

GRE Mathematics Practice Test 56

Advanced Challenge

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

The most demanding form in this five-test cycle. Emphasis on connected ideas, non-routine reasoning, and careful conceptual distinctions. This is cycle 11 of 19, position 1 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(12x) - \sum_{j=0}^4 (-1)^j (12x)^{2j+1} / (2j+1)!}{x^{11}}$.
- (A) $\frac{1}{12} \left(\frac{(-1)^5 12^{11}}{11!} \right)$
(B) $-\frac{(-1)^5 12^{11}}{11!}$
(C) $11 \left(\frac{(-1)^5 12^{11}}{11!} \right)$
(D) $12 \left(\frac{(-1)^5 12^{11}}{11!} \right)$
(E) $\frac{(-1)^5 12^{11}}{11!}$
2. Let $u = (11, 6)^T$ and $v = (1, 0)^T$. Find the squared distance from $(11, 0)^T$ to $\text{span}(u)$.
- (A) $\frac{4199}{157}$
(B) $\frac{4356}{157}$
(C) $\frac{8555}{314}$
(D) $\frac{4513}{157}$
(E) $\frac{8869}{314}$
3. For $D = \{(x, y) : x^2 + y^2 \leq 12\}$, evaluate $\iint_D (x^2 + y^2)^5 dA$.
- (A) 995328π
(B) $\frac{2985984\pi}{11}$
(C) 497664π
(D) 41472π
(E) 2985984π

Questions 4–6

4. Independent trials have success probability $1/23$. Let T count trials up to and including the first success. Find $E[T^2]$.

- (A) 1725
- (B) 1380
- (C) 345
- (D) 1035
- (E) 690

5. The solution of $y' = 12x^5y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{2^{\frac{5}{6}}}{2}$
- (B) $\frac{2^{\frac{2}{3}} \cdot 3^{\frac{5}{6}}}{6}$
- (C) $\frac{1}{2}$
- (D) $\sqrt[6]{2}$
- (E) $\frac{2^{\frac{2}{3}}}{2}$

6. An idempotent 58×58 real matrix P has rank 2. What is $\det(I + P)$?

- (A) 4
- (B) 6
- (C) 5
- (D) 3
- (E) 2

Questions 7–9

7. Let C be the unit circle $x^2 + y^2 = 1$, $z = 6$, counterclockwise viewed from above. Evaluate $\int_C -12y(x^2 + y^2)^5 dx + (12x(x^2 + y^2)^5) dy + 2z dz$.

- (A) -24π
- (B) 24π
- (C) $1 + 24\pi$
- (D) 48π
- (E) 12π

8. A uniformly random subset of 23 objects is selected from a collection with 24 red and 25 blue objects. If X is the number of red objects selected, find $\text{Var}(X)$.

- (A) $\frac{7475}{4802}$
- (B) $\frac{14950}{2401}$
- (C) $\frac{2401}{7475}$
- (D) $\frac{2401}{9876}$
- (E) $\frac{2401}{12277}$

9. If $y' = 24xy$ and $y(0) = 1$, find the coefficient of x^{12} in the Maclaurin series of y .

- (A) $\frac{10368}{5}$
- (B) $\frac{20741}{5}$
- (C) $\frac{20736}{5}$
- (D) $\frac{41472}{5}$
- (E) $-\frac{20736}{5}$

Questions 10–12

10. For $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 52 \\ 10 \\ 62 \end{bmatrix}$, let x minimize $\|Ax - b\|^2$. Find the minimum squared residual.

- (A) $\frac{8833}{6}$
- (B) $\frac{4421}{3}$
- (C) $\frac{8839}{6}$
- (D) $\frac{4418}{3}$
- (E) $\frac{4415}{3}$

11. Evaluate $\int_0^1 \int_x^1 12y^6 \, dy \, dx$.

- (A) 2
- (B) $\frac{3}{14}$
- (C) 6
- (D) $\frac{3}{2}$
- (E) $\frac{12}{7}$

12. X, Y are independent Poisson variables with means 23 and 24. Given $X + Y = 25$, find $\text{Var}(X \mid X + Y)$.

- (A) $\frac{13800}{2209}$
- (B) $\frac{27600}{2209}$
- (C) $\frac{18218}{2209}$
- (D) $\frac{6900}{2209}$
- (E) $\frac{16009}{2209}$

Questions 13–15

13. What is the maximum of x^6y^{12} over $x, y \geq 0$ subject to $x + y = 1$?

- (A) $\frac{2}{9}$
- (B) $\left(\frac{6}{18}\right)^6 \left(\frac{12}{18}\right)^{12}$
- (C) $18 \left(\left(\frac{6}{18}\right)^6 \left(\frac{12}{18}\right)^{12} \right)$
- (D) $\frac{1}{2^{18}}$
- (E) $\left(\frac{6}{18}\right)^{12} \left(\frac{12}{18}\right)^6$

14. What is the coefficient of x^8 in $(x + 11)^8(1 - x)^3$?

- (A) -64637
- (B) -64634
- (C) -64635
- (D) -64636
- (E) -64633

15. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 12x$ and has density $\rho(x, y) = x^4y$. Find its centroid coordinate $\bar{x}$.

