

GRE Mathematics Practice Test 54

Bridge to Standard

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A bridge toward standard practice, with routine multi-step questions and a smaller number of challenging concepts. This is cycle 10 of 19, position 4 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Let $F(x) = \int_0^{x^3} e^{11t} dt$. Find $F'(1)$.

- (A) e^{11}
- (B) e^{33}
- (C) $33e^{11}$
- (D) $3e^{11}$
- (E) $\frac{e^{11}}{3}$

2. A matrix A satisfies $(A - 51I)(A - 53I) = 0$, and its minimal polynomial has degree two. If $A^2 = uA + vI$, find u .

- (A) 102
- (B) 103
- (C) 105
- (D) 106
- (E) 104

3. Evaluate $\int_C (3x^2 + 11y) dx + (11x + 3y^2) dy$, where C is the line segment from $(0, 0)$ to $(5, 1)$.

- (A) 181
- (B) $\frac{181}{2}$
- (C) -181
- (D) 362
- (E) 182

Questions 4–6

4. A permutation of 23 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z]$.

- (A) 0
- (B) 4
- (C) 2
- (D) 1
- (E) 3

5. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^{11}y^2 dV$.

- (A) $6 \left(\frac{11!2!}{16!} \right)$
- (B) $\frac{11!2!}{16!}$
- (C) $\frac{1}{6} \left(\frac{11!2!}{16!} \right)$
- (D) $16 \left(\frac{11!2!}{16!} \right)$
- (E) $\frac{1}{36}$

6. Let $V = \begin{bmatrix} 1 & 50 & 2500 \\ 1 & 52 & 2704 \\ 1 & 55 & 3025 \end{bmatrix}$. Find $\det V$.

- (A) 28
- (B) 29
- (C) 30
- (D) 32
- (E) 31

Questions 7–9

7. If $y'' + 9y = 0$, $y(0) = 11$, and $y'(0) = 5$, find $y(\pi/6)$.

- (A) $\frac{10}{3}$
- (B) $\frac{8}{3}$
- (C) $\frac{5}{3}$
- (D) $\frac{5}{6}$
- (E) $-\frac{5}{3}$

8. A coin success probability p has uniform prior on $[0, 1]$. After observing 21 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?

- (A) $\frac{47}{50}$
- (B) $\frac{3}{25}$
- (C) $\frac{22}{25}$
- (D) $\frac{22}{75}$
- (E) $\frac{11}{25}$

9. What is the maximum of x^3y^{11} over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $14 \left(\left(\frac{3}{14} \right)^3 \left(\frac{11}{14} \right)^{11} \right)$
- (B) $\frac{1}{2^{14}}$
- (C) $\frac{33}{196}$
- (D) $\left(\frac{3}{14} \right)^{11} \left(\frac{11}{14} \right)^3$
- (E) $\left(\frac{3}{14} \right)^3 \left(\frac{11}{14} \right)^{11}$

Questions 10–12

10. Let $u = (1, 2, \dots, 56)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $\det B$.

- (A) 1595
- (B) 1597
- (C) 1596
- (D) 1598
- (E) 1599

11. Solve $y'' - 25y' + 154y = 0$ with $y(0) = 1$, $y'(0) = 0$. Find $y(1)$.

- (A) $\frac{(14 - 11e^3)e^{11}}{3} + 1$
- (B) $\frac{2(14 - 11e^3)e^{11}}{3}$
- (C) $\frac{(14 - 11e^3)e^{11}}{6}$
- (D) $\frac{(14 - 11e^3)e^{11}}{3}$
- (E) $\frac{(-14 + 11e^3)e^{11}}{3}$

12. A random variable has mean 0 and variance 21. What upper bound does the Chebyshev inequality give for $P(|X| \geq 42)$?

- (A) $\frac{1}{168}$
- (B) $\frac{1}{84}$
- (C) $\frac{83}{84}$
- (D) $\frac{1}{252}$
- (E) $\frac{85}{168}$

Questions 13–15

13. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(11+3j)(14+3j)}$.

