

GRE Mathematics Practice Test 57

Advanced Practice

English edition / 66 multiple-choice questions

66

Questions

170

Minutes

A–E

One correct choice

Instructions

Choose exactly one answer for each question. Do not use a calculator. Begin timing at Question 1. Use the space provided or separate scratch paper. The answer key follows Question 66; no solutions are included.

Angles are measured in radians. Unless specified otherwise, logarithms are natural, $\mathbb{N} = \{1, 2, 3, \dots\}$, and distances in $\mathbb{R}^n$ are Euclidean. I_n denotes the $n \times n$ identity matrix. Graphs are finite and simple unless stated otherwise.

Difficulty target

A demanding practice form with frequent multi-step arguments and a mix of computational and conceptual questions. This is cycle 11 of 19, position 2 of 5; difficulty decreases within each cycle.

Difficulty labels are editorial targets, not ETS-equated ratings. No official scaled-score conversion is provided.

Coverage

Calculus / 33	Differential and integral calculus, sequences and series, several variables, vector calculus, and differential equations.
Algebra / 17	Elementary and linear algebra, groups, rings, fields, and number theory.
Additional / 16	Probability, statistics, analysis, topology, complex variables, discrete mathematics, and numerical methods.

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This series uses independently written question families and parameter variants. Related mathematical structures recur across forms; these are training forms, not 95 psychometrically independent official exams.

Scope reference: ETS, *GRE Mathematics Test Practice Book*, ETS official practice-book PDF. Questions are not taken from that book. This booklet is not published or endorsed by ETS.

Questions 1–3

1. What is the radius of convergence about zero of $\sum_{j=1}^{\infty} \frac{(12j)!}{(j!)^{12}} x^{5j}$?
- (A) $\frac{1}{12}$
(B) $12^{-\frac{12}{5}}$
(C) 12^{-12}
(D) $12^{\frac{12}{5}}$
(E) $12^{-\frac{5}{12}}$
2. Let $V = \begin{bmatrix} 1 & 53 & 2809 \\ 1 & 55 & 3025 \\ 1 & 58 & 3364 \end{bmatrix}$. Find $\det(V^T V)$.
- (A) 900
(B) 899
(C) 898
(D) 902
(E) 901
3. Near $(0, 0)$, $z = z(x, y)$ satisfies $z + 12z^2 = x + 5y$ and $z(0, 0) = 0$. Find $z_{xy}(0, 0)$.
- (A) 120
(B) -5
(C) -60
(D) -120
(E) -24

Questions 4–6

4. Independent trials have success probability $1/24$. Let T count trials up to and including the first success. Find $E[T^2]$.

- (A) 1504
- (B) 752
- (C) 1128
- (D) 1880
- (E) 376

5. Evaluate $\int_0^{\log 2} e^x(e^x + 12)^5 dx$.

- (A) $6 \left(\frac{14^6 - 13^6}{6} \right)$
- (B) $14^5 - 13^5$
- (C) $-\frac{14^6 - 13^6}{6}$
- (D) $\frac{14^6 - 13^6}{6}$
- (E) $\frac{14^6}{6}$

6. Let $u = (1, 2, \dots, 59)^T$, $v = (1, 1, \dots, 1)^T$, and $B = I + uv^T$. Find $v^T B^{-1}u$.

- (A) $\frac{5311}{3542}$
- (B) $\frac{8853}{3542}$
- (C) $\frac{1769}{3542}$
- (D) $\frac{3541}{1771}$
- (E) $\frac{1770}{1771}$

Questions 7–9

7. Evaluate $\lim_{x \rightarrow 0} \frac{(1 + 12x)^{1/(4x)} - e^{12/4}}{x}$.

- (A) e^3
- (B) $-18e^3$
- (C) $18e^3$
- (D) $-\frac{3e^3}{2}$
- (E) $-36e^3$

8. A uniformly random subset of 24 objects is selected from a collection with 25 red and 26 blue objects. If X is the number of red objects selected, find $E[X]$.