- (A) $\frac{7}{16}$
- (B) $\frac{15}{8}$
- (C) $\frac{7}{4}$
- (D) $\frac{7}{8}$
- (E) $-\frac{7}{8}$

Questions 16–18

16. X, Y are independent, with variances 23 and 24. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 14
- (B) 7
- (C) 28
- (D) 21
- (E) 35

17. Let $F(x) = \int_0^x (x - t)^5 e^{12t} dt$. Find $F^{(6)}(0)$.

- (A) 0
- (B) 120
- (C) $\frac{1}{120}$
- (D) 720
- (E) 1440

18. A matrix A satisfies $(A - 53I)(A - 55I) = 0$, and its minimal polynomial has degree two. If $A^5 = uA + vI$, find u .

- (A) $5 \cdot 431 \cdot 98711$
- (B) $3 \cdot 431 \cdot 98711$
- (C) $2 \cdot 431 \cdot 98711$
- (D) $431 \cdot 98711$
- (E) 98711

Questions 19–21

19. Find the total outward flux of $\mathbf{F} = (12x^6y, y^6z, xz^6)$ through the boundary of $[0, 1]^3$.

- (A) $\frac{7}{2}$
- (B) 8
- (C) -7
- (D) 7
- (E) 14

20. X has density cx^{23} on $0 < x < 1$ and zero elsewhere. Find $E[X^2]$.

- (A) $\frac{25}{26}$
- (B) $\frac{12}{13}$
- (C) $\frac{4}{13}$
- (D) $\frac{1}{13}$
- (E) $\frac{6}{13}$

21. Evaluate $\lim_{x \rightarrow 0} \frac{e^{12x} - \sum_{j=0}^5 (12x)^j / j!}{x^6}$.

- (A) $720 \left(\frac{12^6}{6!} \right)$
- (B) $6 \left(\frac{12^6}{6!} \right)$
- (C) $\frac{12^6}{6!}$
- (D) $-\frac{12^6}{6!}$
- (E) $\frac{1}{12} \left(\frac{12^6}{6!} \right)$

Questions 22–24

22. Let $V = \begin{bmatrix} 1 & 52 & 2704 \\ 1 & 54 & 2916 \\ 1 & 57 & 3249 \end{bmatrix}$. Find $\det(V^T V)$.

- (A) 902
- (B) 900
- (C) 899
- (D) 901
- (E) 898

23. If $y'' + 36y = 12$ and $y(0) = y'(0) = 0$, find $y(\pi/6)$.

- (A) $\frac{1}{3}$
- (B) $\frac{5}{3}$
- (C) $-\frac{2}{3}$
- (D) $\frac{2}{3}$
- (E) $\frac{4}{3}$

24. Which subset of $\mathbb{R}$ is compact, with 18 fixed as displayed?

- (A) $\mathbb{Q} \cap [0, 18]$
- (B) $18\mathbb{Z}$
- (C) $(0, 18)$
- (D) $\{18/k : k \geq 1\}$
- (E) $\{0\} \cup \{18/k : k \geq 1\}$

Questions 25–27

25. Find the curvature of the helix $r(t) = (12 \cos t, 12 \sin t, 6t)$.

- (A) $\frac{16}{15}$
- (B) $\frac{1}{15}$
- (C) $-\frac{1}{15}$
- (D) $\frac{2}{15}$
- (E) $\frac{1}{30}$

26. Let $u = (1, 2, \dots, 58)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $v^T B^{-1}u$.

- (A) $\frac{3423}{1712}$
- (B) $\frac{1712}{2567}$
- (C) $\frac{1712}{1711}$
- (D) $\frac{1712}{4279}$
- (E) $\frac{855}{1712}$

27. Evaluate $\int_0^{\pi/2} \sin^{13} t \cos^{25} t \, dt$.

- (A) $\frac{6! 12!}{2 \cdot 19!}$
- (B) $\frac{1}{2} \left(\frac{6! 12!}{2 \cdot 19!} \right)$
- (C) $19 \left(\frac{6! 12!}{2 \cdot 19!} \right)$
- (D) $18564 \left(\frac{6! 12!}{2 \cdot 19!} \right)$
- (E) $2 \left(\frac{6! 12!}{2 \cdot 19!} \right)$

