C) $\frac{1}{3} \left(\frac{4\pi 11^{\frac{9}{2}}}{9} \right)$
- (D) $\frac{1}{2} \left(\frac{4\pi 11^{\frac{9}{2}}}{9} \right)$
- (E) $\frac{4\pi 11^{\frac{9}{2}}}{9}$

32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{20 + 1/k : k \geq 1\}$ have?

- (A) 1
- (B) 2
- (C) 3
- (D) 4
- (E) 0

33. Evaluate $\int_0^\infty x^4 e^{-11x} dx$.

- (A) $\frac{4!}{11^5}$
- (B) $\frac{1}{11} \left(\frac{4!}{11^5} \right)$
- (C) $11 \left(\frac{4!}{11^5} \right)$
- (D) 24
- (E) $\frac{1}{4} \left(\frac{4!}{11^5} \right)$

Questions 34–36

34. Let $D = \text{diag}(\underbrace{1, \dots, 1}_9, \underbrace{2, \dots, 2}_{46})$. Find the dimension of the space of real matrices X with $XD = DX$.

- (A) 2198
- (B) 2199
- (C) 2195
- (D) 2197
- (E) 2196

35. Find the upward flux of $\mathbf{F} = (x, y, z)$ through $z = 11x^2 + 4y^2$, $0 \leq x, y \leq 1$.

- (A) -5
- (B) -4
- (C) -10
- (D) 5
- (E) $-\frac{5}{2}$

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 20\}$ have?

- (A) 14
- (B) 21
- (C) 28
- (D) 35
- (E) 7

Questions 37–39

37. Find the coefficient of x^4 in $e^{3x} \log(1 + 11x)$.

- (A) -61248
- (B) $-\frac{14641}{4}$
- (C) 2552
- (D) -2552
- (E) -2541

38. When $x^3 + 1$ is divided by $(x - 50)(x - 51)$, the remainder is $ux + v$. Find u .

- (A) 7651
- (B) 7653
- (C) 7652
- (D) 7650
- (E) 7649

39. Let $F(x) = \int_0^{x^4} e^{11t} dt$. Find $F'(1)$.

- (A) $4e^{11}$
- (B) e^{11}
- (C) $44e^{11}$
- (D) e^{44}
- (E) $\frac{e^{11}}{4}$

Questions 40–42

40. How many zeros, counted with multiplicity, does $z^{15} + 18z + 1$ have in $|z| < 1$?

- (A) 3
- (B) 2
- (C) 0
- (D) 4
- (E) 1

41. For $D = \{(x, y) : x^2 + y^2 \leq 11\}$, evaluate $\iint_D (x^2 + y^2)^3 dA$.

- (A) $\frac{14641\pi}{4}$
- (B) $\frac{1331\pi}{4}$
- (C) 14641π
- (D) $\frac{14641\pi}{7}$
- (E) $\frac{14641\pi}{2}$

42. How many invertible 3×3 matrices over $\mathbb{F}_{29}$ are there whose first column is $(1, 0, 0)^T$?

- (A) $\frac{29^3 29^3 29^3}{29^3 - 1}$
- (B) $\frac{(29^3 - 29^0)(29^3 - 29^1)}{29^3 - 1}$
- (C) $\frac{(29^3 - 29^0)(29^3 - 29^0)(29^3 - 29^0)}{29^3 - 1}$
- (D) $\frac{(29^3 - 29^1)(29^3 - 29^2)(29^3 - 29^3)}{29^3 - 1}$
- (E) $\frac{(29^3 - 29^0)(29^3 - 29^1)(29^3 - 29^2)}{29^3 - 1}$

Questions 43–45

43. Let $F(x) = \int_0^x (x-t)^3 e^{11t} dt$. Find $F^{(4)}(0)$.

- (A) 24
- (B) 6
- (C) $\frac{1}{6}$
- (D) 66
- (E) 0

44. A meromorphic function has 17 zeros and 2 poles inside a positively oriented simple closed contour, counted with multiplicity, and no zeros or poles on it. Find $\frac{1}{2\pi i} \int f'(z)/f(z) dz$.