- (A) $\frac{1}{3}$
- (B) $\frac{1}{14}$
- (C) $\frac{3}{11}$
- (D) $\frac{1}{11}$
- (E) $\frac{1}{33}$

14. Let D be a diagonal 56×56 real matrix with distinct diagonal entries. What is the dimension of the space of matrices X satisfying $XD = DX$?

- (A) 55
- (B) 54
- (C) 58
- (D) 56
- (E) 57

15. Evaluate $\lim_{x \rightarrow 0} (1 + 11x)^{1/(2x)}$.

- (A) $e^{\frac{11}{2}}$
- (B) 11
- (C) e^{12}
- (D) e^{121}
- (E) $e^{\frac{2}{11}}$

Questions 16–18

16. A box is chosen uniformly from two boxes. Box A contains 26 red and 28 blue balls; box B contains 28 red and 26 blue balls. Two balls are drawn without replacement and both are red. What is the conditional probability that A was chosen?

- (A) $\frac{378}{703}$
- (B) $\frac{325}{703}$
- (C) $\frac{1406}{514}$
- (D) $\frac{703}{325}$
- (E) $\frac{2109}{325}$

17. Evaluate $\int_0^{\log 2} e^x(e^x + 11)^3 dx$.

- (A) $13^3 - 12^3$
- (B) $\frac{13^4}{4}$
- (C) $-\frac{13^4 - 12^4}{4}$
- (D) $4 \left(\frac{13^4 - 12^4}{4} \right)$
- (E) $\frac{13^4 - 12^4}{4}$

18. What is the remainder when $x^2 + 1$ is divided by $x - 51$?

- (A) 2600
- (B) 2601
- (C) 2603
- (D) 2602
- (E) 2604

Questions 19–21

19. The solution of $y' = 11x^2y^2$, $y(0) = 1$, has its first positive singularity at which x ?
- (A) $\frac{\sqrt[3]{11} \cdot 3^{\frac{2}{3}}}{3}$
(B) $\frac{22^{\frac{2}{3}} \sqrt[3]{3}}{22}$
(C) $\frac{11^{\frac{2}{3}} \sqrt[3]{3}}{11}$
(D) $\frac{3}{11}$
(E) $\frac{11^{\frac{2}{3}}}{11}$
20. Let $X_1, \dots, X_{26}$ be independent uniform random variables on $[0, 1]$. What is $E[\max_i X_i]$?
- (A) $\frac{26}{81}$
(B) $\frac{26}{27}$
(C) $\frac{53}{54}$
(D) $\frac{1}{27}$
(E) $\frac{13}{27}$
21. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 11x$ and has density $\rho(x, y) = x$. Find its centroid coordinate $\bar{x}$.
- (A) $\frac{3}{2}$
(B) $-\frac{3}{4}$
(C) $\frac{3}{4}$
(D) $\frac{7}{4}$
(E) $\frac{3}{8}$

Questions 22–24

22. Let $P = \text{diag}(\underbrace{1, \dots, 1}_4, \underbrace{0, \dots, 0}_{53})$. What is the dimension of the space of 57×57 real matrices X satisfying $PX = X$?

- (A) 228
- (B) 226
- (C) 227
- (D) 230
- (E) 229

23. For $y' = 11y(1 - y/3)$, what is the derivative of the right-hand side with respect to y at the positive equilibrium?

- (A) -22
- (B) -10
- (C) -11
- (D) $-\frac{11}{2}$
- (E) 11

24. For which real a does $f_m(x) = x^{21m}/m^a$ converge uniformly to zero on $[0, 1]$?

- (A) All real a
- (B) $a > 0$
- (C) $a < 0$
- (D) $a = 0$
- (E) $a > 1$

Questions 25–27

25. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(11j)!}{(j!)^{11}} x^{3j}$?
- (A) $\frac{1}{11}$
(B) $11^{-\frac{11}{3}}$
(C) $11^{\frac{11}{3}}$
(D) 11^{-11}
(E) $11^{-\frac{3}{11}}$
26. A 3×3 matrix A has characteristic polynomial $(t - 50)(t + 1)(t - 2)$. What is $\text{tr}(A^2)$?
- (A) 2504
(B) 2503
(C) 2506
(D) 2505
(E) 2507
27. Find the arc length of $r(t) = (11t, 3t)$ for $0 \leq t \leq 1$.
- (A) $2\sqrt{130}$
(B) $1 + \sqrt{130}$
(C) $\sqrt{130}$
(D) $-\sqrt{130}$
(E) $\frac{\sqrt{130}}{2}$

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{21p}]$ converge?