Questions 28–30

28. For $a_k = (-1)^k + 18/k$, find $\limsup_{k \rightarrow \infty} a_k - \liminf_{k \rightarrow \infty} a_k$.

- (A) 0
- (B) 2
- (C) 3
- (D) 1
- (E) 4

29. Let $f(x, y) = e^{12x+6y}$. Find $\partial^6 f / \partial x^5 \partial y$ at $(0, 0)$.

- (A) $6 \cdot 12^5$
- (B) $\frac{1}{32} (6 \cdot 12^5)$
- (C) $\frac{1}{6} (6 \cdot 12^5)$
- (D) $2 (6 \cdot 12^5)$
- (E) $\frac{1}{16} (6 \cdot 12^5)$

30. Let $D = \text{diag}(\underbrace{1, \dots, 1}_{10}, \underbrace{2, \dots, 2}_{48})$. Find the dimension of the space of real matrices X with $XD = DX$ and $\text{tr } X = 0$.

- (A) 2403
- (B) 2402
- (C) 2401
- (D) 2404
- (E) 2405

Questions 31–33

31. The local inverse $y(x)$ of $x = 12y + y^3$ satisfies $y(0) = 0$. Find $y^{(7)}(0)$.

- (A) $12 \left(-\frac{60480}{12^{10}} \right)$
- (B) $\frac{1}{5040} \left(-\frac{60480}{12^{10}} \right)$
- (C) $\frac{1}{12} \left(-\frac{60480}{12^{10}} \right)$
- (D) $\frac{60480}{12^{10}}$
- (E) $-\frac{60480}{12^{10}}$

32. For which real a is $f(x) = x^a$ in $L^{18}(0, 1)$?

- (A) $a > -1/18$
- (B) All real a
- (C) $a < 1/18$
- (D) $a \geq -1/18$
- (E) $a > 0$

33. Find the arc length of $r(t) = (12(t - t^3/3), 12t^2)$ for $0 \leq t \leq 1/6$.

- (A) $\frac{109}{108}$
- (B) $-\frac{109}{54}$
- (C) $\frac{163}{54}$
- (D) $\frac{109}{27}$
- (E) $\frac{109}{54}$

Questions 34–36

34. When $x^3 + 1$ is divided by $(x - 53)(x - 54)$, the remainder is $ux + v$. Find u .

- (A) 8585
- (B) 8586
- (C) 8587
- (D) 8588
- (E) 8589

35. Let $\mathbf{F} = (12y^6, z^6, x^6)$. Find $(\nabla \times \mathbf{F})(1, 1, 1) \cdot (1, 1, 2)/\sqrt{6}$.

- (A) $-13\sqrt{6}$
- (B) $-52\sqrt{6}$
- (C) $26\sqrt{6}$
- (D) $1 - 26\sqrt{6}$
- (E) $-26\sqrt{6}$

36. $T : \mathbb{R} \rightarrow \mathbb{R}$ has Lipschitz constant at most $1/18$. If $x_1 = T(x_0)$ and $|x_1 - x_0| \leq 1$, what is the sharp upper bound on $|T^3(x_0) - x_*|$, where x_* is its fixed point?

- (A) $\frac{1}{5508}$
- (B) $\frac{5507}{5508}$
- (C) $\frac{1}{11016}$
- (D) $\frac{5509}{11016}$
- (E) $\frac{1}{16524}$

Questions 37–39

37. If $y' + 12y = 6x^6e^{-12x}$ and $y(0) = 2$, find $y(1)$.

- (A) $\frac{10}{7e^{12}}$
- (B) $-\frac{20}{7e^{12}}$
- (C) $\frac{20}{7e^{12}} + 1$
- (D) $\frac{40}{7e^{12}}$
- (E) $\frac{20}{7e^{12}}$

38. Let $P = \text{diag}(\underbrace{1, \dots, 1}_3, \underbrace{0, \dots, 0}_{56})$. What is the dimension of the space of 59×59 real matrices X satisfying $PX = X$ and $XP = 0$?