- (A) 10
- (B) 15
- (C) 5
- (D) 20
- (E) 25

45. The region $0 \leq x \leq 1$, $0 \leq y \leq 11x^4$ is revolved about the line $x = -4$. Find its volume.

- (A) $1 + \frac{319\pi}{15}$
- (B) $\frac{638\pi}{15}$
- (C) $\frac{319\pi}{15}$
- (D) $-\frac{319\pi}{15}$
- (E) $\frac{319\pi}{30}$

Questions 46–48

46. How many permutations in S_{55} consist of one 4-cycle and 51 fixed points?

- (A) $2 \cdot 3^4 \cdot 5 \cdot 11 \cdot 13 \cdot 53$
- (B) $2 \cdot 3^3 \cdot 5 \cdot 11 \cdot 13 \cdot 53$
- (C) $2^2 \cdot 3^3 \cdot 5 \cdot 11 \cdot 13 \cdot 53$
- (D) $3^3 \cdot 5 \cdot 11 \cdot 13 \cdot 53$
- (E) $2 \cdot 3^3 \cdot 5^2 \cdot 11 \cdot 13 \cdot 53$

47. For $\mathbf{F} = (11x^4y, y^4z, xz^4)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) 124
- (B) 62
- (C) 125
- (D) 248
- (E) -124

48. How many truth assignments to $p, q, r_1, \dots, r_{18}$ make $p \Rightarrow q$ false?

- (A) 2^{21}
- (B) 2^{20}
- (C) 2^{17}
- (D) 2^{18}
- (E) 2^{19}

Questions 49–51

49. Find the total outward flux of $\mathbf{F} = (11x^4y, y^4z, xz^4)$ through the boundary of $[0, 1]^3$.

- (A) 13
- (B) $\frac{15}{2}$
- (C) $-\frac{13}{2}$
- (D) $\frac{13}{4}$
- (E) $\frac{13}{2}$

50. How many maximal proper subgroups does the cyclic group of order 636 have?

- (A) 2
- (B) 4
- (C) 3
- (D) 5
- (E) 1

51. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{11t} \cos(4t)$, and $y = e^{11t} \sin(4t)$, find d^2f/dt^2 at $t = 1$.

- (A) 8
- (B) 274
- (C) 22
- (D) 484
- (E) 0

Questions 52–54

52. What is the coefficient of x^2 in $(1 + 2x)^{20}$?

- (A) 760
- (B) 1013
- (C) 254
- (D) 507
- (E) 1266

53. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 11z^2 = x + 4y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.

- (A) -44
- (B) -88
- (C) -22
- (D) 88
- (E) -4

54. How many residues x modulo 72 satisfy $8x \equiv 56 \pmod{72}$ and $2x \equiv 0 \pmod{72}$?

- (A) 2
- (B) 3
- (C) 1
- (D) 0
- (E) 4

Questions 55–57

55. Evaluate $\int_0^{\pi/2} \sin^9 t \cos^{23} t \, dt$.

- (A) $\frac{1}{2} \left(\frac{4! 11!}{2 \cdot 16!} \right)$
- (B) $16 \left(\frac{4! 11!}{2 \cdot 16!} \right)$
- (C) $2 \left(\frac{4! 11!}{2 \cdot 16!} \right)$
- (D) $\frac{4! 11!}{2 \cdot 16!}$
- (E) $1365 \left(\frac{4! 11!}{2 \cdot 16!} \right)$

56. How many edges does the complete bipartite graph $K_{20,22}$ have?

- (A) 732
- (B) 294
- (C) 586
- (D) 148
- (E) 440

57. Let C be the unit circle $x^2 + y^2 = 1$, $z = 4$, counterclockwise viewed from above. Evaluate $\int_C -11y(x^2 + y^2)^3 \, dx + (11x(x^2 + y^2)^3) \, dy + 4z \, dz$.