- (A) $p > 1/21$
- (B) $p \geq 1/21$
- (C) $p < 1/21$
- (D) $p > 0$
- (E) All real p

29. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(11x) - \sum_{j=0}^1 (-1)^j (11x)^{2j+1} / (2j+1)!}{x^5}$.

- (A) $11 \left(\frac{(-1)^2 11^5}{5!} \right)$
- (B) $-\frac{(-1)^2 11^5}{5!}$
- (C) $5 \left(\frac{(-1)^2 11^5}{5!} \right)$
- (D) $\frac{1}{11} \left(\frac{(-1)^2 11^5}{5!} \right)$
- (E) $\frac{(-1)^2 11^5}{5!}$

30. Let N be the 56×56 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\text{rank } N^8$.

- (A) 46
- (B) 50
- (C) 47
- (D) 48
- (E) 49

Questions 31–33

31. Let $F(x) = \int_0^x (x-t)^2 e^{11t} dt$. Find $F^{(3)}(0)$.

- (A) 22
- (B) 0
- (C) 2
- (D) $\frac{1}{2}$
- (E) 6

32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{21 + 1/k : k \geq 1\}$ have?

- (A) 4
- (B) 0
- (C) 1
- (D) 3
- (E) 2

33. Two solutions of $y'' + 11x^1 y' + 3y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $\frac{1}{2e^{\frac{11}{2}}}$
- (B) $-\frac{1}{e^{\frac{11}{2}}}$
- (C) $\frac{2}{e^{\frac{11}{2}}}$
- (D) $e^{-\frac{11}{2}} + 1$
- (E) $e^{-\frac{11}{2}}$

Questions 34–36

34. Let A be the 56×56 matrix whose diagonal entries are 53 and whose off-diagonal entries are 1. What is its smallest eigenvalue?

- (A) 52
- (B) 50
- (C) 54
- (D) 51
- (E) 53

35. Evaluate $\iiint_B 11(x^2 + y^2 + z^2)^{3/2} dV$, where B is the unit ball.

- (A) $\frac{22\pi}{3}$
- (B) $-\frac{22\pi}{3}$
- (C) $1 + \frac{22\pi}{3}$
- (D) $\frac{44\pi}{3}$
- (E) $\frac{11\pi}{3}$

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 21\}$ have?

- (A) 29
- (B) 36
- (C) 8
- (D) 15
- (E) 22

Questions 37–39

37. If $y' = 11y$, $y(0) = 1$, what is the coefficient of x^3 in the Maclaurin series of y ?
- (A) $\frac{1331}{3}$
(B) $\frac{1337}{6}$
(C) $-\frac{1331}{6}$
(D) $\frac{1331}{12}$
(E) $\frac{1331}{6}$
38. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 400$ and $x + y = 58$. Find xy .
- (A) 399
(B) 398
(C) 402
(D) 401
(E) 400
39. For $x > 0$, $xy' = 11y$ and $y(1) = 1$. Find $y(e^3)$.
- (A) $1 + e^{33}$
(B) e^{33}
(C) $\frac{e^{33}}{2}$
(D) $2e^{33}$
(E) $-e^{33}$

Questions 40–42

40. How many zeros, counted with multiplicity, does $z^{16} + 19z + 1$ have in $|z| < 1$?
- (A) 2
(B) 1
(C) 3
(D) 0
(E) 4
41. For $f(x, y) = x^2 + 11xy + 3y^2$, what is its maximum directional derivative at $(1, -1)$?
- (A) 5
(B) -9
(C) 106
(D) 14
(E) $\sqrt{106}$
42. A sequence satisfies $u_0 = 50$ and $u_{k+1} = 1u_k + 8$ for all $k \geq 0$. Find u_4 .
- (A) 84
(B) 81
(C) 83
(D) 80
(E) 82