- (A) $\frac{100}{17}$
- (B) $\frac{217}{17}$
- (C) $\frac{200}{17}$
- (D) $\frac{400}{17}$
- (E) $\frac{234}{17}$

9. Let $f(x) = \log(1 + 12x)$. Find $f^{(5)}(0)$.

- (A) $(-1)^4 4! 12^5$
- (B) $\frac{1}{12} \left((-1)^4 4! 12^5 \right)$
- (C) $\frac{1}{5} \left((-1)^4 4! 12^5 \right)$
- (D) $-(-1)^4 4! 12^5$
- (E) $5 \left((-1)^4 4! 12^5 \right)$

Questions 10–12

10. Let $D = \text{diag}(\underbrace{1, \dots, 1}_{10}, \underbrace{2, \dots, 2}_{49})$. Find the dimension of the space of real matrices X with $XD = DX$.

- (A) 2500
- (B) 2501
- (C) 2503
- (D) 2502
- (E) 2499

11. For $\mathbf{F} = (12x^5y, y^5z, xz^5)$, find $\nabla \cdot \mathbf{F}$ at $(1, 2, 1)$.

- (A) 205
- (B) $\frac{205}{2}$
- (C) -205
- (D) 410
- (E) 206

12. X, Y are independent Poisson variables with means 24 and 25. Given $X + Y = 26$, find $\text{Var}(X \mid X + Y)$.

- (A) $\frac{18001}{2401}$
- (B) $\frac{31200}{2401}$
- (C) $\frac{20402}{2401}$
- (D) $\frac{15600}{2401}$
- (E) $\frac{7800}{2401}$

Questions 13–15

13. Evaluate $\int_C (5x^4 + 12y) dx + (12x + 5y^4) dy$, where C is $r(t) = (t, 3t^2)$, $0 \leq t \leq 1$.

- (A) 281
- (B) 140
- (C) -280
- (D) 280
- (E) 560

14. When $x^3 + 1$ is divided by $(x - 54)(x - 55)$, the remainder is $ux + v$. Find u .

- (A) 8913
- (B) 8909
- (C) 8912
- (D) 8911
- (E) 8910

15. Two solutions of $y'' + 12x^5y' + 5y = 0$ have Wronskian $W(0) = 1$. Find $W(1)$.

- (A) $-\frac{1}{e^2}$
- (B) $\frac{2}{e^2}$
- (C) e^{-2}
- (D) $\frac{1}{2e^2}$
- (E) $e^{-2} + 1$

Questions 16–18

16. X, Y are independent, with variances 24 and 25. Find $\text{Cov}(X + 2Y, 3X - Y)$.

- (A) 29
- (B) 22
- (C) 36
- (D) 15
- (E) 8

17. Let C be $x^2/25 + y^2 = 1$, oriented counterclockwise. Evaluate $\int_C (-4y^3) dx + (\frac{x^3}{3}) dy$.

- (A) $\frac{185\pi}{2}$
- (B) $1 + \frac{185\pi}{4}$
- (C) $\frac{185\pi}{8}$
- (D) $\frac{185\pi}{4}$
- (E) $-\frac{185\pi}{4}$

18. Let $P = \text{diag}(\underbrace{1, \dots, 1}_4, \underbrace{0, \dots, 0}_{56})$. What is the dimension of the space of 60×60 real matrices X satisfying $PX = X$ and $XP = 0$?

- (A) 224
- (B) 222
- (C) 226
- (D) 225
- (E) 223

Questions 19–21

19. If $y'' + 25y = 12$ and $y(0) = y'(0) = 0$, find $y(\pi/5)$.

- (A) $\frac{24}{25}$
- (B) $-\frac{24}{25}$
- (C) $\frac{12}{25}$
- (D) $\frac{49}{25}$
- (E) $\frac{48}{25}$

20. X has density cx^{24} on $0 < x < 1$ and zero elsewhere. Find $E[X^2]$.

- (A) $\frac{25}{54}$
- (B) $\frac{2}{27}$
- (C) $\frac{26}{27}$
- (D) $\frac{25}{81}$
- (E) $\frac{25}{27}$

