

GRE Mathematics Practice Test 55

Standard-Level Target

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A mixed standard-level target, intended to approximate public GRE Mathematics practice in scope and overall demand. This is cycle 10 of 19, position 5 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

zarimion.com/gre

100 practice tests, complete answer keys, and future updates.

Please retain the source attribution when sharing this material.



This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Solve $y' + 11y = x^2e^{-11x}$, $y(0) = 0$, and find $y(1)$.

- (A) $\frac{1}{2e^{11}}$
- (B) $\frac{1}{3e^{11}}$
- (C) $\frac{e^{11}}{3}$
- (D) $\frac{11}{3e^{11}}$
- (E) e^{-11}

2. Let $P = \text{diag}(\underbrace{1, \dots, 1}_1, \underbrace{0, \dots, 0}_{57})$. What is the dimension of the space of 58×58 real matrices X satisfying $PX = X$?

- (A) 60
- (B) 56
- (C) 59
- (D) 57
- (E) 58

3. If $f(x, y) = \log(x^2 + y^2)$, $x = e^{11t} \cos(2t)$, and $y = e^{11t} \sin(2t)$, find d^2f/dt^2 at $t = 1$.

- (A) 484
- (B) 22
- (C) 4
- (D) 250
- (E) 0

Questions 4–6

4. A permutation of 24 objects is chosen uniformly. Let Z be the number of fixed points. Find $E[Z]$.

- (A) 1
- (B) 2
- (C) 4
- (D) 3
- (E) 0

5. The solution of $y' = 11x^1y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{2}{11}$
- (B) $\frac{\sqrt{22}}{2}$
- (C) $\frac{\sqrt{11}}{11}$
- (D) $\frac{2\sqrt{22}}{11}$
- (E) $\frac{\sqrt{22}}{11}$

6. A 3×3 matrix A has characteristic polynomial $(t - 51)(t + 1)(t - 2)$. What is $\text{tr}(A^2)$?

- (A) 2604
- (B) 2607
- (C) 2608
- (D) 2606
- (E) 2605

Questions 7–9

7. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^{11}y^1 dV$.

- (A) $15 \left(\frac{11!1!}{15!} \right)$
- (B) $\frac{1}{24}$
- (C) $\frac{11!1!}{15!}$
- (D) $\frac{1}{6} \left(\frac{11!1!}{15!} \right)$
- (E) $6 \left(\frac{11!1!}{15!} \right)$

8. A coin success probability p has uniform prior on $[0, 1]$. After observing 22 successes and 2 failures in independent tosses conditional on p , what is the predictive probability of success on the next toss?

- (A) $\frac{49}{52}$
- (B) $\frac{52}{23}$
- (C) $\frac{52}{78}$
- (D) $\frac{3}{26}$
- (E) $\frac{23}{26}$

9. Under $x = u + 11v$, $y = u - 2v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) $\frac{1}{13}$
- (B) 169
- (C) 22
- (D) 9
- (E) 13

Questions 10–12

10. Let N be the 57×57 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\text{rank } N^8$.

- (A) 48
- (B) 49
- (C) 50
- (D) 47
- (E) 51

11. Evaluate $\iiint_B 11(x^2 + y^2 + z^2)^1 dV$, where B is the unit ball.

- (A) $1 + \frac{44\pi}{5}$
- (B) $\frac{44\pi}{5}$
- (C) $\frac{88\pi}{5}$
- (D) $\frac{22\pi}{5}$
- (E) $-\frac{44\pi}{5}$

12. A random variable has mean 0 and variance 22. What upper bound does the Chebyshev inequality give for $P(|X| \geq 44)$?

- (A) $\frac{1}{264}$
- (B) $\frac{89}{176}$
- (C) $\frac{87}{88}$
- (D) $\frac{1}{176}$
- (E) $\frac{1}{88}$

Questions 13–15

13. Find the coefficient of x^2 in the Taylor series of $(1 - 11x)^{-2}$ at zero.

- (A) $2(3 \cdot 11^2)$
- (B) $\frac{1}{3}(3 \cdot 11^2)$
- (C) $3 \cdot 11^2$
- (D) $\frac{2}{3}(3 \cdot 11^2)$
- (E) $\frac{1}{11}(3 \cdot 11^2)$

14. Let A be the 57×57 matrix whose diagonal entries are 54 and whose off-diagonal entries are 1. What is its smallest eigenvalue?