- (A) 169
- (B) 166
- (C) 168
- (D) 170
- (E) 167

39. Evaluate $\iiint_B 12z^2(x^2 + y^2 + z^2)^6 dV$, where B is the unit ball.

- (A) $-\frac{16\pi}{17}$
- (B) $1 + \frac{16\pi}{17}$
- (C) $\frac{8\pi}{17}$
- (D) $\frac{32\pi}{17}$
- (E) $\frac{16\pi}{17}$

Questions 40–42

40. Let $\zeta = e^{2\pi i/15}$. Find $\sum_{k=0}^{14} \zeta^k$.

- (A) 4
- (B) 0
- (C) 1
- (D) 3
- (E) 2

41. Let $f(x, y) = x^2 + 24xy + (150)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 24
- (B) 600
- (C) 150
- (D) 12
- (E) 6

42. How many elements z of $\mathbb{F}_{11^{12}}$ satisfy $z^{121} = z$?

- (A) 122
- (B) 120
- (C) 123
- (D) 119
- (E) 121

Questions 43–45

43. Evaluate $\int_C (6x^5 + 12y) dx + (12x + 6y^5) dy$, where C is $r(t) = (t, 2t^2)$, $0 \leq t \leq 1$.

- (A) -89
- (B) 178
- (C) 89
- (D) $\frac{89}{2}$
- (E) 90

44. An analytic function has real part $u(x, y) = x^2 - y^2 + 13x$ and satisfies $f(0) = 0$. Find $\text{Im } f(1 + i)$.

- (A) 10
- (B) 20
- (C) 15
- (D) 5
- (E) 25

45. Evaluate $\sum_{j=0}^{\infty} \frac{1}{(12 + 6j)(18 + 6j)}$.

- (A) $\frac{1}{18}$
- (B) $\frac{1}{2}$
- (C) $\frac{1}{72}$
- (D) $\frac{1}{6}$
- (E) $\frac{1}{12}$

Questions 46–48

46. How many generators does the cyclic group of order 887503681 have?

- (A) $31^5(31 - 1)$
- (B) $31^4(31 - 1)$
- (C) $31^6(31 - 1)$
- (D) $31^8(31 - 1)$
- (E) $31^7(31 - 1)$

47. Evaluate $\iiint_{x^2+y^2+z^2 \leq 12} (x^2 + y^2 + z^2)^5 dV$.

- (A) $\frac{1}{2} \left(\frac{4\pi 12^{\frac{13}{2}}}{13} \right)$
- (B) $\frac{1}{3} \left(\frac{4\pi 12^{\frac{13}{2}}}{13} \right)$
- (C) $\frac{13}{132} \left(\frac{4\pi 12^{\frac{13}{2}}}{13} \right)$
- (D) $\frac{4\pi 12^{\frac{13}{2}}}{13}$
- (E) $13 \left(\frac{4\pi 12^{\frac{13}{2}}}{13} \right)$

48. A tree has 23 vertices of degree 3; every other vertex has degree 1. How many leaves does it have?

- (A) 41
- (B) 33
- (C) 25
- (D) 9
- (E) 17

Questions 49–51

49. For $x > 0$, $x^2y'' + (1 - 18)xy' + 72y = 0$, $y(1) = 0$, $y'(1) = 12$. Find $y(e)$.

- (A) $-e^6 + e^{12}$
- (B) $2(1 - e^6)e^6$
- (C) $\frac{e^{12}}{18}$
- (D) $e^6 + e^{12}$
- (E) $-2(1 - e^6)e^6$

50. Let $R = (\mathbb{Z}/65\mathbb{Z})/([13])$, where $([13])$ is the ideal generated by $[13]$. How many units does R have?

- (A) 13
- (B) 12
- (C) 10
- (D) 11
- (E) 14

51. Evaluate $\int_0^1 x^{11}(\log x)^5 dx$.

- (A) $\frac{10368}{5} \left(\frac{(-1)^5 5!}{12^6} \right)$
- (B) $\frac{(-1)^5 5!}{12^6}$
- (C) $12 \left(\frac{(-1)^5 5!}{12^6} \right)$
- (D) $\frac{1}{6} \left(\frac{(-1)^5 5!}{12^6} \right)$
- (E) $-\frac{(-1)^5 5!}{12^6}$

Questions 52–54

52. A connected planar graph has 26 vertices and 47 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 37
- (B) 9
- (C) 16
- (D) 23
- (E) 30

53. The solution of $y' = 12x^6y^3$ has $y(0) = 1/13$. Find $y(1)$.

- (A) $\frac{\sqrt{8113}}{1159}$
- (B) $\frac{2\sqrt{8113}}{1159}$
- (C) $\frac{\sqrt{8113}}{1159} + 1$
- (D) $\frac{\sqrt{8113}}{2318}$
- (E) $-\frac{\sqrt{8113}}{1159}$

54. What is the largest integer k such that 3^k divides $61!$?

- (A) 30
- (B) 27
- (C) 26
- (D) 28
- (E) 29

Questions 55–57

55. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(12j)!}{(j!)^{12}} x^{6j}$?