21. Let $f(x, y) = x^2 + 24xy + (149)y^2$. What is the product of the eigenvalues of the Hessian of f ?

- (A) 20
- (B) 596
- (C) 149
- (D) 10
- (E) 5

Questions 22–24

22. A 3×3 matrix A has characteristic polynomial $(t - 53)(t + 1)(t - 2)$. What is $\text{tr}(A^3 - 2A + I)$?

- (A) 148778
- (B) 148777
- (C) 148781
- (D) 148780
- (E) 148779

23. Find the curvature of the helix $r(t) = (12 \cos t, 12 \sin t, 5t)$.

- (A) $\frac{6}{169}$
- (B) $\frac{181}{169}$
- (C) $\frac{12}{169}$
- (D) $-\frac{12}{169}$
- (E) $\frac{24}{169}$

24. Which subset of $\mathbb{R}$ is compact, with 19 fixed as displayed?

- (A) $19\mathbb{Z}$
- (B) $\{19/k : k \geq 1\}$
- (C) $\mathbb{Q} \cap [0, 19]$
- (D) $\{0\} \cup \{19/k : k \geq 1\}$
- (E) $(0, 19)$

Questions 25–27

25. Find the upward flux of $\mathbf{F} = (x, y, z)$ through $z = 12x^2 + 5y^2$, $0 \leq x, y \leq 1$.

- (A) $\frac{17}{3}$
- (B) $-\frac{17}{3}$
- (C) $-\frac{34}{3}$
- (D) $-\frac{14}{3}$
- (E) $-\frac{17}{6}$

26. Let N be the 59×59 matrix with ones immediately above the diagonal and zeros elsewhere. Find $\dim(\ker N^{18} / \ker N^9)$.

- (A) 9
- (B) 10
- (C) 11
- (D) 8
- (E) 7

27. Find the total outward flux of $\mathbf{F} = (12x^5y, y^5z, xz^5)$ through the boundary of $[0, 1]^3$.

- (A) $\frac{7}{2}$
- (B) -7
- (C) 14
- (D) 8
- (E) 7

Questions 28–30

28. For $a_k = (-1)^k + 19/k$, find $\limsup_{k \rightarrow \infty} a_k - \liminf_{k \rightarrow \infty} a_k$.

- (A) 2
- (B) 0
- (C) 4
- (D) 3
- (E) 1

29. Find the coefficient of x^5 in the Taylor series of $(1 - 12x)^{-2}$ at zero.

- (A) $\frac{1}{12} (6 \cdot 12^5)$
- (B) $6 \cdot 12^5$
- (C) $\frac{5}{6} (6 \cdot 12^5)$
- (D) $-6 \cdot 12^5$
- (E) $\frac{1}{6} (6 \cdot 12^5)$

30. Let A be the 59×59 matrix whose diagonal entries are 56 and whose off-diagonal entries are 1. What is its largest eigenvalue?

- (A) 113
- (B) 115
- (C) 112
- (D) 116
- (E) 114

Questions 31–33

31. For $x > 0$, $x^2y'' + (1 - 17)xy' + 60y = 0$, $y(1) = 0$, $y'(1) = 12$. Find $y(e)$.

- (A) $\frac{12(1 - e^7)e^5}{7}$
- (B) $-e^5 + e^{12}$
- (C) $e^5 + e^{12}$
- (D) $-\frac{12(1 - e^7)e^5}{7}$
- (E) $\frac{e^{12}}{17}$

32. For which real a is $f(x) = x^a$ in $L^{19}(0, 1)$?

- (A) All real a
- (B) $a \geq -1/19$
- (C) $a < 1/19$
- (D) $a > -1/19$
- (E) $a > 0$

33. Under $x = u + 12v$, $y = u - 5v$, what is the area of the image of $0 \leq u, v \leq 1$?

- (A) 289
- (B) 60
- (C) 7
- (D) 17
- (E) $\frac{1}{17}$

Questions 34–36

34. Positive real numbers x, y satisfy $\log_2 x + \log_2 y = \log_2 424$ and $x + y = 61$. Find $x^2 + y^2$.

- (A) 2872
- (B) 2873
- (C) 2875
- (D) 2871
- (E) 2874

35. The solution of $y' = 12x^4y^2$, $y(0) = 1$, has its first positive singularity at which x ?

- (A) $\frac{\sqrt[5]{648}}{6}$
- (B) $\frac{18^{\frac{2}{5}} \sqrt[5]{5}}{6}$
- (C) $\frac{5}{12}$
- (D) $\frac{\sqrt[5]{12} \cdot 5^{\frac{4}{5}}}{5}$
- (E) $\frac{\sqrt[5]{3240}}{6}$