- (A) 51
- (B) 55
- (C) 53
- (D) 52
- (E) 54

15. Let $F(x) = \int_0^x (x - t)^1 e^{11t} dt$. Find $F^{(2)}(0)$.

- (A) 1
- (B) 2
- (C) 11
- (D) 0
- (E) $\frac{1}{2}$

Questions 16–18

16. A box is chosen uniformly from two boxes. Box A contains 27 red and 29 blue balls; box B contains 29 red and 27 blue balls. One ball is drawn and is red. What is the conditional probability that A was chosen?

- (A) $\frac{27}{56}$
- (B) $\frac{27}{112}$
- (C) $\frac{9}{56}$
- (D) $\frac{83}{112}$
- (E) $\frac{29}{56}$

17. For $f(x) = x^2 e^{11x}$, find $f^{(4)}(0)$.

- (A) $\frac{1}{12} \left(\frac{4! 11^2}{2} \right)$
- (B) $2 \left(\frac{4! 11^2}{2} \right)$
- (C) $\frac{1}{2} \left(\frac{4! 11^2}{2} \right)$
- (D) $\frac{4! 11^2}{2}$
- (E) $\frac{2}{11} \left(\frac{4! 11^2}{2} \right)$

18. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 306$ and $x + y = 57$. Find xy .

- (A) 307
- (B) 306
- (C) 304
- (D) 305
- (E) 308

Questions 19–21

19. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 11x$ and has density $\rho(x, y) = 1$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{5}{3}$
- (B) $-\frac{2}{3}$
- (C) $\frac{2}{3}$
- (D) $\frac{1}{3}$
- (E) $\frac{4}{3}$

20. Let $X_1, \dots, X_{27}$ be independent uniform random variables on $[0, 1]$. What is $E[\max_i X_i]$?

- (A) $\frac{27}{56}$
- (B) $\frac{1}{28}$
- (C) $\frac{28}{9}$
- (D) $\frac{27}{28}$
- (E) $\frac{55}{56}$

21. Evaluate $\int_0^1 x^{11}(1-x)^1 dx$.

- (A) $\frac{1}{2} \left(\frac{11! 1!}{13!} \right)$
- (B) $13 \left(\frac{11! 1!}{13!} \right)$
- (C) $\frac{11! 1!}{13!}$
- (D) $\frac{13}{2} \left(\frac{11! 1!}{13!} \right)$
- (E) $12 \left(\frac{11! 1!}{13!} \right)$

Questions 22–24

22. A sequence satisfies $u_0 = 51$ and $u_{k+1} = 1u_k + 6$ for all $k \geq 0$. Find u_3 .

- (A) 69
- (B) 68
- (C) 71
- (D) 70
- (E) 67

23. Let C be the unit circle $x^2 + y^2 = 1$, $z = 2$, counterclockwise viewed from above. Evaluate $\int_C -11y(x^2 + y^2) dx + (11x(x^2 + y^2)) dy$.

- (A) $1 + 22\pi$
- (B) -22π
- (C) 44π
- (D) 22π
- (E) 11π

24. For which real a does $f_m(x) = x^{22m}/m^a$ converge uniformly to zero on $[0, 1]$?

- (A) $a > 0$
- (B) $a = 0$
- (C) All real a
- (D) $a > 1$
- (E) $a < 0$

Questions 25–27

25. If $y' = 11y$, $y(0) = 1$, what is the coefficient of x^2 in the Maclaurin series of y ?

- (A) $-\frac{121}{2}$
- (B) $\frac{121}{2}$
- (C) $\frac{123}{2}$
- (D) $\frac{121}{4}$
- (E) 121