- (A) 12^{-12}
- (B) 12^2
- (C) $12(12^{-2})$
- (D) 12^{-2}
- (E) $12^{-\frac{1}{2}}$

56. How many integers in $\{1, \dots, 138\}$ are divisible by 2 or 3?

- (A) 152
- (B) 122
- (C) 62
- (D) 32
- (E) 92

57. Evaluate $\int_0^{\log 2} e^x (e^x + 12)^6 dx$.

- (A) $-\frac{14^7 - 13^7}{7}$
- (B) $14^6 - 13^6$
- (C) $\frac{14^7}{7}$
- (D) $7 \left(\frac{14^7 - 13^7}{7} \right)$
- (E) $\frac{14^7 - 13^7}{7}$

Questions 58–60

58. In the ring $\mathbb{Z}/336\mathbb{Z}$, how many elements satisfy $x^2 = 1$?

- (A) 18
- (B) 15
- (C) 14
- (D) 16
- (E) 17

59. Evaluate $\int_0^1 \frac{x^5}{1 + 12x^6} dx$.

- (A) $\frac{\log(13)}{12}$
- (B) $\frac{\log(13)}{6}$
- (C) $-\frac{\log(13)}{72}$
- (D) $\log(13)$
- (E) $\frac{\log(13)}{72}$

60. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 23 have no adjacent 1s?

- (A) F_{25}
- (B) F_{26}
- (C) F_{23}
- (D) F_{27}
- (E) F_{24}

Questions 61–63

61. The region $0 \leq x \leq 1$, $0 \leq y \leq 12x^6$ is revolved about the line $x = -2$. Find its volume.
- (A) $\frac{69\pi}{7}$
(B) $\frac{69\pi}{14}$
(C) $\frac{138\pi}{7}$
(D) $1 + \frac{69\pi}{7}$
(E) $-\frac{69\pi}{7}$
62. A permutation consists of disjoint cycles of lengths 5 and 7, together with a disjoint transposition when the sum of these two lengths is odd. It fixes all other points. What is its order?
- (A) 33
(B) 37
(C) 35
(D) 34
(E) 36
63. Find the coefficient of x^6 in the Taylor series of $(1 - 12x)^{-2}$ at zero.
- (A) $\frac{1}{7} (7 \cdot 12^6)$
(B) $7 \cdot 12^6$
(C) $\frac{6}{7} (7 \cdot 12^6)$
(D) $2 (7 \cdot 12^6)$
(E) $\frac{1}{12} (7 \cdot 12^6)$

Questions 64–66

64. For Newton iteration on $f(x) = x^2 - 529$ near the root $r = 23$, find $\lim_{k \rightarrow \infty} (x_{k+1} - r)/(x_k - r)^2$ for nonterminating convergent iterates.

- (A) $\frac{1}{138}$
- (B) $\frac{1}{46}$
- (C) $\frac{46}{92}$
- (D) $\frac{1}{92}$
- (E) $\frac{45}{46}$

65. Two solutions of $y'' + 12x^6y' + 6y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $-\frac{1}{e^{\frac{12}{7}}}$
- (B) $e^{-\frac{12}{7}} + 1$
- (C) $\frac{1}{2e^{\frac{12}{7}}}$
- (D) $\frac{2}{e^{\frac{12}{7}}}$
- (E) $e^{-\frac{12}{7}}$

66. In $R = \mathbb{F}_{11}[x]/(x^{12})$, how many elements are units?

- (A) $11^{13} - 11^{12}$
- (B) $11^{14} - 11^{13}$
- (C) $11^{12} - 11^0$
- (D) $11^{12} - 11^{11}$
- (E) $11^{13} - 11^{11}$

Answer Key

Practice Test 56 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	E	23	D	45	C
2	B	24	E	46	A
3	C	25	B	47	D
4	D	26	C	48	C
5	A	27	A	49	E
6	A	28	B	50	B
7	B	29	A	51	B
8	C	30	A	52	D
9	C	31	E	53	A
10	D	32	A	54	D
11	D	33	E	55	D
12	A	34	C	56	E
13	B	35	E	57	E
14	C	36	A	58	D
15	D	37	E	59	E
16	D	38	C	60	A
17	B	39	E	61	A
18	D	40	B	62	C
19	D	41	A	63	B
20	B	42	E	64	B
21	C	43	C	65	E
22	B	44	C	66	D

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