36. $T : \mathbb{R} \rightarrow \mathbb{R}$ has Lipschitz constant at most $1/19$. If $x_1 = T(x_0)$ and $|x_1 - x_0| \leq 1$, what is the sharp upper bound on $|T^3(x_0) - x_*|$, where x_* is its fixed point?

- (A) $\frac{1}{19494}$
- (B) $\frac{1}{6498}$
- (C) $\frac{1}{12996}$
- (D) $\frac{1}{6499}$
- (E) $\frac{1}{6497}$

Questions 37–39

37. Let C be the unit circle $x^2 + y^2 = 1$, $z = 5$, counterclockwise viewed from above. Evaluate $\int_C -12y(x^2 + y^2)^4 dx + (12x(x^2 + y^2)^4) dy + 3z dz$.

- (A) 48π
- (B) $1 + 24\pi$
- (C) -24π
- (D) 24π
- (E) 12π

38. A sequence satisfies $u_0 = 53$ and $u_{k+1} = 2u_k + 8$ for all $k \geq 0$. Find u_7 .

- (A) 7799
- (B) 7798
- (C) 7800
- (D) 7802
- (E) 7801

39. For $f(x, y) = x^2 + 12xy + 5y^2$, what is its maximum directional derivative at $(1, -1)$?

- (A) 12
- (B) -10
- (C) $2\sqrt{26}$
- (D) 104
- (E) 2

Questions 40–42

40. Let $\zeta = e^{2\pi i/16}$. Find $\sum_{k=0}^{15} \zeta^k$.

- (A) 4
- (B) 1
- (C) 2
- (D) 0
- (E) 3

41. The solution of $y' = 12x^5y^3$ has $y(0) = 1/13$. Find $y(1)$.

- (A) $-\frac{\sqrt{165}}{165}$
- (B) $\frac{\sqrt{165}}{165} + 1$
- (C) $\frac{\sqrt{165}}{165}$
- (D) $\frac{2\sqrt{165}}{165}$
- (E) $\frac{\sqrt{165}}{330}$

42. Integers x satisfy $x \equiv 2 \pmod{5}$ and $x \equiv 7 \pmod{8}$. Find the sum of all such x with $0 \leq x < 80$.

- (A) 54
- (B) 55
- (C) 52
- (D) 56
- (E) 53

Questions 43–45

43. Evaluate $\int_0^1 \int_x^1 12y^5 \, dy \, dx$.

- (A) 2
- (B) $\frac{12}{5}$
- (C) 6
- (D) $\frac{2}{7}$
- (E) $\frac{12}{7}$

44. An analytic function has real part $u(x, y) = x^2 - y^2 + 14x$ and satisfies $f(0) = 0$. Find $\text{Im } f(1 + i)$.

- (A) 16
- (B) 21
- (C) 11
- (D) 26
- (E) 6

45. If $y' = 24xy$ and $y(0) = 1$, find the coefficient of x^{10} in the Maclaurin series of y .

- (A) $\frac{5184}{5}$
- (B) $\frac{20736}{5}$
- (C) $-\frac{10368}{5}$
- (D) $\frac{10373}{5}$
- (E) $\frac{10368}{5}$

Questions 46–48

46. The symmetric group S_{59} acts on the two-element subsets of $\{1, \dots, 59\}$. What is the index of the setwise stabilizer of $\{1, 2\}$?