26. A linear map $T : \mathbb{R}^{58} \rightarrow \mathbb{R}^{60}$ has rank 3. What is its nullity?

- (A) 53
- (B) 55
- (C) 57
- (D) 54
- (E) 56

27. Solve $y'' - 24y' + 143y = 0$ with $y(0) = 1$, $y'(0) = 0$. Find $y(1)$.

- (A) $\frac{(13 - 11e^2)e^{11}}{2} + 1$
- (B) $(13 - 11e^2)e^{11}$
- (C) $\frac{(13 - 11e^2)e^{11}}{4}$
- (D) $\frac{(13 - 11e^2)e^{11}}{2}$
- (E) $\frac{(-13 + 11e^2)e^{11}}{2}$

Questions 28–30

28. For which real p does $\sum_{k=2}^{\infty} 1/[k(\log k)^{22p}]$ converge?

- (A) All real p
- (B) $p \geq 1/22$
- (C) $p > 0$
- (D) $p > 1/22$
- (E) $p < 1/22$

29. Evaluate $\int_0^{\log 2} e^x(e^x + 11)^2 dx$.

- (A) $13^2 - 12^2$
- (B) $\frac{13^3}{3}$
- (C) $3 \left(\frac{13^3 - 12^3}{3} \right)$
- (D) $\frac{13^3 - 12^3}{3}$
- (E) $-\frac{13^3 - 12^3}{3}$

30. What is the dimension of the real vector space of 57×57 symmetric matrices?

- (A) 1651
- (B) 1652
- (C) 1653
- (D) 1654
- (E) 1655

Questions 31–33

31. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(11j)!}{(j!)^{11}} x^{2j}$?

- (A) $11^{-\frac{2}{11}}$
- (B) $11^{\frac{11}{2}}$
- (C) $\frac{1}{11}$
- (D) $11^{-\frac{11}{2}}$
- (E) 11^{-11}

32. How many accumulation points in $\mathbb{R}$ does $S = \{1/k : k \geq 1\} \cup \{22 + 1/k : k \geq 1\}$ have?

- (A) 1
- (B) 0
- (C) 2
- (D) 3
- (E) 4

33. Evaluate $\int_C (2x + 11y) dx + (11x + 2y) dy$, where C is the line segment from $(0, 0)$ to $(6, 1)$.

- (A) 206
- (B) -103
- (C) $\frac{103}{2}$
- (D) 103
- (E) 104

Questions 34–36

34. Let $A = \begin{bmatrix} 1 & 51 & 0 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix}$. What is the $(1, 2)$ entry of A^6 ?

- (A) 304
- (B) 305
- (C) 307
- (D) 308
- (E) 306

35. For $f(x, y) = x^2 + 11xy + 2y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) -9
- (B) 7
- (C) $\sqrt{130}$
- (D) 16
- (E) 130

36. How many connected components does $\mathbb{R} \setminus \{1, 2, \dots, 22\}$ have?

- (A) 23
- (B) 37
- (C) 30
- (D) 9
- (E) 16

Questions 37–39

37. For $x > 0$, $xy' = 11y$ and $y(1) = 1$. Find $y(e^2)$.

- (A) e^{22}
- (B) $-e^{22}$
- (C) $2e^{22}$
- (D) $1 + e^{22}$
- (E) $\frac{e^{22}}{2}$

38. An invertible 57×57 matrix A has determinant 3. Find $\det(A^T A)$.

- (A) 7
- (B) 10
- (C) 11
- (D) 9
- (E) 8

39. Evaluate $\iiint_{x^2+y^2+z^2 \leq 11} (x^2 + y^2 + z^2)^1 dV$.

- (A) $\frac{4\pi 11^{\frac{5}{2}}}{5}$
- (B) $\frac{1}{2} \left(\frac{4\pi 11^{\frac{5}{2}}}{5} \right)$
- (C) $\frac{5}{33} \left(\frac{4\pi 11^{\frac{5}{2}}}{5} \right)$
- (D) $5 \left(\frac{4\pi 11^{\frac{5}{2}}}{5} \right)$
- (E) $\frac{1}{3} \left(\frac{4\pi 11^{\frac{5}{2}}}{5} \right)$

Questions 40–42

40. How many zeros, counted with multiplicity, does $z^{17} + 20z + 1$ have in $|z| < 1$?

- (A) 3
- (B) 1
- (C) 2
- (D) 4
- (E) 0

41. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(11+2j)(13+2j)}$.