- (A) 22π
- (B) $1 + 22\pi$
- (C) 44π
- (D) 11π
- (E) -22π

Questions 58–60

58. The field K has 5^{11} elements. Find the largest possible multiplicative order of a nonzero element of K .

- (A) $5^{11} - 1$
- (B) 5^{11}
- (C) $5^{12} - 1$
- (D) $5^{10} - 1$
- (E) $5^{11} + 1$

59. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(11j)!}{(j!)^{11}} x^{4j}$?

- (A) $11^{-\frac{4}{11}}$
- (B) $\frac{1}{11}$
- (C) $11^{-\frac{11}{4}}$
- (D) 11^{-11}
- (E) $11^{\frac{11}{4}}$

60. How many functions from a set of 25 elements onto a set of 3 elements exist?

- (A) $3^{25} - 3(2^{25})$
- (B) $3^{25} - 3(2^{25}) + 3$
- (C) 3^{25}
- (D) $3^{25} - 2^{25}$
- (E) $3^{25} - 3(2^{25}) - 3$

Questions 61–63

61. Find the coefficient of x^4 in the Taylor series of $(1 - 11x)^{-2}$ at zero.

- (A) $\frac{1}{5} (5 \cdot 11^4)$
- (B) $2 (5 \cdot 11^4)$
- (C) $\frac{1}{11} (5 \cdot 11^4)$
- (D) $\frac{4}{5} (5 \cdot 11^4)$
- (E) $5 \cdot 11^4$

62. Let $G = \mathbb{Z}/324\mathbb{Z} \times \mathbb{Z}/220\mathbb{Z}$. Find the order of G/H , where $H = \{g \in G : 4g = 0\}$.

- (A) 4456
- (B) 4455
- (C) 4454
- (D) 4453
- (E) 4457

63. If $y' = 22xy$ and $y(0) = 1$, find the coefficient of x^8 in the Maclaurin series of y .

- (A) $\frac{14665}{24}$
- (B) $-\frac{14641}{24}$
- (C) $\frac{14641}{24}$
- (D) $\frac{14641}{12}$
- (E) $\frac{14641}{48}$

Questions 64–66

64. How many nonnegative integer triples sum to 50, if each coordinate is at most 25?

- (A) 468
- (B) 234
- (C) 117
- (D) 351
- (E) 585

65. Let $f(x, y) = e^{11x+4y}$. Find $\partial^4 f / \partial x^3 \partial y$ at $(0, 0)$.

- (A) $\frac{1}{4} (4 \cdot 11^3)$
- (B) $\frac{11}{4} (4 \cdot 11^3)$
- (C) $4 \cdot 11^3$
- (D) $\frac{64}{1331} (4 \cdot 11^3)$
- (E) $\frac{16}{121} (4 \cdot 11^3)$

66. How many group homomorphisms $\mathbb{Z}/324\mathbb{Z} \rightarrow \mathbb{Z}/220\mathbb{Z}$ are there?

- (A) 2
- (B) 5
- (C) 4
- (D) 6
- (E) 3

Answer Key

Practice Test 53 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	E	23	B	45	C
2	A	24	E	46	B
3	E	25	C	47	A
4	C	26	A	48	D
5	B	27	C	49	E
6	A	28	D	50	C
7	D	29	B	51	E
8	C	30	E	52	A
9	E	31	E	53	B
10	D	32	B	54	D
11	B	33	A	55	D
12	B	34	D	56	E
13	D	35	A	57	A
14	C	36	B	58	A
15	B	37	D	59	C
16	D	38	A	60	B
17	C	39	A	61	E
18	E	40	E	62	B
19	C	41	A	63	C
20	D	42	E	64	D
21	A	43	B	65	C
22	D	44	B	66	C

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