- (A) 1711
- (B) 1712
- (C) 1709
- (D) 1713
- (E) 1710

47. Find the coefficient of x^4 in $e^{4x} \log(1 + 12x)$.

- (A) -5184
- (B) -79872
- (C) -3328
- (D) -3316
- (E) 3328

48. A tree has 24 vertices of degree 3; every other vertex has degree 1. How many leaves does it have?

- (A) 18
- (B) 42
- (C) 10
- (D) 34
- (E) 26

Questions 49–51

49. For $f(x) = x^5 e^{12x}$, find $f^{(7)}(0)$.

- (A) $\frac{1}{6} \left(\frac{7! 12^2}{2} \right)$
- (B) $\frac{7! 12^2}{2}$
- (C) $\frac{1}{2520} \left(\frac{7! 12^2}{2} \right)$
- (D) $\frac{1}{42} \left(\frac{7! 12^2}{2} \right)$
- (E) $2 \left(\frac{7! 12^2}{2} \right)$

50. In $\mathbb{Z}/1626\mathbb{Z}$, how many residues are not idempotent ($x^2 \neq x$)?

- (A) 1619
- (B) 1616
- (C) 1618
- (D) 1617
- (E) 1620

51. A lamina occupies $0 \leq x \leq 1$, $0 \leq y \leq 12x$ and has density $\rho(x, y) = x^3 y$. Find its centroid coordinate $\bar{x}$.

- (A) $-\frac{6}{7}$
- (B) $\frac{13}{7}$
- (C) $\frac{6}{7}$
- (D) $\frac{3}{7}$
- (E) $\frac{12}{7}$

Questions 52–54

52. A connected planar graph has 27 vertices and 49 edges. How many faces, including the unbounded face, are in a planar embedding?

- (A) 16
- (B) 24
- (C) 8
- (D) 32
- (E) 40

53. Evaluate $\int_0^1 x^{12}(1-x)^4 dx$.

- (A) $476 \left(\frac{12! 4!}{17!} \right)$
- (B) $\frac{12! 4!}{17!}$
- (C) $\frac{1}{5} \left(\frac{12! 4!}{17!} \right)$
- (D) $13 \left(\frac{12! 4!}{17!} \right)$
- (E) $17 \left(\frac{12! 4!}{17!} \right)$

54. In the additive group $\mathbb{Z}/78\mathbb{Z}$, find the index of the subgroup generated by $[3]$.

- (A) 1
- (B) 2
- (C) 3
- (D) 4
- (E) 5

Questions 55–57

55. For the curve $x = t^5$, $y = t^6 + 12t$, find d^2y/dx^2 at $t = 1$.

- (A) $\frac{18}{5}$
- (B) $-\frac{42}{5}$
- (C) $-\frac{42}{25}$
- (D) $-\frac{12}{5}$
- (E) $\frac{54}{25}$

56. How many integers in $\{1, \dots, 144\}$ are divisible by 2 or 3?

- (A) 32
- (B) 128
- (C) 64
- (D) 160
- (E) 96

57. For $x > 0$, $x^2y'' + (-28)xy' + 204y = 0$, $y(1) = 0$, $y'(1) = 1$. Find $y(e)$.

- (A) $-\frac{e^{12}}{5} + 1 + \frac{e^{17}}{5}$
- (B) $-\frac{(1 - e^5)e^{12}}{10}$
- (C) $-\frac{2(1 - e^5)e^{12}}{5}$
- (D) $-\frac{(1 - e^5)e^{12}}{5}$
- (E) $\frac{(1 - e^5)e^{12}}{5}$

Questions 58–60

58. How many invertible 3×3 matrices over $\mathbb{F}_{31}$ are there whose first column is $(1, 0, 0)^T$?

- (A) $\frac{(31^3 - 31^0)(31^3 - 31^1)}{31^3 - 1}$
(B) $\frac{(31^3 - 31^0)(31^3 - 31^0)(31^3 - 31^0)}{31^3 - 1}$
(C) $\frac{(31^3 - 31^1)(31^3 - 31^2)(31^3 - 31^3)}{31^3 - 1}$
(D) $\frac{(31^3 - 31^0)(31^3 - 31^1)(31^3 - 31^2)}{31^3 - 1}$
(E) $\frac{31^3 31^3 31^3}{31^3 - 1}$