- (A) $\frac{1}{13}$
- (B) $\frac{1}{2}$
- (C) $\frac{2}{11}$
- (D) $\frac{1}{11}$
- (E) $\frac{1}{22}$

42. On the space P_{58} of real polynomials of degree at most 58, define $T(p) = p'$. What is the rank of T ?

- (A) 60
- (B) 57
- (C) 58
- (D) 59
- (E) 56

Questions 43–45

43. If $y' + 11y = 2e^{-11x}$ and $y(0) = 6$, find $y(1)$.

- (A) $\frac{16}{e^{11}}$
- (B) $\frac{8}{e^{11}}$
- (C) $-\frac{8}{e^{11}}$
- (D) $\frac{4}{e^{11}}$
- (E) $\frac{8}{e^{11}} + 1$

44. A meromorphic function has 19 zeros and 2 poles inside a positively oriented simple closed contour, counted with multiplicity, and no zeros or poles on it. Find $\frac{1}{2\pi i} \int f'(z)/f(z) dz$.

- (A) 12
- (B) 27
- (C) 22
- (D) 17
- (E) 7

45. Evaluate $\int_0^1 \frac{x^1}{1 + 11x^2} dx$.

- (A) $\frac{\log(12)}{2}$
- (B) $\frac{\log(12)}{22}$
- (C) $\log(12)$
- (D) $-\frac{\log(12)}{22}$
- (E) $\frac{\log(12)}{11}$

Questions 46–48

46. Subspaces U, V of $\mathbb{R}^{59}$ have dimensions 2 and 3. What is the largest possible value of $\dim(U \cap V)$?

- (A) 2
- (B) 3
- (C) 4
- (D) 1
- (E) 0

47. The region $0 \leq x \leq 1$, $0 \leq y \leq 11x^2$ is revolved about the x -axis. Find its volume.

- (A) $\frac{121\pi}{5}$
- (B) $1 + \frac{121\pi}{5}$
- (C) $\frac{121\pi}{10}$
- (D) $\frac{242\pi}{5}$
- (E) $-\frac{121\pi}{5}$

48. How many truth assignments to $p, q, r_1, \dots, r_{20}$ make $p \Rightarrow q$ false?

- (A) 2^{19}
- (B) 2^{23}
- (C) 2^{22}
- (D) 2^{21}
- (E) 2^{20}

Questions 49–51

49. Find the directional derivative of $f(x, y, z) = x^2 + 11y^2 + 2z^2$ at $(1, 1, 1)$ in the direction $(1, 6, 0)$.

- (A) $\frac{268\sqrt{37}}{37}$
- (B) $1 + \frac{134\sqrt{37}}{37}$
- (C) $-\frac{134\sqrt{37}}{37}$
- (D) $\frac{67\sqrt{37}}{37}$
- (E) $\frac{134\sqrt{37}}{37}$

50. A permutation consists of disjoint cycles of lengths 3 and 4 and fixes all other points. What is its order?

- (A) 11
- (B) 14
- (C) 12
- (D) 10
- (E) 13

51. Find the coefficient of x^4 in $\cos(11x)/(1 - x^2)$.

- (A) $\frac{14641}{24}$
- (B) $-\frac{13213}{24}$
- (C) $\frac{13189}{24}$
- (D) 13213
- (E) $\frac{13213}{24}$

Questions 52–54

52. What is the coefficient of x^2 in $(1 + 2x)^{22}$?

- (A) 1232
- (B) 1540
- (C) 924
- (D) 616
- (E) 308

53. Find the coefficient of x^2 in $e^{1x} \log(1 + 11x)$.

- (A) $-\frac{77}{2}$
- (B) $-\frac{99}{2}$
- (C) -99
- (D) $-\frac{121}{2}$
- (E) $\frac{99}{2}$