Questions 43–45

43. Let $f(x, y) = e^{11x+3y}$. Find $\partial^3 f / \partial x^2 \partial y$ at $(0, 0)$.

- (A) $\frac{9}{121} (3 \cdot 11^2)$
- (B) $\frac{1}{3} (3 \cdot 11^2)$
- (C) $\frac{11}{3} (3 \cdot 11^2)$
- (D) $3 \cdot 11^2$
- (E) $\frac{3}{11} (3 \cdot 11^2)$

44. A meromorphic function has 18 zeros and 2 poles inside a positively oriented simple closed contour, counted with multiplicity, and no zeros or poles on it. Find $\frac{1}{2\pi i} \int f'(z)/f(z) dz$.

- (A) 6
- (B) 11
- (C) 21
- (D) 16
- (E) 26

45. For $f(x) = x^3 e^{11x}$, find $f^{(5)}(0)$.

- (A) $2 \left(\frac{5! 11^2}{2} \right)$
- (B) $\frac{1}{60} \left(\frac{5! 11^2}{2} \right)$
- (C) $\frac{1}{20} \left(\frac{5! 11^2}{2} \right)$
- (D) $\frac{5! 11^2}{2}$
- (E) $\frac{2}{11} \left(\frac{5! 11^2}{2} \right)$

Questions 46–48

46. How many 1-dimensional subspaces does $\mathbb{F}_{29}^4$ have?

- (A) $\frac{29^4 - 1}{29 - 1}$
- (B) $\frac{29^6 - 1}{29 - 1}$
- (C) $\frac{29^7 - 1}{29 - 1}$
- (D) $\frac{29^5 - 1}{29 - 1}$
- (E) $\frac{29^3 - 1}{29 - 1}$

47. Find the total outward flux of $\mathbf{F} = (11x^3, y^3, z^3)$ through the boundary of $[0, 1]^3$.

- (A) 14
- (B) 26
- (C) 13
- (D) -13
- (E) $\frac{13}{2}$

48. How many truth assignments to $p, q, r_1, \dots, r_{19}$ make $p \Rightarrow q$ false?

- (A) 2^{20}
- (B) 2^{21}
- (C) 2^{22}
- (D) 2^{19}
- (E) 2^{18}

Questions 49–51

49. Find the directional derivative of $f(x, y, z) = x^2 + 11y^2 + 3z^2$ at $(1, 1, 1)$ in the direction $(1, 5, 0)$.

- (A) $\frac{112\sqrt{26}}{13}$
- (B) $\frac{28\sqrt{26}}{13}$
- (C) $1 + \frac{56\sqrt{26}}{13}$
- (D) $\frac{56\sqrt{26}}{13}$
- (E) $-\frac{56\sqrt{26}}{13}$

50. How many monic irreducible polynomials of degree 2 are there over $\mathbb{F}_{29}$?

- (A) 407
- (B) 406
- (C) 405
- (D) 404
- (E) 408

51. Let $f(x) = \log(1 + 11x)$. Find $f^{(3)}(0)$.

- (A) $(-1)^2 2! 11^3$
- (B) $3 \left((-1)^2 2! 11^3 \right)$
- (C) $\frac{1}{3} \left((-1)^2 2! 11^3 \right)$
- (D) $-(-1)^2 2! 11^3$
- (E) $\frac{1}{11} \left((-1)^2 2! 11^3 \right)$

Questions 52–54

52. What is the coefficient of x^2 in $(1 + 2x)^{21}$?

- (A) 840
- (B) 280
- (C) 1400
- (D) 1120
- (E) 560

53. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{11t} \cos(3t)$, and $y = e^{11t} \sin(3t)$, find d^2f/dt^2 at $t = 1$.