59. Find the directional derivative of $f(x, y, z) = x^2y + 5xz^2 + 12y^2z$ at $(1, 1, 1)$ in the direction $(1, 3, 0)$.

- (A) $1 + \frac{41\sqrt{10}}{5}$
(B) $\frac{41\sqrt{10}}{10}$
(C) $\frac{82\sqrt{10}}{5}$
(D) $-\frac{41\sqrt{10}}{5}$
(E) $\frac{41\sqrt{10}}{5}$

60. Let $F_0 = 0$, $F_1 = 1$, and $F_{k+2} = F_{k+1} + F_k$. How many binary strings of length 24 have no adjacent 1s?

- (A) F_{26}
(B) F_{25}
(C) F_{28}
(D) F_{24}
(E) F_{27}

Questions 61–63

61. For $T = \{x, y, z \geq 0 : x + y + z \leq 1\}$, evaluate $\iiint_T x^{12} y^4 dV$.
- (A) $6 \left(\frac{12! 4!}{19!} \right)$
(B) $\frac{12! 4!}{19!}$
(C) $\frac{1}{6} \left(\frac{12! 4!}{19!} \right)$
(D) $\frac{1}{65}$
(E) $19 \left(\frac{12! 4!}{19!} \right)$
62. How many permutations in S_{59} consist of one 5-cycle and 54 fixed points?
- (A) $2^4 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 19 \cdot 29 \cdot 59$
(B) $2^5 \cdot 3 \cdot 7 \cdot 11 \cdot 19 \cdot 29 \cdot 59$
(C) $2^3 \cdot 3 \cdot 7 \cdot 11 \cdot 19 \cdot 29 \cdot 59$
(D) $2^4 \cdot 3^2 \cdot 7 \cdot 11 \cdot 19 \cdot 29 \cdot 59$
(E) $2^4 \cdot 3 \cdot 7 \cdot 11 \cdot 19 \cdot 29 \cdot 59$
63. Evaluate $\iiint_{x^2+y^2+z^2 \leq 12} (x^2 + y^2 + z^2)^4 dV$.
- (A) $11 \left(\frac{4\pi 12^{\frac{11}{2}}}{11} \right)$
(B) $\frac{11}{108} \left(\frac{4\pi 12^{\frac{11}{2}}}{11} \right)$
(C) $\frac{1}{3} \left(\frac{4\pi 12^{\frac{11}{2}}}{11} \right)$
(D) $\frac{1}{2} \left(\frac{4\pi 12^{\frac{11}{2}}}{11} \right)$
(E) $\frac{4\pi 12^{\frac{11}{2}}}{11}$

Questions 64–66

64. For Newton iteration on $f(x) = x^2 - 576$ near the root $r = 24$, find $\lim_{k \rightarrow \infty} (x_{k+1} - r)/(x_k - r)^2$ for nonterminating convergent iterates.

- (A) $\frac{1}{48}$
- (B) $\frac{1}{96}$
- (C) $\frac{47}{48}$
- (D) $\frac{49}{96}$
- (E) $\frac{1}{144}$

65. Find the arc length of $r(t) = (12(t - t^3/3), 12t^2)$ for $0 \leq t \leq 1/5$.

- (A) $\frac{608}{125}$
- (B) $\frac{304}{125}$
- (C) $-\frac{304}{125}$
- (D) $\frac{152}{125}$
- (E) $\frac{429}{125}$

66. How many maximal proper subgroups does the cyclic group of order 684 have?

- (A) 5
- (B) 2
- (C) 3
- (D) 1
- (E) 4

Answer Key

Practice Test 57 / Questions 1–66

Question	Answer	Question	Answer	Question	Answer
1	B	23	C	45	E
2	A	24	D	46	A
3	D	25	B	47	C
4	C	26	A	48	E
5	D	27	E	49	B
6	E	28	A	50	C
7	B	29	B	51	C
8	C	30	E	52	B
9	A	31	D	53	B
10	B	32	D	54	C
11	A	33	D	55	C
12	D	34	B	56	E
13	D	35	E	57	D
14	D	36	B	58	D
15	C	37	D	59	E
16	B	38	C	60	A
17	D	39	C	61	B
18	A	40	D	62	E
19	A	41	C	63	E
20	E	42	A	64	A
21	A	43	E	65	B
22	E	44	A	66	C

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