54. In $R = \mathbb{F}_2[x]/(x^{11})$, how many elements are there?

- (A) 2^{10}
- (B) 2^{12}
- (C) 2^{11}
- (D) 2^{13}
- (E) 2^{14}

Questions 55–57

55. Evaluate $\lim_{x \rightarrow 0} \frac{e^{11x} - \sum_{j=0}^1 (11x)^j / j!}{x^2}$.

- (A) $\frac{11^2}{2!}$
- (B) $-\frac{11^2}{2!}$
- (C) $\frac{1}{11} \left(\frac{11^2}{2!} \right)$
- (D) $\frac{1}{2} \left(\frac{11^2}{2!} \right)$
- (E) $2 \left(\frac{11^2}{2!} \right)$

56. How many edges does the complete bipartite graph $K_{22,24}$ have?

- (A) 176
- (B) 528
- (C) 880
- (D) 352
- (E) 704

57. The solution of $y' = 11x^2y^2$ has $y(0) = 1/12$. Find $y(1)$.

- (A) $\frac{3}{50}$
- (B) $\frac{6}{25}$
- (C) $\frac{28}{25}$
- (D) $\frac{3}{25}$
- (E) $-\frac{3}{25}$

Questions 58–60

58. Integers x satisfy $x \equiv 1 \pmod{7}$ and $x \equiv 3 \pmod{9}$. Find the least nonnegative solution.

- (A) 58
- (B) 56
- (C) 55
- (D) 59
- (E) 57

59. Evaluate $\int_0^1 \int_0^x x^{11} y^2 \, dy \, dx$.

- (A) $\frac{1}{42}$
- (B) $\frac{1}{45}$
- (C) $\frac{1}{60}$
- (D) $\frac{1}{36}$
- (E) $\frac{1}{15}$

60. How many functions from a set of 27 elements onto a set of 2 elements exist?

- (A) $2^{27} - 2$
- (B) 2^{26}
- (C) $2^{27} - 1$
- (D) $2^{27} - 3$
- (E) 2^{27}

Questions 61–63

61. Evaluate $\lim_{x \rightarrow 0} (1 + 11x)^{1/(1x)}$.

- (A) 11
- (B) $e^{\frac{1}{11}}$
- (C) e^{12}
- (D) e^{121}
- (E) e^{11}

62. The symmetric group S_{57} acts on $\{1, \dots, 57\}$. What is the index of the stabilizer of 1?

- (A) 57
- (B) 58
- (C) 55
- (D) 59
- (E) 56

63. Evaluate $\int_0^\infty x^2 e^{-11x} dx$.

- (A) $\frac{1}{11} \left(\frac{2!}{11^3} \right)$
- (B) $\frac{1}{2} \left(\frac{2!}{11^3} \right)$
- (C) $11 \left(\frac{2!}{11^3} \right)$
- (D) $\frac{2!}{11^3}$
- (E) $1331 \left(\frac{2!}{11^3} \right)$

Questions 64–66

64. How many nonnegative integer triples (a, b, c) satisfy $a + b + c = 27$?

- (A) 676
- (B) 136
- (C) 541
- (D) 271
- (E) 406

65. For $\mathbf{F} = (11x^2, y^2, z^2)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) 56
- (B) 28
- (C) 14
- (D) 29
- (E) -28

66. In $\mathbb{Z}/1578\mathbb{Z}$, how many residues are idempotent ($x^2 = x$)?

- (A) 10
- (B) 9
- (C) 8
- (D) 6
- (E) 7

Answer Key

Practice Test 55 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	B	23	D	45	B
2	E	24	A	46	A
3	E	25	B	47	A
4	A	26	B	48	E
5	E	27	D	49	E
6	D	28	D	50	C
7	C	29	D	51	E
8	E	30	C	52	C
9	E	31	D	53	B
10	B	32	C	54	C
11	B	33	D	55	A
12	E	34	E	56	B
13	C	35	C	57	D
14	C	36	A	58	E
15	A	37	A	59	B
16	A	38	D	60	A
17	D	39	A	61	E
18	B	40	B	62	A
19	C	41	E	63	D
20	D	42	C	64	E
21	C	43	B	65	B
22	A	44	D	66	C

zarimion.com/gre

100 practice tests, complete answer keys, and future updates.
Please retain the source attribution when sharing this material.