- (A) 260
- (B) 0
- (C) 22
- (D) 6
- (E) 484

54. How many elements z of $\mathbb{F}_{3^{11}}$ satisfy $z^3 = z$?

- (A) 1
- (B) 5
- (C) 3
- (D) 2
- (E) 4

Questions 55–57

55. Evaluate $\int_0^1 \int_x^1 11y^3 \, dy \, dx$.

- (A) $\frac{11}{20}$
- (B) $\frac{11}{5}$
- (C) $\frac{11}{2}$
- (D) $\frac{11}{4}$
- (E) $\frac{11}{3}$

56. How many edges does the complete bipartite graph $K_{21,23}$ have?

- (A) 644
- (B) 161
- (C) 322
- (D) 805
- (E) 483

57. For the curve $x = t^3$, $y = t^4 + 11t$, find d^2y/dx^2 at $t = 1$.

- (A) -6
- (B) $-\frac{11}{3}$
- (C) $\frac{26}{9}$
- (D) 5
- (E) -2

Questions 58–60

58. How many generators does the cyclic group of order 29 have?

- (A) 30
- (B) 27
- (C) 28
- (D) 26
- (E) 29

59. Evaluate $\lim_{x \rightarrow 0} \frac{e^{11x} - \sum_{j=0}^2 (11x)^j / j!}{x^3}$.

- (A) $6 \left(\frac{11^3}{3!} \right)$
- (B) $\frac{1}{11} \left(\frac{11^3}{3!} \right)$
- (C) $\frac{11^3}{3!}$
- (D) $-\frac{11^3}{3!}$
- (E) $3 \left(\frac{11^3}{3!} \right)$

60. How many functions from a set of 26 elements onto a set of 3 elements exist?

- (A) $3^{26} - 3(2^{26}) + 3$
- (B) $3^{26} - 3(2^{26})$
- (C) $3^{26} - 2^{26}$
- (D) $3^{26} - 3(2^{26}) - 3$
- (E) 3^{26}

Questions 61–63

61. Find the coefficient of x^6 in $\cos(11x)/(1 - 2x^2)$.

- (A) $-\frac{1771561}{720}$
- (B) $-\frac{1062301}{720}$
- (C) $-\frac{1061581}{720}$
- (D) $\frac{1061581}{720}$
- (E) -1061581

62. Let $R = (\mathbb{Z}/30\mathbb{Z})/([15])$, where $([15])$ is the ideal generated by $[15]$. How many elements does R have?

- (A) 16
- (B) 17
- (C) 14
- (D) 15
- (E) 13

63. Evaluate $\int_0^1 \frac{x^2}{1 + 11x^3} dx$.

- (A) $\frac{\log(12)}{33}$
- (B) $\frac{\log(12)}{3}$
- (C) $-\frac{\log(12)}{33}$
- (D) $\frac{\log(12)}{11}$
- (E) $\log(12)$

Questions 64–66

64. How many nonnegative integer triples (a, b, c) satisfy $a + b + c = 26$?

- (A) 378
- (B) 630
- (C) 504
- (D) 126
- (E) 252

65. Evaluate $\int_0^1 x^{10}(\log x)^2 dx$.

- (A) $\frac{121}{2} \left(\frac{(-1)^2 2!}{11^3} \right)$
- (B) $\frac{(-1)^2 2!}{11^3}$
- (C) $-\frac{(-1)^2 2!}{11^3}$
- (D) $11 \left(\frac{(-1)^2 2!}{11^3} \right)$
- (E) $\frac{1}{3} \left(\frac{(-1)^2 2!}{11^3} \right)$

66. What is the largest integer k such that 2^k divides $59!$?

- (A) 54
- (B) 52
- (C) 53
- (D) 56
- (E) 55

Answer Key

Practice Test 54 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	D	23	C	45	D
2	E	24	B	46	A
3	A	25	B	47	C
4	D	26	D	48	D
5	B	27	C	49	D
6	C	28	A	50	B
7	C	29	E	51	A
8	C	30	D	52	A
9	E	31	C	53	B
10	B	32	E	54	C
11	D	33	E	55	B
12	B	34	A	56	E
13	E	35	A	57	E
14	D	36	E	58	C
15	A	37	E	59	C
16	B	38	E	60	A
17	E	39	B	61	C
18	D	40	B	62	D
19	C	41	E	63	A
20	B	42	E	64	A
21	C	43	D	65	B
22	A	44	D	66	A

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